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Number 109

## Ergodic Theory, Groups, and Geometry

NSF-CBMS Regional Research Conferences in  
the Mathematical Sciences

June 22–26, 1998

University of Minnesota

Robert J. Zimmer

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## Preface

The study of group actions on manifolds is the meeting ground of a variety of mathematical areas. In particular, interesting geometric insights can be obtained by applying measure theoretic techniques. These notes provide an introduction to some of the important methods, major developments, and open problems in the subject. They are slightly expanded from lectures of R. J. Z. at a CBMS Conference at the University of Minnesota, Minneapolis, in June, 1998. The main text presents a perspective on the field as it was at that time, and comments after the notes of each lecture provide suggestions for further reading, including references to recent developments, but the content of these notes is by no means exhaustive.



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