

OSCILLATION MATRICES
AND KERNELS AND SMALL VIBRATIONS
OF MECHANICAL SYSTEMS

REVISED EDITION

F. R. GANTMACHER
M. G. KREIN

AMS CHELSEA PUBLISHING
American Mathematical Society • Providence, Rhode Island



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Preface to the Revised English Edition

In preping this publication, the following sources were used:

1. Ф. Р. Гантмахер, М. Г. Крейн, Осцилляционные матрицы ядра и малые колебания механических систем, ГИИТЛ, Москва–Ленинград, 1950;
2. F. R. Gantmakher, M. G. Krein, Oscillation matrices and kernels and small vibrations of mechanical systems, AEC-tr-4481 (physics) Office of technical documentation, Department of Commerce, Washington DC, April 1961;
3. F. R. Gantmacher, M. G. Krein, Oszillationsmatrizen, Oszillationskerne und kleine Schwingungen machanischer Systeme, Akademie-Verlag, Berlin, 1960.

I tried to preserve the terminology and the unique style of the authors (for example, they never use set-theoretic terminology). The only important exception was made for systematic use of the words “eigenvalue” and “eigenvector” in this translation, instead of “characteristic numbers” and “proper vectors” used in the original. We warn the reader that an eigenvalue of a Fredholm integral equation is not the same as an eigenvalue of the corresponding Fredholm operator, but rather its reciprocal. This terminology is traditional and well established in the theory of integral equations (see, for example, [8]) and it is consistent with physical interpretation of these eigenvalues as squares of frequencies.

The modern literature on the subjects originated with this book is enormous, so no attempt was made to compose a complete up-to-date bibliography. Only few modern books and survey papers have been added.

This new English edition was made possible by generous support of Purdue University and Humboldt Foundation. David Drasin helped very much with correction of English, and Betty Gick typed the manuscript in TeX.

Alex Eremenko, Editor of Translation

Foreword to the Second Edition

The present edition of this book differs from the first edition, published in 1941, at the beginning of the Great Patriotic War, under the title “Oscillatory Matrices and Small Vibrations of Mechanical Systems” in the following respects:

Chapter II, devoted to the theory of oscillatory matrices, has been substantially revised. The treatment of the theory is now made more accessible and purposeful. We retain in Chapter II only the material that has direct relation to oscillatory matrices and is used in the application of these matrices to the theory of oscillations of mechanical systems (Chapters III and IV).

In Chapter III, devoted to small oscillations of systems with n degrees of freedom, there has been a substantial revision in the section that explains the mechanical properties that cause the oscillatory nature of the matrix of the influence coefficients of a linear continuum (a string or a rod). In addition, this chapter contains a new section (Sec. 5), in which the properties of Chebyshev systems of functions are explained. These systems of functions are used in Chapters III and IV.

Chapter IV is essentially new. The main results of this chapter were contained as Appendix I in the first edition in outline form. Chapter IV is a natural continuation and generalization of Chapters II and III. It treats problems of vibrations of systems with infinite number of degrees of freedom. Whereas the mathematical basis for Chapter III is the theory of oscillatory matrices, the natural mathematical tool of Chapter IV is the theory of loaded integral equations with symmetric oscillatory kernel. This theory in its complete form is presented in this book for the first time.

In Chapter V various generalizations and supplements to the algebraic investigations of the preceding chapters are gathered. In the first edition, these results were partially covered in Chapter II, and partially in appendices.

Two appendices at the end of this book contain new material which was absent in the old edition.

In Appendix I we give a development of the iteration method of approximate calculation of eigenvalues and eigenvectors for a class of oscillatory matrices.

Appendix II is devoted to the application of continued fractions to the inverse problems of the theory of oscillations – the construction of a mechanical system with finite number of degrees of freedom from its spectral characteristics.

We shall not mention here various less important additions, refinements and corrections in various parts of the book.

All that is required to understand the entire material of the book, with the exception of Chapter IV, is that the reader be familiar with the principles of calculus and the theory of determinants. Chapter IV requires familiarity with the theory of linear integral equations (at least with symmetric kernels).

In Chapter V the authors used many valuable comments from the late doctoral candidate of the Academy of Sciences, U.S.S.R., Vitold Lvovich Shmulyan, who was killed in action in the Great Patriotic War. These remarks concerning the first edition of the book were sent by him from the front in August 1942.

The authors take this opportunity to pay our last homage to the cherished memory of this talented mathematician and patriot.

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