Lectures on Mathematics

Felix Klein

AMS CHELSEA PUBLISHING
American Mathematical Society • Providence, Rhode Island

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To reproduce after a lapse of seventeen years lectures which at the time they were delivered were in such close contact with the most recent work of that day, may well call for a word of justification. Has mathematics not advanced since then, and are the questions here treated of first importance at the present time? I reply by asking: What *is* important in the development of mathematics? Is it solely the attainment of new results of potential value, or must not an essential part of the best scientific efforts of each new age be devoted to possessing itself of the heritage of the age that has just preceded it?

It is a noble example that Professor Klein has set all ages of mathematicians that, beginning his activity at a time when the contributions of the immediate past were so rich and so unrelated, he was able to uncover the essential bonds that connect them and to discern the fields to whose development the new methods were best adapted.

His instinct for that which is vital in mathematics is sure, and the light with which his treatment illumines the problems here considered may well serve as a guide for the youth who is approaching the study of the problems of a later day.

WILLIAM F. OSGOOD.

CAMBRIDGE, December 31, 1910.

PREFACE.

THE Congress of Mathematics held under the auspices of the World's Fair Auxiliary in Chicago, from the 21st to the 26th of August, 1893, was attended by Professor Felix Klein of the University of Göttingen, as one of the commissioners of the German university exhibit at the Columbian Exposition. After the adjournment of the Congress, Professor Klein kindly consented to hold a *colloquium* on mathematics with such members of the Congress as might wish to participate. The Northwestern University at Evanston, Ill., tendered the use of rooms for this purpose and placed a collection of mathematical books from its library at the disposal of the members of the colloquium. The following is a list of the members attending the colloquium: —

W. W. BEMAN, A.M., professor of mathematics, University of Michigan.
E. M. BLAKE, Ph.D., instructor in mathematics, Columbia College.
O. BOLZA, Ph.D., associate professor of mathematics, University of Chicago.
H. T. EDDY, Ph.D., president of the Rose Polytechnic Institute.
A. M. ELY, A.B., professor of mathematics, Vassar College.
F. FRANKLIN, Ph.D., professor of mathematics, Johns Hopkins University.
T. F. HOLGATE, Ph.D., instructor in mathematics, Northwestern University.
L. S. HULBURT, A.M., instructor in mathematics, Johns Hopkins University.
F. H. LOUD, A.B., professor of mathematics and astronomy, Colorado College.
J. MCMAHON, A.M., assistant professor of mathematics, Cornell University.
H. MASCHKE, Ph.D., assistant professor of mathematics, University of Chicago.
E. H. MOORE, Ph.D., professor of mathematics, University of Chicago.

J. E. OLIVER, A.M., professor of mathematics, Cornell University.
A. M. SAWIN, Sc.M., Evanston.
W. E. STORY, Ph.D., professor of mathematics, Clark University.
E. STUDY, Ph.D., professor of mathematics, University of Marburg.
H. TABER, Ph.D., assistant professor of mathematics, Clark University.
H. W. TYLER, Ph.D., professor of mathematics, Massachusetts Institute of Technology.
J. M. VAN VLECK, A.M., LL.D., professor of mathematics and astronomy, Wesleyan University.
E. B. VAN VLECK, Ph.D., instructor in mathematics, University of Wisconsin.
C. A. WALDO, A.M., professor of mathematics, De Pauw University.
H. S. WHITE, Ph.D., associate professor of mathematics, Northwestern University.
M. F. WINSTON, A.B., honorary fellow in mathematics, University of Chicago.
A. ZIWET, assistant professor of mathematics, University of Michigan.

The meetings lasted from August 28th till September 9th; and in the course of these two weeks Professor Klein gave a daily lecture, besides devoting a large portion of his time to personal intercourse and conferences with those attending the meetings. The lectures were delivered freely, in the English language, substantially in the form in which they are here given to the public. The only change made consists in obliterating the conversational form of the frequent questions and discussions by means of which Professor Klein understands so well to enliven his discourse. My notes, after being written out each day, were carefully revised by Professor Klein himself, both in manuscript and in the proofs.

As an appendix it has been thought proper to give a translation of the interesting historical sketch contributed by Professor Klein to the work *Die deutschen Universitäten.* The translation was prepared by Professor H. W. Tyler, of the Massachusetts Institute of Technology.

It is to be hoped that the proceedings of the Chicago Congress of Mathematics, in which Professor Klein took a leading

part, will soon be published in full. The papers presented to this Congress, and the discussions that followed their reading, form an important complement to the Evanston colloquium. Indeed, in reading the lectures here published, it should be kept in mind that they followed immediately upon the adjournment of the Chicago meeting, and were addressed to members of the Congress. This circumstance, in addition to the limited time and the informal character of the colloquium, must account for the incompleteness with which the various subjects are treated.

In concluding, the editor wishes to express his thanks to Professors W. W. Beman and H. S. White for aid in preparing the manuscript and correcting the proofs.

ALEXANDER ZIWET.

ANN ARBOR, MICH., November, 1893.

The new edition is printed from the original plates. The only changes in the text consist in the correction of a few misprints.

A. Z.

ANN ARBOR, January, 1911.

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