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Dedicated to

MIKIO SATO

and

LUDWIG D. FADDEEV

Commemorating the Fruitful Exchange and Interaction
Between Their Schools in Kyoto and Leningrad

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Contents

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Preface

The aim of the present volume is to give a survey of the recent development on the interplay between solvable lattice models in statistical mechanics and representation theory of quantum affine algebras. The original papers on this subject were published in the form of a series and the results are all scattered around. We thus felt that a systematic account was necessary, which develops the materials from scratch, focusing attention on the most fundamental case and without assuming prior knowledge about lattice models nor representation theory.

Schematically, the basic problems of integrable models in field theory or statistical mechanics are to diagonalize the given Hamiltonian, and to compute the correlation functions. By correlation functions we mean a system of functions $\langle\phi_\alpha(x)\rangle$, $\langle\phi_\alpha(x)\phi_\beta(y)\rangle$, $\cdots$ obtained as vacuum expectations of the operators in the theory. In the context of lattice statistics they are functions of the lattice sites $x, y, \cdots$; in field theory they are functions of the space-time coordinates or momenta. In principle the totality of the correlation functions has enough information to determine the theory completely.

In a naïve way the Hamiltonian is an infinte dimensional matrix acting on some infinite dimensional space. For instance, in the lattice models the latter is typically given as an infinite tensor product of 'local' spaces, e.g.

$$\cdots \otimes \mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \cdots.$$

Obviously such a Hamiltonian cannot be defined literally because of the difficulty of divergence. In fact, an arbitrary vector in this huge space is not meaningful; what make sense are only those eigenvectors which have finite energy (= finite eigenvalues). They can be thought of as constituting a non-trivial space, which we will refer to as the *space of states*.

At present one knows very few systems whose correlation functions can be described explicitly. The representative examples are 1) the Ising model, and 2) conformal field theory. The Ising model is a two-dimensional lattice model. Its correlations (on the lattice or in the continuum limit) can be characterized by classical non-linear systems such as the Painlevé equations or soliton equations. The conformal field theory deals with critical, or massless, systems in the continuum. Their correlation functions belong to the linear world, giving a good class of generalized hypergeometric functions. The success in the Ising model or conformal field theory is largely related to the fact that their spaces of states have clear mathematical structures: in the Ising model they are the fermion Fock spaces, and in conformal field theory they are the highest weight representations of infinite-dimensional Lie algebras.

Beyond the Ising model, a large class of solvable lattice models have been known; they are built on the solutions of the Yang-Baxter equation. Our main example— the six-vertex model and its spin-chain equivalent, the XXZ model— is one of the most typical models of this sort. However, until recently the space of states and correlation functions have not been understood very well for these more general class of models.

One of the key insights to this problem came from the corner transfer matrix method introduced by Baxter in 1976. The calculation of the one-point functions is reduced to counting the multiplicities of the eigenvalues of the corner transfer matrix. Among others, in the study of the Hard Hexagon model, it led to a remarkable connection with the Rogers-Ramanujan identities. It was then recognized that, in many interesting cases including the Hard-Hexagon model, the spectra of the corner transfer matrices can be described in terms of the characters of affine Lie algebras. Despite the close similarity to certain structure in conformal field theory, this finding has remained a curiosity for some years. Its combinatorial aspect was subsequently clarified by the theory of crystal bases for quantum affine algebras.

Another key emerged through the recent symmetry approach to massive integrable field theories. Bernard and others realized that these theories possess hidden non-Abelian symmetries by the Yangians. It was hoped to exploit these symmetries to understand the integrability in the massive case, following the spirit of conformal field theory. In the latter case a central role was played by the notion of vertex operators and the Knizhnik-Zamolodchikov (KZ) equations for the correlation functions. It was then found that these structures admit a remarkable deformation: by Smirnov, who showed that the form factors he has constructed over the years satisfy the deformed KZ equations; and by Frenkel and Reshetikhin, who studied the vertex operators for quantum affine algebras and derived the q-deformed KZ equations for their matrix elements.

For lattice models the space on which the corner transfer matrix is acting can be viewed as 'half' of the space of states. The appearance of the Lie algebra characters suggests that this half can be identified with a highest weight representation of the quantum affine algebra, which we expect to govern the symmtries of the models. Our first goal in this volume is to explain that it is indeed so. Let $\mathcal{H} = V(\Lambda_0) \oplus V(\Lambda_1)$ be the direct sum of level one integrable representations of the quantum affine algebra $U_q(\widehat{sl}_2)$. Then the space of states for the six-vertex model has the structure $\mathcal{H} \otimes \mathcal{H}^*$, the tensor product being understood in a certain completed sense. Thus we are upgrading the dimension counting by the characters to a structural understanding of the space of states. This picture will lead to the description of the correlation functions and the form factors in terms of the q-deformed vertex operators, and, via bosonization, to the integral formulas for them. This will be our second goal.

Our expositions are organized as follows. In Chapter 0 we shall give a brief account of basic principles in statstical mechanics. We also touch upon the history of solvable models. The first three Chapters are devoted to the standard subjects concerning solvable lattice models in statistical mechanics. Our main examples are the spin 1/2 XXZ chain and the six-vertex model. The setting for these models and their mutual equivalence are explained in Chapter 1 and Chapter 2, respectively. In Chapter 3 we discuss the integrability of the models. The role of the Yang-Baxter equation and the commuting transfer matrices are clarified. The rest of the Chapter is devoted to the introduction of the quantum affine algebra $U_q(\widehat{sl}_2)$, and the representation theoretical interpretation of the Yang-Baxter equation. In Chapter 4 we introduce the main objects, the corner transfer matrices and the vertex operators. By a physical argument we then show how the correlation functions can be written as the trace of products of the vertex operators, and derive difference equations for them. Having these as physical motivations, we restart our mathematical discussions from the next Chapters. Chapter 5 is devoted to the Frenkel-Jing bosonization of the level 1 module of $U_q(\widehat{sl}_2)$. In Chapter 6 we derive the formulas for the vertex operators using bosons. In Chapter 7 we reformulate the physical setting in representation theoretical terms, such as the space of states, vacuum, translation, Hamiltonian and its eigenstates. To derive the formulas for the correlation functions and the form factors we need to calculate the trace of products of vertex operators. This computation is carried out in Chapter 8, and its application is given in Chapter 9. The limit of the XXX model is briefly discussed in Chapter 10. We note that the formulas in Chapters 8–10 are presented here for the first time in such details. The last Chapter 11 is devoted to the discussion of the other types of models, and related works. In the Appendix we collect basic formulas for reader's reference. The bibliography is far from being exhaustive. We have limited the citations to only those which are directly related to the discussions.

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Michio Jimbo and Tetsuji Miwa

Kyoto, Japan
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