A Course in Ring Theory

Donald S. Passman

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Preface

These are the somewhat expanded notes from a course in ring theory that I have been giving for about ten years. The nature of the course has evolved over time; I am now relatively happy with the choices made.

In this book, we use the underlying theme of projective and injective modules to touch upon various aspects of commutative and noncommutative ring theory. In particular, we highlight and prove a number of rather major results.

In Part I, "Projective Modules," we begin with basic module theory and a brief study of free and projective modules. We then consider Wedderburn rings and, more general, Artinian rings. Next, come hereditary rings and, in particular, Dedekind domains. With this, we are ready for the key concepts of the projective dimension of a module and of the global dimension of a ring. Finally, we introduce the tensor product of modules and we determine all projective modules of local rings.

In Part II, "Polynomial Rings," we study these rings in a mildly noncommutative setting. We start with skew polynomial rings, determine their global dimension and then compute their Grothendieck and projective Grothendieck groups. In particular, we obtain the Hilbert Syzygy Theorem in the commutative case. Next, we offer an affirmative solution to the Serre Conjecture and, in fact, we determine all the projective modules of these polynomial rings. Finally, we use generic flatness to prove the Hilbert Nullstellensatz for almost commutative algebras.

In Part III, "Injective Modules," we start with injective analogs of projective results, but quickly move on to intrinsically injective properties. In particular, we study the maximal ring of quotients and use it to prove the existence of the classical ring of quotients. We then obtain the

Goldie Theorems, study uniform dimension, and characterize the injective modules of Noetherian rings. We close with basic properties of reduced rank and determine when Artinian quotient rings exist.

This book contains numerous exercises for the student and ends with a list of suggested additional reading.

In closing, I would like to express my thanks to a number of people. First, to my friends Larry Levy, Martin Lorenz, Jim Osterburg, and Lance Small for their input and helpful criticism. Second, to Mike Slattery, who attended the first course I gave on this subject and who offered me a copy of his class notes. I suspect he will be rather surprised at the direction in which this course evolved. Third, to Irving Kaplansky, who introduced noncomputational homological algebra. These notes are written in the spirit of his book "Fields and Rings." Finally, my love and appreciation to my family Marj, Barbara, and Jon for their enthusiastic support of this project. I couldn't have done it without them.

Donald S. Passman

Madison, Wisconsin
November, 1990

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Suggested Additional Reading

1. F. Anderson and K. Fuller, "*Rings and Categories of Modules*", Springer-Verlag, Berlin, 1974.
2. M. F. Atiyah and I. G. Macdonald, "*Introduction to Commutative Algebra*", Addison-Wesley, Reading, 1969.
3. K. R. Goodearl and R. B. Warfield, Jr., "*An Introduction to Noncommutative Noetherian Rings*", Cambridge Univ. Press, Cambridge, 1989.
4. I. N. Herstein, "*Noncommutative Rings*", Carus Math. Monograph, AMS, Providence, 1968.
5. N. Jacobson, "*Basic Algebra*", Freeman, San Francisco, 1974 and 1980.
6. I. Kaplansky, "*Commutative Rings*", Univ. of Chicago Press, Chicago, 1974.
7. I. Kaplansky, "*Fields and Rings*", Univ. of Chicago Press, Chicago, 1972.
8. T. Y. Lam, "*Serre's Conjecture*", Springer-Verlag, Berlin, 1978.
9. J. Lambek, "*Lectures on Rings and Modules*", Blaisdell, Waltham, 1966.
10. J. C. McConnell and J. C. Robson, "*Noncommutative Noetherian Rings*", Wiley-Interscience, New York, 1987.
11. D. S. Passman, "*Infinite Crossed Products*", Academic Press, Boston, 1989.
12. J. Rotman, "*An Introduction to Homological Algebra*", Academic Press, New York, 1979.
13. L. Rowen, "*Ring Theory*", Academic Press, Boston, 1988.

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