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Number 95

Single Orbit Dynamics

Benjamin Weiss

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Preface

These notes represent a mildly expanded version of a series of ten lectures that I gave at a CBMS conference organized by Kamel Haddad at California State University, Bakersfield, CA in June, 1995. Due to external circumstances their publication has been delayed for a few years but I hope that they still give a timely presentation of a novel point of view in dynamical systems. I have not made any effort to update the exposition, but I would like to point out some recent developments that are relevant to the reader who wishes to pursue matters further.

First of all I would like to recommend the new book by Paul Shields, *The ergodic theory of discrete sample paths*, Grad. Studies in Math. v. 13 (AMS), 1996. In this book there is a very careful treatment of some central issues in ergodic theory and information from a point of view that is close to that expounded in these lectures. There is a particularly good treatment there of entropy related matters and of various characterizations of Bernoulli processes.

Following up on an idea proposed by M. Gromov, Elon Lindenstrauss and I have developed a new invariant in topological dynamics which refines the classical notion of topological entropy. This invariant, called the mean topological dimension, vanishes for all systems with finite topological entropy but distinguishes between various systems with infinite topological entropy. There is a single orbit interpretation of this invariant which should shed some new light on spaces of meromorphic functions, solutions of dynamical systems with infinitely many degrees of freedom etc. The basic theory is set out in a joint paper *Mean Topological Dimension*, (to appear in the Israel J. of Math).

The style of these notes is that of a lecture. When proofs are given they are meant to be complete, but not every i is dotted nor every t crossed. Most of the material appears elsewhere and I have given references at the end of each chapter to guide the reader who wants to pursue matters in more detail. Chapter 4 contains

results that haven't appeared before in print. They originate in discussions that I had with Don Ornstein fifteen years ago when we were traveling on a weekly basis from Stanford to MSRI. The main result in chapter 5 is due to Y. Katznelson, I thank him for his permission to include it in these lectures. Hillel Furstenberg's influence on these lectures began with a course that I took with him thirty five years ago at Princeton during which he gave the first exposition of his ideas on disjointness which he then called absolutely independent. It has continued ever since and culminated in a careful reading that he, and Eli Glasner, gave of these notes. Naturally the responsibility for all remaining errors is mine alone.

Finally I would like to express my thanks to Kamel Haddad who organized the wonderful conference that made these notes possible. Last, but not least my thanks to Stanford University and Willene Perez, who typed there the first draft of these notes, and to Shani Ben David, the Hebrew University of Jerusalem, who is responsible for the final version.

Benjamin Weiss, July 1999
Jerusalem (TVBBA), Israel

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Single Orbit Dynamics

Benjamin Weiss

This book presents the expanded notes from ten lectures given by the author at the NSF/CBMS conference held at California State University (Bakersfield). The author describes what he calls *single orbit dynamics*, which is an approach to the analysis of dynamical systems via the study of single orbits, rather than the study of a system as a whole. He presents single orbit interpretations of several areas of topological dynamics and ergodic theory and some new applications of dynamics to graph theory.

In the concluding lectures, single orbit approaches to generalizations of the Shannon-Breiman-McMillan theorem and related problems of compression and universal coding are presented. Complete proofs and illuminating discussions are included and references for further study are given. Some of the material appears here for the first time in print.

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