

# COLLEGE ALGEBRA

HENRY BURCHARD FINE

AMS CHELSEA PUBLISHING  
American Mathematical Society • Providence, Rhode Island



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## PREFACE

In this book I have endeavored to develop the theory of the algebraic processes in as elementary and informal a manner as possible, but connectedly and rigorously, and to present the processes themselves in the form best adapted to the purposes of practical reckoning.

The book is meant to contain everything relating to algebra that a student is likely to need during his school and college course, and the effort has been made to arrange this varied material in an order which will properly exhibit the logical interdependence of its related parts.

It has seemed to me best to divide the book into two parts, a preliminary part devoted to the number system of algebra and a principal part devoted to algebra itself.

I have based my discussion of number on the notion of cardinal number and the notion of order as exhibited in the first instance in the natural scale  $1, 2, 3, \dots$ . There are considerations of a theoretical nature in favor of this procedure into which I need not enter here. But experience has convinced me that from a pedagogical point of view also this method is the best. The meaning of the ordinal definition of an irrational number, for example, can be made clear even to a young student, whereas any other real definition of such a number is too abstract to be always correctly understood by advanced students.

My discussion of number may be thought unnecessarily elaborate. But in dealing with questions of this fundamental character a writer cannot with a good conscience omit points which properly belong to his discussion, or fail to give proofs

of statements which require demonstration. I hope the details of the discussion will interest the more thoughtful class of students; but all that the general student need be asked to learn from it is the ordinal character of the real numbers and of the relations of equality and inequality among them, and that for all numbers, real and complex, the fundamental operations admit of definitions which conform to the commutative, associative, and distributive laws.

In the second or main part of the book I begin by observing that in algebra, where numbers are represented by letters, the laws just mentioned are essentially the definitions of the fundamental operations. These algebraic definitions are stated in detail, and from them the entire theory of the algebraic processes and the practical rules of reckoning are subsequently derived deductively.

I shall not attempt to describe this part of the book minutely. It will be found to differ in essential features from the text-books in general use. I have carefully refrained from departing from accepted methods merely for the sake of novelty. But I have not hesitated to depart from these methods when this seemed to me necessary in order to secure logical consistency, or when I saw an opportunity to simplify a matter of theory or practice. I have given little space to special devices either in the text or in the exercises. On the other hand, I have constantly sought to assist the student to really master the general methods of the science.

Thus, instead of relegating to the latter part of the book the method of undetermined coefficients, the principal method of investigation in analysis, I have introduced it very early and have subsequently employed it wherever this could be done to advantage. This has naturally affected the arrangement of topics. In particular I have considered partial fractions in the chapter on fractions. They belong there logically, and when adequately treated, supply the best practice in elementary reckoning that algebra affords.

Again, I have laid great stress upon the division transformation and its consequences, and in connection with it have introduced the powerful method of synthetic division.

The earlier chapters on equations will be found to contain a pretty full discussion of the reasoning on which the solution of equations depends, a more systematic treatment than is customary of systems of equations which can be solved by aid of the quadratic, and a somewhat elaborate consideration of the graphs of equations of the first and second degrees in two variables.

The binomial theorem for positive integral exponents is treated as a special case of continued multiplication, experience having convinced me that no other method serves so well to convey to the student the meaning of this important theorem. I have introduced practice in the use of the general binomial theorem in the chapter on fractional exponents, but have deferred the proof of the theorem itself, together with all that relates to the subject of infinite series, until near the end of the book.

In the chapters on the theory of equations and determinants there will be found proofs of the fundamental theorems regarding symmetric functions of the roots of an equation and a discussion of the more important properties of resultants. These subjects do not belong in an elementary course in algebra, but the college student who continues his mathematical studies will need them. The like is to be said of the chapters on infinite series and of the chapter on properties of continuous functions with which the book ends.

The ideas which underlie the first part of the book are those of Rowan Hamilton, Grassmann, Helmholtz, Dedekind, and Georg Cantor. But I do not know that any one hitherto has developed the doctrine of ordinal number from just the point of view I have taken, and in the same detail.

In preparing the algebra itself I have profited by suggestions from many books on the subject. I wish in particular to acknowledge my indebtedness to the treatises of Chrystal.

The book has been several years in preparation. Every year since 1898 the publishers have done me the courtesy to issue for the use of the freshmen at Princeton a pamphlet containing what at the time seemed to me the most satisfactory treatment of the more important parts of algebra. With the assistance of my colleagues, Mr. Eisenhart and Mr. Gillespie, I endeavored after each new trial to select what had proved good and to discard what had proved unsatisfactory. As a consequence, much of the book has been rewritten a number of times. No doubt subsequent experience will bring to light many further possibilities of improvement; but I have hopes that as the book stands it will serve to show that algebra is not only more intelligible to the student, but also more interesting and stimulating, when due consideration is given to the reasoning on which its processes depend.

PRINCETON UNIVERSITY  
June, 1905

HENRY P. FINE

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# ANSWERS

## I. Page 89

1. Seven, four, eight; ten; seventeen.
2. Seven.
3.  $n = 7$ ,  $a_0 = 3$ ,  $a_1 = 1$ ,  $a_2 = 0$ ,  $a_3 = -4$ ,  $a_4 = 1$ ,  $a_5 = a_6 = 0$ ,  $a_7 = -12$ .
4.  $f(0) = 3$ ,  $f(-1) = 0$ ,  $f(3) = 48$ ,  $f(8) = 963$ .
5.  $f(0) = 2/5$ ,  $f(-2) = 12$ ,  $f(6) = 20/17$ .
6.  $f(1) = 5$ ,  $f(4) = 9$ ,  $f(5) = 8 + \sqrt{5}$ .
7.  $f(x-2) = 2x-1$ ,  $f(x^2+1) = 2x^2+5$ .
8.  $f(0, 0) = 8$ ,  $f(1, 0) = 10$ ,  $f(0, 1) = 7$ ,  $f(1, 1) = 9$ ,  $f(-2, -3) = 1$

## II. Page 97

1.  $2x^2y(b-a)$ .
2.  $a^2 + a + b^2$ .
3.  $-9$ .
4.  $-4a^3 - 4a^2b - 21ab^2 + 11b^3$ .
5.  $-a + 3b - 7c$ .
6.  $x^3 + 4x^2 + 5x - 2$ .
7.  $b^3 - 5a^2b$ .
8.  $y^3 + 3y - 1$ .
9.  $-10a + 24b$ .
10.  $8x + 11$ .
11.  $-5a + 5b + 9c$ .
12.  $2y - 4x - 2z$ .
13.  $x^3 - x^2 - 8x - 12$ .
14.  $-x^4 + 9x^2 + y^2 + x - 3y - 7$ .

## § 316. Page 105

1.  $a^2 + b^2 + 4c^2 + 9d^2 - 2ab + 4ac - 6ad - 4bc + 6bd - 12cd$ .
2.  $1 + 4x + 10x^2 + 12x^3 + 9x^4$ .
3.  $x^6 - 2x^5y + 3x^4y^2 - 4x^3y^3 + 3x^2y^4 - 2xy^5 + y^6$ .

## III. Page 106

1.  $6x^7 - 13x^6 + 7x^5 + 15x^4 - 34x^3 + 35x^2 - 21x + 5$ .
2.  $15x^5 - 14x^4a - x^3a^2 + 7x^2a^3 - 5xa^4 - 2a^5$ .
3.  $x^6 - y^6$ .
4.  $6x^6 - 4x^5 - 9x^4 + 35x^3 - 10x^2 - 21x + 35$ .
5.  $28x^2 - 43xy + 10y^2$ .
6.  $a^2b + (a^2 - ab + b^2)x - ax^2 - x^3$ .
7.  $x^6 - 2x^5 + 9x^4 - 10x^3 + 17x^2 - 6x$ .
8.  $2x^{2n-2} - 2x^{2n-3} - 3x^{2n-4} + 8x^{2n-5} - 5x^{2n-6}$ .
9.  $a^4 - a^2b^2 + 6ab^3 - 9b^4$ .
10.  $x^2 - 9y^2 - 4z^2 + 12yz$ .
11.  $x^8 - y^8 - 3xy - 1$ .
12.  $a^3 + b^3 - c^3 + 3abc$

13.  $3x^2 - 14xy + 8y^2 + 23x - 32y + 30$ .  
 14.  $2x^2 + 7y^2 + 24z^2 + 15xy - 59yz - 14zx$ .  
 15.  $b^4 - x^4$ .  
 16.  $x^8 + x^4 + 1$ .  
 17.  $2x^2y^2 + 2y^2z^2 + 2z^2x^2 - x^4 - y^4 - z^4$ .  
 18.  $1 + 1 + 1, 1 + 2 + 3 + 2 + 1, 1 + 3 + 6 + 7 + 6 + 3 + 1,$   
 $1 + 4 + 10 + 16 + 19 + 16 + 10 + 4 + 1$ .  
 19.  $1 + 5 + 10 + 10 + 5 + 1$   
 $1 + 6 + 15 + 20 + 15 + 6 + 1$   
 $1 + 7 + 21 + 35 + 35 + 21 + 7 + 1$   
 $1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1$   
 $1 + 9 + 36 + 84 + 126 + 126 + 84 + 36 + 9 + 1$   
 $1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 + 1$ .  
 20.  $16x^2 - 24xy + 9y^2, 64x^3 - 144x^2y + 108xy^2 - 27y^3$ .  
 21.  $x^2 + 4y^2 + 9z^2 + 16u^2 + 4xy + 6xz - 8xu + 12yz - 16yu - 24zu$ .  
 22.  $x^3 + 8y^3 + 27z^3 + 6x^2y + 12xy^2 + 9x^2z + 27xz^2 + 36y^2z + 54yz^2$   
 $+ 36xyz, x^3 + 8y^3 - 27z^3 + 6x^2y + 12xy^2 - 9x^2z + 27xz^2 - 36y^2z$   
 $+ 54yz^2 - 36xyz$ .  
 23.  $a^4 - 8a^2b^2 + 16b^4$ .  
 24.  $a_0b_{17} + a_1b_{16} + \cdots + a_{17}b_0, a_{12}b_{19} + a_{13}b_{18} + \cdots + a_{27}b_4$ .  
 25.  $-4, -14, -55$ .  
 27.  $32a^{10}x^{15}y^{35}, -x^{35}y^{56}z^{63}, a^4nb^{2mn}c^{6n}, a^{mn}b^{n^2}c^{2n^2}$ .  
 28.  $a^{12}b^4c^{18}, -8a^2x^{16}y^{34}$ .

## IV. Page 110

1.  $3a^2/2b$ .  
 2.  $-3y^4z/4ax^5$ .  
 3.  $-5x^m/4y^m$ .  
 4.  $3a^7b^8/c$ .  
 5.  $xy/(x+y)$ .  
 6.  $(x^2 + xy + y^2)(x+y)$ .  
 7.  $(a-b)(b-c)(c-a)$ .  
 8.  $-6abc + 5a^2c^2 - 4a^3b^2c^4$ .  
 9.  $3(x-y)^2 - 2(x-y) + 5$ .  
 10.  $6a^7b^3c$ .  
 11.  $a^3$ .  
 12.  $-2ax^2/y^2$ .

## V. Page 119

1.  $-1/2$ .  
 2.  $-2$ .  
 3.  $-5/2$ .  
 4.  $16$ .  
 5.  $6$ .  
 6.  $1$ .  
 7.  $5$ .  
 8.  $-13/4$ .  
 9.  $-39/5$ .  
 10.  $779/1439$ .  
 11.  $(4a+5b)/3(b+c)$ .  
 12.  $1$ .  
 13.  $a$ .  
 14.  $(a+b)/a$ .  
 15.  $(m-n)/(m+n)$ .  
 16.  $1/2, 1/3, -1/4, -2/5$ .  
 17.  $0, 1, 14$ .  
 18.  $0, 8/3$ .  
 19.  $c/b, 0$ .  
 20.  $1, 1, 2$ .

## VI. Page 124

1. 68.  
 2. 14.  
 3. 26 and 324.  
 4. 84.  
 5. 5.  
 6. 40 and 10; 5 years hence.  
 7. 2 hours and 24 minutes.  
 8. A, 25 days; B,  $16\frac{2}{3}$  days.

9. In the same direction at  $43\frac{7}{11}$  minutes after 8; in opposite directions at  $10\frac{9}{11}$  minutes after 8.
10. In  $5\frac{5}{11}$  minutes. 11. 30 seconds per hour.
12. A, \$540; B, \$360; C, \$240; D, \$160. 13. \$42,000.
14. 576 square feet. 15. 80 feet.
16. 5 dollar pieces, 10 half-dollar pieces, 15 dimes.
17. \$3750 at 6%, \$1250 at 4%. 18. In the ratio 2:3.
19.  $\frac{1}{3}$  of a pound.
20. One gallon the first time, two gallons the second time.  
The original liquid was 60% alcohol.
21. The trains pass at 11:12 $\frac{1}{3}$  o'clock, 35 $\frac{1}{9}$  miles from Jersey City.
22. Their rates are 60 and 36 miles per hour. They pass at 11:15 A.M.
23. 250. 24. 57 of gold, 330 of silver. 25. \$28. 26. 75 square inches.
27. Units digit is  $(9a + b)/18$ , tens digit is  $(9a - b)/18$ .
28.  $(bc - a)/(1 - c)$  years from the present time.

## VII. Page 134

1.  $x = 37, y = 25$ . 2.  $x = 10, y = 7$ . 3.  $x = 3, y = -2$ .
4.  $x = 5, y = -7$ . 5.  $x = 1/3, y = 1/2$ . 6.  $x = 4, y = 6$ .
7.  $x = -3/20, y = 4/5$ . 8.  $x = 5/2, y = 1$ . 9.  $x = -2, y = 1$ .
10.  $x = (a^2 + b^2)/(a + b) + 2, y = (a^2 + b^2)/(a + b)$ .
11.  $x = cq/(aq + bp), y = cp/(aq + bp)$ .
12.  $x = a + b, y = a - b$ . 13.  $x = 10, y = 8$ . 14.  $x = -1, y = 1/2$ .
15.  $x = (bc' + b'c)aa'/(ab' + a'b)cc', y = (ac' - a'c)bb'/(ab' + a'b)cc'$ .
16.  $x = y = ab/(a + b - ab)$ .

## VIII. Page 136

1.  $x = 48, y = -32/7$ . 2.  $x = 1/5, y = 2$ .
3.  $x = 2, y = 3$ . 4.  $x, y = 0, 1/2; 0, 2; 1, 0; -2/3, 0$ .
5.  $x, y = 1, 1; -5/3, 0$ . 6.  $x, y = 0, 5/2; 5/3, 5/3; -5, 5$ .
7.  $x, y = 1, 3; 2, 2$ . 8.  $x, y = 2, 1; -4, 5$ .
9.  $x, y = -1/3, 4/3; -3, 2; -3/5, 8/5; -7/5, 2/5$ .
10.  $x, y = -3, 1; 2, 2; -2, -1; 4, -3$ .

## X. Page 147

1.  $x = 5, y = 6, z = 7$ . 2.  $x = 1, y = -3, z = 3$ .
3.  $x = 5, y = 2, z = 2$ . 4.  $x = -7, y = -1, z = -3$ .
5.  $x = 33/5, y = 22/5, z = 11/10$ . 6.  $x = 1, y = -1, z = 7$ .
7.  $x = 1/8, y = 1/7, z = 1$ . 8.  $x = 2, y = 3, z = 5, u = -4$ .
9.  $x = 0, y = 4, z = -3, u = 0$ .
10.  $x = \frac{l - m + n}{2c}, y = \frac{l + m - n}{2b}, z = \frac{-l + m + n}{2a}$ .

11.  $x = \frac{mnd}{amn + bnl + clm}$ ,  $y = \frac{nld}{amn + bnl + clm}$ ,  $z = \frac{lmd}{amn + bnl + clm}$ .  
 12.  $x = -12$ ,  $y = -8$ ,  $z = -4$ .  
 13.  $(x - y - 3) + (y - z + 5) + (z - x - 2) \equiv 0$ .  
 14.  $2(3x - 8y + 7z - 10) + 5(2x + 5y - 3z - 12) - (16x + 9y - z - 80) \equiv 0$ .

**XI. Page 150**

1. 5, 6, and 9.    2. 33, 14, and 4.    3. 87.    4. A, \$4600; B, \$600.  
 5. A,  $(p - q + r)/2$ ; B,  $(p + q - r)/2$ ; C,  $(-p + q + r)/2$ .  
 6. \$2345,  $4\frac{1}{2}\%$ .    7. \$15,500.    8. 12 square inches.  
 9. A, \$27; B, \$13.    10. A, 12 days; B, 9 days; C, 8 days.  
 11. 18 and 12 feet per second.    12. 20 and 16 miles per hour.  
 13. A's, 15 miles per hour; B's, 10 miles per hour.    14.  $10\frac{1}{4}\frac{2}{3}$  yards.  
 15. A's,  $7\frac{1}{2}\frac{7}{9}$  yards per second; B's, 7 yards per second.    16. 100 pounds.  
 17. A,  $1/3$ ; B,  $11/3$ ; C, 5.    18. In A, 64%; in B, 32%.  
 19. Velocity of sound 1100 feet per second; of bullet,  $1447\frac{7}{9}$  feet per second.  
 20. Capacity of tank, 18,000 gallons. 300 gallons per minute pass through A, 200 through B, 600 through C.

**XII. Page 154**

1.  $3(x - 2)^3 + 17(x - 2)^2 + 34(x - 2) + 19$ .  
 2.  $(2x + 3)^2 - 2(2x + 3) + 4$ .    3.  $f(x) = 43x^2/24 - 8x + 29/24$ .  
 4.  $f(x) = 3x^3 + x + 5$ .    5.  $f(x, y) = 2x - 3y + 4$ .  
 6.  $2x + y - 7 = 0$ .    7. No.    8.  $x + 3y - 11 = 0$ .  
 9.  $c = 3$ .    10.  $2x - 5y + 11 = 0$ , and  $4x - y - 5 = 0$ .  
 11.  $x^3 - 2x^2 - 5x + 6 = 0$ .    12.  $x^2 + xy - x - 4y = 0$ .  
 13.  $3x + 2y - 3 \equiv -(x + y - 1) + (2x - y + 2) + 2(x + 2y - 3)$ .

**§ 401. Page 158**

2.  $Q = x^2 + 3$ ,  $R = -4x + 4$ .

**§ 404. Page 161**

2.  $Q = 3x^3 + 5x^2 - 11x/2 + 9/4$ ,  $R = -9x/4 + 5/2$ .

**§ 405. Page 161**

2.  $l = -11$ ,  $m = -6$ .

**§ 408. Page 164**

2.  $2 - x + 6x^2 - 7x^3 + 13x^4$ .

## § 409. Page 164

2.  $2x + 3y - 1$ .

## XIII. Page 165

1.  $Q = 2x^2 - 3x + 4$ ,  $R = 2x + 4$ .    2.  $Q = 3x^2 - 8x + 5$ ,  $R = 0$ .  
 3.  $Q = 2x^3 - x^2 + 3x - 5$ ,  $R = 20$ .  
 4.  $Q = 4x^4 + x^2 - x + 3$ ,  $R = -2x^2 + 4x - 8$ .  
 5.  $Q = 2x + 3$ ,  $R = 6x - 11$ .  
 6.  $Q = 2x^2 - 3x + 6$ ,  $R = -12x^2 + 24x - 17$ .  
 7.  $3x^3 - 5x^2 - 7x + 12 \equiv (x - 2)(3x^2 + x - 5) + 2$ .  
 8.  $a = -3/2$ ,  $b = -3/2$ .    9.  $a = -2$ ,  $b = 5$ .  
 10.  $Q = x^4 + x^2 + 1$ ,  $R = 0$ .    11.  $Q = 2x - y + 3$ ,  $R = 0$ .  
 12.  $Q = a - b + 2c$ ,  $R = 0$ .    13.  $Q = a + b - c$ ,  $R = 0$ .  
 14.  $Q = x^2 + ax + 2a$ ,  $R = 0$ .    15.  $Q = 4x^2 + 6xy + 9y^2$ ,  $R = 0$ .  
 16.  $Q = x^3 + x^2y + xy^2 - 3y^3$ ,  $R = 0$ .    17.  $Q = 3a^3 + 2a^2b - ab^2 + 4b^3$ ,  $R = 0$ .  
 18.  $Q = x^2 - yx/2 + 3y^2/4$ ,  $R = y^3/4$ ;  $Q' = y^2 - xy + 2x^2$ ,  $R' = -2x^3$ .  
 19.  $Q = 1 - 4x + 6x^2$ ,  $R = 6x^3 - 18x^4$ .  
 20.  $Q = 1 + 4x + 10x^2$ ,  $R = 10x^3 - 50x^4$ .  
 21.  $1 + 2x + 4x^2 + 8x^3$ .    22.  $2 + 5x + 5x^2 - 5x^3$ .

## § 412. Page 169

3.  $Q = 5x^4 + 15x^3 + 44x^2 + 132x + 397$ ,  $R = 1193$ .  
 4.  $Q = x^2 + 3x + 2$ ,  $R = 0$ .    5.  $Q = x^2 + 4$ ,  $R = 0$ .

## § 414. Page 170

2.  $f(2) = 48$ ,  $f(-2) = 96$ ,  $f(4) = 756$ ,  $f(-2/3) = 112/9$ .

## XIV. Page 173

1.  $Q = x^3 + x^2 + 3x + 1$ ,  $R = 0$ .  
 2.  $Q = 5x^4 + 9x^3 + 19x^2 + 64x + 198$ ,  $R = 597$ .  
 3.  $Q = 3x^3 - 6x^2 + 13x - 26$ ,  $R = 51$ .  
 4.  $Q = x^2 + 5x - 6$ ,  $R = 0$ .    5.  $Q = x^2 - 5x/3 - 2/9$ ,  $R = 16/9$ .  
 6.  $Q = x^2 - (b + c)x + bc$ ,  $R = 0$ .    7.  $Q = 2x^3 + 3x^2y - xy^2 + 5y^3$ ,  $R = 0$ .  
 8.  $f(1) = 0$ ,  $f(2) = 9$ ,  $f(5) = 228$ ,  $f(-1) = 6$ ,  $f(-3) = -36$ ,  $f(-6) = -399$ .  
 9.  $m = 3$ .    10.  $l = -22$ ,  $m = -24$ .    13.  $2x^3 - 6x^2 - 12x + 16$ .  
 14. Misprint. For  $x = -1$  read  $x = 1$ .  
 The answer is then  $5x^3 - 24x^2 + 25x + 6$ .

**XV. Page 176**

1.  $(x^2 + 1)^2 + (x - 2)(x^2 + 1) - x$ .
2.  $(2x^2 + 1)^2 + x(2x^2 + 1) + 5$ .
3.  $(2x + 1)(x^3 - x^2 + x + 3)^2 - (2x^2 + 8x)(x^3 - x^2 + x + 3) + (6x^2 - 3)$ .
4.  $(x - 2y)(x^2 + xy + y^2)^2 + (2x + 3y)y^2(x^2 + xy + y^2) - 2xy^4$ .
5.  $2(x - 3)^3 + 10(x - 3)^2 + 7(x - 3) - 9$ .
6.  $(x + 2)^5 - 7(x + 2)^4 + 10(x + 2)^3 + 30(x + 2)^2 - 99(x + 2) + 85$ .
7.  $(x + 3)^3 - 27$ .
8.  $(x + 1)^3 - 2(x + 1)$ .

**XVI. Page 180**

1.  $2x^2y^3(3x^2z^2 - 6yz + 4)$ .
2.  $3n(n - 1)$ .
3.  $(a + 1)(b - 1)$ .
4.  $(m - n)(x - n)$ .
5.  $(3y - 2)(x - 4)$ .
6.  $(2x + y)(5y + 3)$ .
7.  $xy(x - y)(xy + 2)$ .
8.  $x(x + 1)(x^2 + 1)$ .
9.  $(a - b)(c - d)$ .
10.  $a(c + d)(a - b)$ .
11.  $(d + e)(a + b + c)$ .
12.  $(a + d)(a - b + c)$ .

**§ 435. Page 181. Ex. 3**

1.  $(x + 7)^2$ .
2.  $(3 - a)^2$ .
3.  $(3xy + 5)^2$ .
4.  $(x - 2y + 3)^2$ .
5.  $(8a^4 - 3)^2$ .
6.  $(a - b + c)^2$ .

**§ 436. Page 181**

1.  $(x^2 + y^3)(x^2 - y^3)$ .
2.  $6a(a + b)(a - b)$ .
3.  $3ax(2ax + 5y)(2ax - 5y)$ .
4.  $(5x^n + 7x^m)(5x^n - 7x^m)$ .
5.  $(6x^2 + 1)(\sqrt{6}x + 1)(\sqrt{6}x - 1)$ .
6.  $(x^2 + xy - y^2)(x^2 - xy - y^2)$ .

**§ 438. Page 182**

1.  $(4x - 5y)(16x^2 + 20xy + 25y^2)$ .
2.  $(3x + 1)(9x^2 - 3x + 1)$ .
3.  $(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$ .

**§ 439. Page 184**

1.  $(x^2 + \sqrt{2}xy + y^2)(x^2 - \sqrt{2}xy + y^2)$ .
2.  $(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$ .
3.  $(x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$ .

**XVII. Page 184**

1.  $xy(2x - 5y)^2$ .
2.  $7t(2x + 3y)(2x - 3y)$ .
3.  $(x - 2y + 3z)^2$ .
4.  $45(a + b)(a - b)(a^2 + b^2)$ .
5.  $48x(x + 1)^2(x - 1)$ .
6.  $(2 + a + 2b)(2 - a - 2b)$ .
7.  $(x^2 + x + 1)(x^2 - x + 1)$ .
8.  $(a^2 + 2ab - b^2)(a^2 - 2ab - b^2)$ .
9.  $(a^2 + 2a + 4)(a^2 - 2a + 4)$ .
10.  $(3x^2 + 3xy + 4y^2)(3x^2 - 3xy + 4y^2)$ .

- 11.**  $(-a + b + c + d)(a - b + c + d)(a + b - c + d)(a + b + c - d)$ .  
**12.**  $9y^3(2x + y^2)(2x - y^2)(4x^2 + 2xy^2 + y^4)(4x^2 - 2xy + y^4)$ .  
**13.**  $(x - y)(x^2 + xy + y^2)(x^6 + x^3y^3 + y^6)$ .  
**14.**  $(x + y)(x - y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$ .  
**15.**  $(x^2 + y^2)(x^8 - x^6y^2 + x^4y^4 - x^2y^6 + y^8)$ .  
**16.**  $(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$ .  
**17.**  $(x + y^2)(x^6 - x^6y^2 + x^4y^4 - x^3y^6 + x^2y^8 - xy^{10} + y^{12})$ .

**XVIII. Page 185**

- 1.**  $(x + 1)(x - 1)(x^2 - x + 1)$ .      **2.**  $(x + 1)(x - 1)(x - 2)(x^2 + 2x + 4)$ .  
**3.**  $(x + 1)(x - 1)^3$ .      **4.**  $(x + 2)(x - 2)(x - 7)$ .  
**5.**  $(x + y)^2(x - y)^2(x^2 + y^2)$ .      **6.**  $(x + 1)(x^2 + x + 2)$ .  
**7.**  $(x + 1)(x^4 + x^3 + 2x^2 + x + 1)$ .      **8.**  $(x^2 + 2x + 3)^2$ .

**§ 442. Page 186. Ex. 4**

- 1.**  $(x + 1)(x + 2)$ .      **2.**  $(x - 1)(x - 15)$ .      **3.**  $(x + 2)(x - 6)$ .  
**4.**  $(x + 6)(x - 5)$ .      **5.**  $(x + 8)(x + 12)$ .      **6.**  $(x - 5)(x - 16)$ .

**§ 443. Page 187. Ex. 4**

- 1.**  $(3x - 2)(2x - 3)$ .      **2.**  $(x + 3)(5x - 1)$ .      **3.**  $(2x + 1)(7x - 3)$ .  
**4.**  $(3x + 1)(6x + 5)$ .      **5.**  $(7x + 4)(7x + 11)$ .  
**6.** Misprint.  $abx^2 - (ac - b^2)x - bc = (ax + b)(bx - c)$ .

**§ 444. Page 188. Ex. 4**

- 1.**  $(x + 5 + \sqrt{2})(x + 5 - \sqrt{2})$ .      **2.**  $(x - 4)(x - 6)$ .  
**3.**  $(x - 6 + 3i)(x - 6 - 3i)$ .      **4.**  $[x + (1 + \sqrt{3}i)/2][x + (1 - \sqrt{3}i)/2]$ .  
**5.**  $2[x + (3 + \sqrt{7}i)/4][x + (3 - \sqrt{7}i)/4]$ .      **6.**  $(x - 2b)(x - 4a + 2b)$ .

**§ 445. Page 189. Ex. 2**

- 1.**  $(x + y)(x + 4y)$ .  
**2.**  $\left(x - \frac{1 + \sqrt{3}i}{2}y\right)\left(x - \frac{1 - \sqrt{3}i}{2}y\right)$ .

**§ 446. Page 190**

- 2.**  $(2x - y + z)(x - 3y + 2z)$ .



**XIX. Page 190**

1.  $(x - 6)(x - 8)$ .
2.  $[x - (21 + \sqrt{921})/2][x - (21 - \sqrt{921})/2]$ .
3.  $(x - 11)(5x + 2)$ .
4.  $(4x + 7)(4x + 9)$ .
5.  $(6x - 1)(9x - 2)$ .
6.  $4(x + 2y)(3x - y)$ .
7.  $(x + 2)(x + 3)(x - 2)(x - 3)$ .
8.  $xy(x + 3y)(x - 6y)$ .
9.  $[x - (3 + \sqrt{3}i)/2][x - (3 - \sqrt{3}i)/2]$ .
10.  $3[x + (1 + \sqrt{10})/3][x + (1 - \sqrt{10})/3]$ .
11.  $[x - (2 + \sqrt{6})y][x - (2 - \sqrt{6})y]$ .
12.  $(x + 3b)(x - 6a - 3b)$ .
13.  $(ax + a - b)(bx - a - b)$ .
14.  $[(1 + c)x + b][x + d]$ .
15.  $(x - 3y - 1)(x - 5y + 3)$ .
16.  $(x + y + 2z)(x + 2y + z)$ .

**XX. Page 194**

1.  $(x - 1)(x - 2)(x + 3)$ .
2.  $(x + 1)(x + 2)(x + 3)$ .
3.  $(x - 1)(x - 2)(x - 3)(x - 4)$ .
4.  $(x - 1)(x + 2)(x^2 - x + 1)$ .
5.  $(x - 3)(2x + 1)(3x + 1)$ .
6.  $[x - (1 + \sqrt{3})y][x - (1 - \sqrt{3})y](2x - y)$ .
7.  $(x - 1)^3(2x + 5)$ .
8.  $(x + 1)(x - 1)(x + 3)(x - 3)(2x + 1)(2x - 1)$ .
9.  $(x + 2)(2x + 1)(3x + 2)(x^2 + 1)$ .
10.  $(x + 1)(x - 1)(x - 2)(5x - 2)(x^2 + x + 1)$ .
11.  $x = -2, 6$ .
12.  $x = 1/2, 2/3$ .
13.  $x = 7, -2$ .
14.  $x = -3 \pm \sqrt{11}$ .
15.  $x = 2, 3, 4$ .
16.  $x = 1, 1, -1, -3$ .
17.  $x = 1, (-1 \pm \sqrt{3}i)/2$ .
18.  $x = 1, 2/5, -1/2$ .

**XXI. Page 195**

1.  $(3x - 2)(2y + 5)$ .
2.  $(ac - bd)(ab - cd)$ .
3.  $(a - b)^2(a^2 + ab + b^2)$ .
4.  $a(a + 3b)(a - 3b)(a^2 + 9b^2)$ .
5.  $ab(a - b)(a + b)^2$ .
6.  $(3ax + y)(bx - 2y)$ .
7.  $3(x + 2y)(x - 2y)(x^2 + 2xy + 4y^2)(x^2 - 2xy + 4y^2)$ .
8.  $(x - 1)(x + 2)(x^4 + 2x^3 + 3x^2 + 2x + 4)$ .
9.  $y^3(2x + y^2)(2x - y^2)(4x^2 + 2xy^2 + y^4)(4x^2 - 2xy^2 + y^4)$ .
10.  $(x - a)(x + b)$ .
11.  $(x^n + 3)(x^n - 6)$ .
12.  $-(x + 6)(x - 7)$ .
13.  $3(x + 1)(x - 2)(x^2 + 2x + 4)$ .
14.  $(x + 2)(x + 3)(x - 3)(x^2 + 2x + 4)$ .
15.  $(x + a + b + c)(x - a - b + c)$ .
16.  $(x + 2)(x + 4)(x - 2)(x - 4)$ .
17.  $(a - b - 2)(a - b - 3)$ .
18.  $(x + y)(x + 3y)(x - y)(x - 3y)$ .
19.  $(2x + y)(3x - 5y - 2)$ .
20.  $(x + a)(x - a)(x + b)(x - b)$ .
21.  $(-x + y + z + u)(x - y + z + u)(x + y - z + u)(x + y + z - u)$ .
22.  $(2x + 3)(7x - 1)$ .
23.  $(1 + 22y)(1 - 3y)$ .
24.  $xy(y + 4x)(y + 51x)$ .
25.  $(a + 3bc)^2(a - 3bc)^2$ .

- 26.**  $x(x-1)(x-6)(x-7)$ .      **27.**  $(5y-x)(7y^2-10xy+19x^2)$ .  
**28.**  $(x+y)(x-y)^3$ .      **29.**  $(x+a)(x+a-b)$ .  
**30.**  $5xy(x-y)(x^2-xy+y^2)$ .      **31.**  $(x+1)^2(x-1)^3$ .  
**32.**  $(b^2+b+1)(b^2-b+1)$ .      **33.**  $(2x+y-1)(x+3y+5)$ .  
**34.**  $(a^2+2a+2)(a^2-2a+2)$ .      **35.**  $(x+y+3z)(x-2y+z)$ .  
**36.**  $(2a^2+3ab+3b^2)(2a^2-3ab+3b^2)$ .      **37.**  $(x+5b)(x-8a-5b)$ .  
**38.**  $(x^2+x+1)(x^2-x+1)(x^4-x^2+1)$ .      **39.**  $4x^2(2x-1)$ .  
**40.**  $(a+b)(a-b)(x+y)(x-y)$ .      **41.**  $(x-a)(x+b)(x-b)$ .  
**42.**  $(x-a)(x+b)(x^2+ax+a^2)$ .      **43.**  $(a+3b-2c)(a-3b+2c)$ .  
**44.**  $(2a+1)^3$ .      **45.**  $(x^2-x+1)^2$ .  
**46.**  $(a^2+b^2)(x^2+y^2)$ .  
**47.**  $(x+1)(x+3)(x-3)(2x+1)(2x-1)$ .  
**48.**  $(x-1)^2(x^2+2x+3)$ .      **49.**  $(x+3a+b)(x+2a-b)$ .  
**50.**  $(x+2)(3x-2)(5x+3)$ .      **51.**  $(cx+ab)(abx+c)$ .  
**52.**  $(x+a)(x-2a)(2x+a)$ .      **53.**  $(x+1)[(a-b)x+(a+b)]$ .  
**54.**  $(x-y)(x^2+xy+y^2)(x^4+x^3y+x^2y^2+xy^3+y^4)$   
 $(x^8-x^7y+x^6y^2-x^5y^3+x^4y^4+x^3y^5-xy^7+y^8)$ .  
**55.**  $(x+1)(x-1)(x-2)(x-4)$ .      **56.**  $(x-1)(2x+1)^2$ .  
**57.**  $(x-1)(x-4)(3x+2)(x^2+x+1)$ .      **58.**  $(x-2)(x+3)(x+4)(5x-1)$ .  
**59.**  $(ab+c+d)(ac-d)$ .  
**60.**  $(x+y+z)(x-y-z)(-x+y-z)(-x-y+z)$ .

**§ 459. Page 197. Ex. 2**

1.  $x^3y^2$ .      2.  $x+y$ .      3.  $x+2$ .      4.  $(x-2)(x-3)$ .

**§ 460. Page 198. Ex. 3**

1.  $x+3$ .      2.  $(x+2)(x+3)$ .      3.  $(x-1)(x-3)^2$ .

**§ 462. Page 199. Ex. 3**

1.  $x-2$ .      2.  $x+4$ .

**§ 468. Page 203**

2.  $2x^2+x+1$ .

**XXII. Page 204**

1.  $x^3yz^3$ .      2.  $(a+b)(a-o)$ .      3.  $y^2-y+1$ .  
 4.  $a+1$ .      5.  $x-1$ .      6.  $x^2+y^2$ .  
 7.  $x+2$ .      8.  $(x-1)(x-2)$ .      9.  $x^2+x+1$ .

- |                    |                       |                      |
|--------------------|-----------------------|----------------------|
| 10. $(x+1)^3$ .    | 11. $(x-1)(x-2)^2$ .  | 12. $x+2$ .          |
| 13. $x^2+x+1$ .    | 14. $x^2+x-6$ .       | 15. $x^3+x^2-x-1$ .  |
| 16. $3x-7$ .       | 17. $3x+5$ .          | 18. $x(6x^2+7x-3)$ . |
| 19. $2x^2-x+3$ .   | 20. $2x^3-4x^2+x-1$ . | 21. $x^2+x+1$ .      |
| 22. $x+2$ .        | 23. $x^2-3$ .         | 24. $y(x-2y)$ .      |
| 25. $(x-1)(x+3)$ . | 26. $(2x-3)^2(x+1)$ . |                      |

**XXIII. Page 207**

- |   |                               |                        |
|---|-------------------------------|------------------------|
| 1. $(9x^2+1)(9x^2-1)$ .                                 | 2. $(a^5+b^5)(a^5-b^5)$ .     | 3. $a^3(a+1)(a^3-1)$ . |
| 4. $(x-y)^4(x+y)^2(x^2+xy+y^2)(x^2+y^2)$ .              | 5. $(x-1)(x-2)(x-3)$ .        |                        |
| 6. $(x+y+z)(x-y-z)(-x+y-z)(-x-y+z)$ .                   |                               |                        |
| 7. $(2x-3y)(x+3y)(3x-y)$ .                              | 8. $(x+1)(x-1)(x^2+1)$ .      |                        |
| 9. $(a^2+xy)(2x+3y)(2x-3y)$ .                           |                               |                        |
| 10. $x(2x+3y)(2x-3y)(2x-y)(4x+5y)$ .                    | 11. $(x^3+y^3)(x^3-y^3)$ .    |                        |
| 12. $(x^2-1)(x^4+x^2+1)(3x-5)(x^2+1)$ .                 |                               |                        |
| 13. $(2x+3)(4x^2-6x+9)(4x^2+6x+9)(3x-2)$ .              |                               |                        |
| 14. $(x^2+4a^2)(x+2a)(x-2a)$ .                          | 15. $x(x+2)(x+b)(x-b)(x+a)$ . |                        |
| 16. $(x+1)(x+2)(x+3)(x+4)(2x-5)$ .                      |                               |                        |
| 17. $(x-2)(x^2+2x+4)(3x+1)(9x^2-3x+1)(2x+1)(x^2+x+1)$ . |                               |                        |
| 18. $(x-1)(x-2)(x-3)(x+3)(2x-1)$ .                      |                               |                        |
| 19. $(2x^4-x^3+10x^2+4x+5)(x^2+x+1)(x^2+3)$ .           |                               |                        |
| 20. $(2x^4+3x^3-4x^2+13x-6)(x+1)^2$ .                   |                               |                        |

**§ 491. Page 212**Ex.  $m=5$ ,  $n=-14$ .**XXIV. Page 215**

- |                                      |   |                                |
|--------------------------------------|---|--------------------------------|
| 1. $xy(x+2y)$ .                      | 2. $\frac{x^2+xy+y^2}{x^2+y^2}$ .       | 3. $\frac{x+3}{x+9}$ .         |
| 4. $\frac{x-3}{x+2}$ .               | 5. $\frac{3(x-3b)}{2(x+3b)}$ .          | 6. $\frac{5x+a}{5x-3a}$ .      |
| 7. $\frac{(x+5)(x-5)}{(x+3)(x-2)}$ . | 8. $\frac{3x-5}{2x-3}$ .                | 9. $\frac{1}{x^2-y^2}$ .       |
| 10. $\frac{x-y+z}{x+y-z}$ .          | 11. $1-y^2$ .                           | 12. $\frac{m-6n}{3m-n}$ .      |
| 13. $\frac{2x^2+5x-12}{2x^2+x-15}$ . | 14. $\frac{x-1}{2x+1}$ .                | 15. $\frac{x^2+4}{2(x^2+6)}$ . |
| 16. $\frac{x-3}{x^2+2x+4}$ .         | 17. $\frac{x^2-2bx+c^2}{x^2+2bx-c^2}$ . | 18. 3.                         |

**XXV. Page 222**

- |  |   |                                   |
|--|---|-----------------------------------|
| 1. $\frac{2}{2a+3b}$ .                 | 2. $\frac{x^3-x^2+2x-1}{(x^3+1)(x-1)}$ .      | 3. $\frac{3}{(x-1)(x-3)}$ .       |
| 4. $\frac{3x^2-11}{(x-1)(x-2)(x-3)}$ . | 5. $\frac{2x(b^2-c^2)}{(x^2-b^2)(x^2-c^2)}$ . | 6. 0.                             |
| 7. $a$ .                               | 8. $-\frac{8x^2+6x-9}{8x^3-27}$ .             | 9. 4.                             |
| 10. $3a^2+3b^2+3c^2+4ab+4ac+4bc$ .     | 11. $\frac{7}{x^2-x-3}$ .                     |                                   |
| 12. $\frac{2}{x^4-4x^2-x+2}$ .         | 13. $\frac{a^6+a^4+a^2+1}{a^3}$ .             | 14. $a^3+1$ .                     |
| 15. $\frac{x+1}{x-1}$ .                | 16. $\frac{2x-3}{x(x+1)}$ .                   | 17. $\frac{x^3}{x+a}$ .           |
| 18. $x^2-y^2+z^2-2xz$ .                | 19. $\frac{4ab}{(a-b)^2}$ .                   | 20. $\frac{-x+y+z}{x-y+z}$ .      |
| 21. $-\frac{a^2b^2}{(a-b)^2}$ .        | 22. $\frac{x^2-3x+1}{x^2-4x+1}$ .             | 23. $\frac{x^4+3x^2+1}{x^3+2x}$ . |

**XXVI. Page 230**

- |               |               |                          |             |
|---------------|---------------|--------------------------|-------------|
| 1. $1/2$ .    | 2. $-3$ .     | 3. $\infty$ .            | 4. 0.       |
| 5. $-5/4$ .   | 6. $\infty$ . | 7. $3/2, \infty, 0, 2$ . | 8. $-1/9$ . |
| 9. $\infty$ . | 10. 3.        | 11. $2/3$ .              |             |

**XXVII. Page 235**

- |  |                       |                |
|--|-----------------------|----------------|
| 1. $x=1$ .   | 2. $x=15/31$ .        | 3. $x=6$ .     |
| 4. $x=4$ .   | 5. $x=5/6$ .          | 6. $x=-5/3$ .  |
| 7. $x=0$ .   | 8. $x=-(a+b)$ .       | 9. $x=-10$ .   |
| 10. $x=-2$ .   | 11. $x=-2/3$ .        | 12. $x=-7$ .   |
| 13. $x=3/4$ .  | 14. $x=3$ .           | 15. $x=-4$ .   |
| 16. $x = \frac{cq+dp+(a+b)pq}{c+d+ap+bq}$ .  | 17. $x = -71/33$ .    |                |
| 18. $x = 2bc/(b+c)$ .  | 19. $x = (a+b+c)/3$ . | 20. $x = -5$ . |
| 21. No root.   | 22. $x=3, y=-2$ .     | 23. $x=1, y=3$ |
| 24. $x = \frac{2abc}{ab+bc-ca}, y = \frac{2abc}{-ab+bc+ca}, z = \frac{2abc}{ab-bc+ca}$ . |                       |                |
| 25. $x=2, y=-2, z=4$ .   |                       |                |

**§ 534. Page 239**

$$2. -\frac{4}{x} + \frac{4x^2+4x+9}{x^3+x^2+x-1}$$

## XXVIII. Page 244

1.  $\frac{3}{x-2} - \frac{1}{x+3}$ .
2.  $\frac{8}{5(2x+1)} + \frac{3}{5(3x-1)}$ .
3.  $-\frac{2}{x+1} + \frac{8}{x+2} - \frac{6}{x+3}$ .
4.  $-\frac{1}{x-1} + \frac{11}{2(x-2)} - \frac{9}{x-3} + \frac{9}{2(x-4)}$ .
5.  $\frac{1}{x+1} + \frac{1}{x^2-x+1}$ .
6.  $\frac{2}{x} + \frac{3}{1+x} + \frac{5}{1-x}$ .
7.  $x+2 + \frac{1}{x-1} - \frac{2}{x-2}$ .
8.  $\frac{2}{x-2} + \frac{11}{(x-2)^2} + \frac{20}{(x-2)^3} + \frac{13}{(x-2)^4}$ .
9.  $\frac{1}{12x} + \frac{3}{44(x-4)} - \frac{10}{33(2x+3)}$ .
10.  $-\frac{4}{2x^2+1} - \frac{1}{x+1} + \frac{1}{x-1}$ .
11.  $\frac{2}{(x+3)^2} - \frac{21}{(x+3)^3} + \frac{76}{(x+3)^4} - \frac{98}{(x+3)^5}$ .
12.  $\frac{x}{x^2+1} - \frac{x-1}{x^2+2}$ .
13.  $\frac{3}{2(x-1)} - \frac{1}{2(x-3)} + \frac{13}{(x-3)^2}$ .
14.  $\frac{1}{x-2} - \frac{x-1}{x^2+1}$ .
15.  $\frac{2x-4}{x^2+x+1} + \frac{2x+6}{(x^2+x+1)^2} - \frac{3x+1}{(x^2+x+1)^3}$ .
16.  $\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x-1} + \frac{2}{(x-1)^2}$ .
17.  $\frac{1}{x^2+2} + \frac{2}{3(x-2)} - \frac{2}{3(x+1)}$ .
18.  $\frac{a^2+pa+q}{(a-b)(a-c)} \cdot \frac{1}{x-a} + \frac{b^2+pb+q}{(b-a)(b-c)} \cdot \frac{1}{x-b} + \frac{c^2+pc+q}{(c-a)(c-b)} \cdot \frac{1}{x-c}$ .
19.  $-\frac{2}{27x} + \frac{25}{256(x-1)} - \frac{3}{64(x-1)^2} - \frac{163}{6912(x+3)} - \frac{35}{288(x+3)^2} - \frac{25}{48(x+3)^3}$ .
20.  $\frac{4x+3}{2(x^2+x+1)} - \frac{2x-3}{2(x^2-x+1)}$ .

## XXIX. Page 249

1.  $x$  and  $z$ .
2.  $\Sigma a^2b^2 = a^2b^2 + b^2c^2 + c^2a^2$ .  
 $\Sigma a^3b^4 = a^3b^4 + b^3a^4 + b^3c^4 + c^3b^4 + c^3a^4 + a^3c^4$   
 $\Sigma a^2/b = a^2/b + b^2/a + b^2/c + c^2/b + c^2/a + a^2/c$ .  
 $\Sigma a^2b^3c^5 = a^2b^3c^5 + a^2c^3b^5 + b^2c^3a^5 + b^2a^3c^5 + c^2a^3b^5 + c^2b^3a^5$ .  
 $\Sigma a^2b^2c^4 = a^2b^2c^4 + a^2c^2b^4 + b^2c^2a^4$ .  
 $\Sigma (a+b)c = (a+b)c + (b+c)a + (c+a)b$ .  
 $\Sigma (a+b^2)c^3 = (a+b^2)c^3 + (b+a^2)c^3 + (b+c^2)a^3 + (c+b^2)^3$   
 $+ (c+a^2)b^3 + (a+c^2)b^3$ .  
 $\Sigma (a+2b+3c) = (a+2b+3c) + (a+2c+3b) + (b+2c+3a)$   
 $+ (b+2a+3c) + (c+2a+3b) + (c+2b+3a)$ .
4. No.
5.  $y^2 - x^2, x^2 - z^2, z^2 - y^2; a^2bc, b^2cd, c^2da, d^2ab; (a-c)(b-a),$   
 $(b-a)(c-b), (c-b)(a-c)$ .

$$6. \frac{ab^3c^2 + bc^3d^2 + cd^3a^2 + da^3b^2}{(a-b)(a-c)} + \frac{a(b-c) + b(c-d) + c(d-a) + d(a-b)}{(b+2c)(a+d) + (c+2d)(b+a) + (d+2a)(c+b) + (a+2b)(d+c)} + \frac{b^2}{(b-c)(b-d)} + \frac{c^2}{(c-d)(c-a)} + \frac{d^2}{(d-a)(d-b)}.$$

**XXX. Page 251**

1.  $-(x-y)(y-z)(z-x)$ .
2.  $-(x-y)(y-z)(z-x)$ .
3.  $3(x-y)(y-z)(z-x)$ .
4.  $(x-y)(y-z)(z-x)(x+y+z)$ .
5.  $(x-y)(y-z)(z-x)(xy+yz+zx)$ .
6.  $-(x+y)(y+z)(z+x)(x-y)(y-z)(z-x)$ .
7.  $3(x+y)(y+z)(z+x)$ .
8.  $5(x-y)(y-z)(z-x)(x^2+y^2+z^2-xy-yz-zx)$ .
9.  $80xyz(x^2+y^2+z^2)$ .
10.  $-2(x-y)(y-z)(z-x)(x+y+z)$ .
11.  $(x+y)(y+z)(z+x)$ .
12.  $-(x-y)(y-z)(z-x)(x^3+y^3+z^3+x^2y+y^2x+x^2z+z^2x+y^2z+z^2y+xyz)$ .
13.  $a^2+b^2+c^2+ab+bc+ca$ .
14. 0.
15. 0.
16. 1.
17.  $\frac{x^2}{(x-a)(x-b)(x-c)}$ .

**XXXI. Page 259**

1.  $27x^3 + 54x^2y + 36xy^2 + 8y^3$ .
2.  $a^8 - 8a^7b + 28a^6b^2 - 56a^5b^3 + 70a^4b^4 - 56a^3b^5 + 28a^2b^6 - 8ab^7 + b^8$ .
3.  $1 + 14x^2 + 84x^4 + 280x^6 + 560x^8 + 672x^{10} + 448x^{12} + 128x^{14}$ .
4.  $16 + \frac{32}{x} + \frac{24}{x^2} + \frac{8}{x^3} + \frac{1}{x^4}$ .
5.  $x^6 - 18x^4 + 135x^2 - 540 + \frac{1215}{x^2} - \frac{1458}{x^4} + \frac{729}{x^6}$ .
6.  $\frac{x^5}{y^5} - 5\frac{x^3}{y^3} + 10\frac{x}{y} - 10\frac{y}{x} + 5\frac{y^3}{x^3} - \frac{y^5}{x^5}$ .
7.  $1 - 4x + 14x^2 - 28x^3 + 49x^4 - 56x^5 + 56x^6 - 32x^7 + 16x^8$ .
8.  $a^6 + 3a^5x - 5a^3x^3 + 3ax^5 - x^6$ .
9.  $231x^5/16$ .
10.  $-3153199104a^5b^7$ .
11.  $-8064a^{10}b^5c^5$ .
12.  $126x^4, -126x^5$ .
13. 56.
14. 15120.
15. 15.
16.  $-648$ .
17. 924.
18. 2,795,520.
19.  $x^3 - 6x^2y - xy^2 + 30y^3$ .
20.  $x^4 - 4x^3 - 19x^2 + 46x + 120$ .
21. 96.
22. 64, 400.
23. 16, 16, 24.

**§ 568. Page 260**

1.  $\frac{8a^2b^3}{5c^4d^6}$ .
2.  $3xy^2z^3$ .
3.  $\frac{2x^2y^6}{az^5}$ .

**§ 569. Page 262**

3.  $2x^2 - x + 1$ .

## § 570. Page 264

$$5x^2 - 4x + 3.$$

## § 572. Page 264

$$2. 2 - \frac{x}{4} + \frac{15x^2}{64}.$$

## XXXII. Page 269

- |                                      |  |                                       |
|--------------------------------------|--|---------------------------------------|
| 1. $-\frac{3x^2y^5}{5a^3z^4}.$       | 2. $\frac{23a^2b^3}{25cd^4}.$            | 3. $xy(x-y).$                         |
| 4. $x^2 - x + 1.$                    | 5. $x^3 - x + 3.$                        | 6. $2x^3 + 3x^2y - y^3.$              |
| 7. $2x - 5 - 3/x.$                   | 8. $7 - 6x - 5x^2.$                      | 9. $x^4 + x^3 - x^2 + x - 2.$         |
| 10. $x^2 - 2x - 1.$                  | 11. $2x^2 - 3xy + 4.$                    | 12. $\frac{x}{y} + xy + \frac{y}{x}.$ |
| 13. $1 - x - x^2/2 - x^3/2.$         | 14. $2 - x/4 + 47x^2/64 + 47x^3/512.$    |                                       |
| 15. $x^2 + x + 1.$                   | 16. $3x^4 + x^2 - 1.$                    | 17. $2x^2 - 3ax + 3a^2.$              |
| 18. $\frac{x}{y} + 1 + \frac{y}{x}.$ | 19. $1 - \frac{1}{3}x + \frac{2}{9}x^2.$ | 20. $x^2 - x + 1.$                    |
| 21. $x^2 + x + 1.$                   | 22. $a = 6, b = 1.$                      | 23. 167.                              |
24. Misprint. The number should be 2313.61. Its square root is 48.1.  
 25. 24.15.    26. 2037.    27. .0566.    28. 3.004.    29. 1.414.  
 30. 7.449.    31. 15.315.    32. 123.    33. 55.1.    34. 10.12.

## XXXIII. Page 274

- |  |   |  |                              |
|--|---|--|------------------------------|
| 1. $3\sqrt{2}.$                            | 2. $14\sqrt{3}.$                            | 3. $-9.$                                       | 4. $-\sqrt[3]{10}.$          |
| 5. $\sqrt{6}/2.$                           | 6. $\sqrt[3]{12}/2.$                        | 7. $\sqrt[3]{6}/2.$                            | 8. $\sqrt[5]{6}/2.$          |
| 9. $ab^2c^3d^5\sqrt[5]{25d}.$              | 10. $2c\sqrt[5]{2a^2b^4c^2}.$               | 11. $z\sqrt[3]{2x^2y^3z}.$                     | 12. $\sqrt[3]{5ab^2c^3}.$    |
| 13. $\sqrt[3]{ab^2c^3}.$                   | 14. $a^2b^3c^4\sqrt[3]{ab^2}.$              | 15. $x\sqrt{y^2 - z^2}.$                       | 16. $(x+y)\sqrt{(x-y)}.$     |
| 17. $x\sqrt[3]{x^3 - y^3}.$                | 18. $b\sqrt{a(a-b)}.$                       | 19. $\frac{1}{4ab}\sqrt[3]{2a^2b(a^3 + b^3)}.$ |                              |
| 20. $\frac{1}{a-b}\sqrt{a^2 - b^2}.$       | 21. $\frac{1}{3(x+1)}\sqrt[3]{3(x^3 + 1)}.$ | 22. $\sqrt[3]{b^3 - a^3}/b.$                   |                              |
| 23. $\frac{c}{a^n b^n + 1}\sqrt[3]{bc^n}.$ | 24. $\frac{ax - b}{b^2}\sqrt{b}.$           | 25. $\sqrt{27a^3}.$                            | 26. $\sqrt{\frac{a+b}{a-b}}$ |
| 27. $\sqrt[3]{3ax}.$                       | 28. $3\sqrt{2}, 5\sqrt{2}, \sqrt{2}/4.$     |  |                              |
29. Misprint. Replace  $\sqrt[5]{192}$  by  $\sqrt[3]{192}$ . The three radicals then reduce to  $2\sqrt[3]{3}, 4\sqrt[3]{3}, 2\sqrt[3]{3}/3.$   
 30.  $(x-y)\sqrt{x^2 + xy + y^2}, xy\sqrt{x^2 + xy + y^2}.$

## XXXIV. Page 277

1.  $\sqrt[30]{243}$ ,  $\sqrt[30]{27}$ ,  $\sqrt[30]{9}$ .
2.  $\sqrt[12]{a^8}$ ,  $\sqrt[12]{8a^9b^6}$ ,  $\sqrt[12]{49b^{10}}$ .
3.  $3\sqrt{2} = \sqrt[6]{5832}$ ,  $2\sqrt[3]{3} = \sqrt[6]{576}$ .  $\therefore 3\sqrt{2} > 2\sqrt[3]{3}$ .
4.  $\sqrt{3} = \sqrt[12]{729}$ ,  $\sqrt[3]{4} = \sqrt[12]{256}$ ,  $\sqrt[4]{5} = \sqrt[12]{125}$ .  $\therefore \sqrt{3} > \sqrt[3]{4} > \sqrt[4]{5}$ .
5. 5.
6.  $2\sqrt{5}$ .
7.  $2\sqrt[3]{4}$ .
8. 30.
9.  $30\sqrt[3]{3}$ .
10.  $\sqrt{6}/3$ .
11.  $2\sqrt[12]{2}$ .
12.  $\sqrt[12]{17578125/5}$ .
13. 10.
14.  $a^2b^3c^6\sqrt[6]{ab^5c}$ .
15.  $\sqrt[6]{a^5}$ .
16.  $\sqrt[3]{a^2b^2}$ .
17.  $abc^2$ .
18.  $\sqrt[6n]{a^{2n+1}}$ .
19.  $\sqrt[18]{ab^{17}/b}$ .
20.  $\frac{1}{ab}\sqrt[3]{b^2}$ .
21.  $24\sqrt{3}$ .
22.  $a^4$ .
23.  $64xy^3z^4\sqrt{xz}$ .
24.  $\sqrt[6]{a}$ .
25.  $\sqrt{2}$ .
26.  $\sqrt[10]{ab^2c^7}/c$ .
27.  $\sqrt[3]{4}$ .
28.  $\sqrt[4]{8}$ .
29.  $\sqrt[3]{4}$ .
30.  $\sqrt[12]{32}$ .
31.  $\sqrt[2n]{a}$ .
32.  $\sqrt[4]{a^p}$ .
33.  $10\sqrt{3}$ .
34.  $\frac{51}{10}\sqrt{5} + 3\sqrt{7}$ .
35.  $\frac{5}{2}\sqrt[3]{4}$ .
36.  $\frac{a+b+c}{abc}\sqrt{abc}$ .
37.  $\frac{7}{2}\sqrt{2} - \frac{2}{5}\sqrt[3]{3}$ .
38. 0.
39.  $(3+2a)\sqrt{ax}$ .
40.  $\frac{1}{x^2-y^2}\sqrt{x^2-y^2}$ .
41.  $2\sqrt{3} + 3\sqrt{2} + 6$ .
42.  $\sqrt{3} + \sqrt{5} + \sqrt{7}$ .
43.  $7\sqrt{3} + 4\sqrt{10}$ .
44.  $\sqrt{17}$ .
45.  $10 + 6\sqrt{3}$ .
46.  $a + \sqrt{a} + 1$ .

## XXXV. Page 282

1.  $a^{\frac{1}{3}}$ .
2.  $c^{\frac{2}{3}}$ .
3.  $a^{\frac{2}{3}}$ .
4.  $b^{\frac{1}{2}}$ .
5.  $\sqrt[3]{a^2}$ .
6.  $\frac{1}{c^2}\sqrt{c}$ .
7.  $\frac{1}{d^4}$ .
8.  $\sqrt{e}$ .
9.  $\frac{b^3c^2}{a}$ .
10.  $\frac{1}{x^{\frac{1}{2}}y^{\frac{3}{2}}}$ .
11.  $\frac{1}{x^{10}}$ .
12.  $\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ .
13.  $bc - c^2b^{-1}$ .
14.  $125.2^{\frac{1}{2}}/32$ .
15. 27.
16. 9.
17.  $\frac{2^{\frac{1}{2}}}{256}$ .
18.  $a^{\frac{2}{3}}$ .
19. 1.
20.  $(ab)^{\frac{2}{3}}$ .
21.  $a^4b^{-3}$ .
22.  $a^{\frac{1}{3}}$ .
23.  $a^2b^4c^{-6}$ .
24.  $-8a^6$ .
25.  $b^6a^{-4}$ .
26.  $b^{\frac{1}{2}}$ .
27.  $a^{-1}b^{\frac{1}{2}}c^{\frac{3}{2}}$ .
28.  $5a^{11}/4$ .
29.  $a^{\frac{1}{2}}b^{-1}c$ .
30.  $a^{-\frac{1}{2}}$ .
31.  $a^2$ .
32.  $x^{x^3}$ .
33.  $x^{x^2}y^{xy}$ .
34.  $x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{3}{2}}$ .
35.  $x^{\frac{1}{2}} + x^{\frac{1}{2}}y^{\frac{1}{2}} + y^{\frac{1}{2}}$ .
36.  $a^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{1}{2}} + ab + a^{\frac{3}{2}}b^{\frac{3}{2}} + a^{\frac{1}{2}}b^2 + b^{\frac{5}{2}}$ .
37.  $x^2 - 4x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} + 6xy^{\frac{1}{2}}z^{\frac{1}{2}} - 4x^{\frac{1}{2}}y^{\frac{3}{2}}z^{\frac{3}{2}} + yz$ .
38.  $e^x - e^{-x}$ .
39.  $x + 2x^{\frac{1}{2}}y^{\frac{1}{2}} + 3x^{-\frac{1}{2}}y^{\frac{1}{2}}$ .
40.  $x + 1 + x^{-1}$ .



## XXXVI. Page 284

1.  $1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81}$ .
2.  $a^{-\frac{1}{2}} - \frac{a^{-1}x^{-\frac{1}{2}}}{2} + \frac{3a^{-\frac{3}{2}}x^{-\frac{1}{2}}}{8} - \frac{5a^{-\frac{5}{2}}x^{-\frac{1}{2}}}{16}$
3.  $9 - \frac{4x}{3^2} - \frac{4x^2}{3^6} - \frac{32x^3}{3^{11}}$ .
4.  $a + \frac{1}{m}a^{1-m}x + \frac{1-m}{m^2 2!}a^{1-2m}x^2 + \frac{(1-m)(1-2m)}{m^3 3!}a^{1-3m}x^3$ .
5.  $a^4 + 4a^5b^{-\frac{1}{2}} + 10a^6b^{-1} + 20a^7b^{-\frac{3}{2}}$ .
6.  $x^{-3} - 6x^{-\frac{7}{2}}y^{\frac{1}{2}} + 21x^{-4}y^{\frac{3}{2}} - 56x^{-\frac{5}{2}}y$ .
7.  $\frac{1}{2} - \frac{3x}{4} + \frac{9x^2}{8} - \frac{27x^3}{16}$ .
8.  $1 - \frac{2x}{5} + \frac{7x^2}{25} - \frac{28x^3}{125}$ .
9.  $1 - \frac{9x^{\frac{1}{2}}}{2} + \frac{135x}{8} - \frac{945x^{\frac{3}{2}}}{16}$ .
10.  $-55x^9$ .
11.  $-\frac{663}{2^{10}}x^{\frac{1}{2}}y^2$ .
12.  $-\frac{19 \cdot 23 \cdot 31 \cdot 33}{2^{25}}x^{\frac{3}{2}}$ .
13.  $-\frac{5}{2^6}x^{-2}$ .
14. 1. 9.9498. 2. 3.9578. 3. 1.9873.

## XXXVII. Page 287

1.  $a^{\frac{2}{3}}$ .
2.  $a^{\frac{1}{2}}b^{\frac{1}{2}}$ .
3.  $x^{\frac{1}{2}}$ .
4.  $\sqrt{a} - \sqrt{bc}$ .
5.  $(\sqrt{x} + \sqrt{y} - \sqrt{z})(x + y - z - 2\sqrt{xy})$ .
6.  $(\sqrt{xy} + \sqrt{yz} - \sqrt{zx})(xy + yz - zx - 2y\sqrt{xz})$ .
7.  $[\sqrt{x} + \sqrt{y} + \sqrt{z} + \sqrt{u}][x + y - u - z - 2(\sqrt{xy} - \sqrt{uz})]$   
 $[x^2 + y^2 + z^2 + u^2 - 2xy - 2xz - 2xu - 2yz - 2yu - 2zu - 8\sqrt{xyzu}]$ .
8.  $(\sqrt{x} - \sqrt[4]{x} + 1)(x - \sqrt{x} + 1)$ .
9.  $x^{\frac{3}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}} + y^{\frac{3}{2}}$ .
10.  $a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{2}{3}}$ .
11.  $(x^{\frac{1}{2}} + y^{\frac{1}{2}})(x^{\frac{1}{2}} + y^{\frac{1}{2}})$ .
12.  $(x^9)^{\frac{1}{2}} - (x^9)^{\frac{1}{2}}(y^8)^{\frac{1}{2}} + (x^9)^{\frac{1}{2}}(y^8)^{\frac{1}{2}} - \dots + (y^8)^{\frac{1}{2}}$ .
13.  $(1 - x^{\frac{1}{2}}y^{\frac{1}{2}})(1 + xy^{\frac{3}{2}} + x^2y^{\frac{3}{2}})$ .
14.  $(x^{\frac{1}{2}} - 1)$ .
15.  $3 + \sqrt{5}$ .
16.  $(1 + \sqrt{2} - \sqrt{3})\sqrt{2}$ .
17.  $(1 - \sqrt[3]{2} + \sqrt[3]{4})$ .
18.  $(1 - \sqrt[3]{3})$ .
19.  $\sqrt[3]{3^2}(1 - \sqrt[3]{2})$ .
20.  $\frac{\sqrt{a} \cdot \sqrt[5]{b^3}}{ab}$ .
21.  $\frac{a^2 + 2a\sqrt{b} + b}{a^2 - b}$ .
22.  $\frac{3 - \sqrt{6}}{15}$ .
23.  $\frac{b - \sqrt{b^2 - a^2}}{a^2}$ .
24.  $\frac{x + \sqrt{x^2 - y^2}}{y}$ .
25.  $\sqrt{2} + \sqrt{3}$ .
26.  $\frac{1 - \sqrt{2} - \sqrt{3} + \sqrt{6}}{2}$ .
27.  $\frac{(\sqrt{x} + \sqrt{y})(\sqrt{x} + \sqrt{y} - \sqrt{x+y})}{2}$ .
28.  $\frac{\sqrt[3]{3}(\sqrt[3]{3^4} + \sqrt[3]{3^2} + 1)}{4}$ .
29. .447.
30. 2.756
31. 1.732.

**XXXVIII. Page 290**

1. 256.      2.  $1/9$ .      3.  $\pm 16\sqrt{2}$ .      4. 14.      5. 1.  
 6.  $\left(\frac{d}{\sqrt{a} + \sqrt{b} + \sqrt{c}}\right)^2$ .      7. 3.      8. 5.      9. -1.  
 10.  $34/15$ .      11. 2.      12. 2.      13. 6.      14.  $5/4$ .  
 15.  $\begin{cases} x = -1, y = 3, \\ x = 3, y = (73 - 10\sqrt{10})/9. \end{cases}$       16.  $x = 10, y = 3$ .

**XXXIX. Page 293**

1.  $\sqrt{7} + \sqrt{2}$ .      2.  $2(\sqrt{2} + \sqrt{3})$ .      3.  $5 - \sqrt{7}$ .      4.  $(\sqrt{10} + \sqrt{15})/5$ .  
 5.  $(3\sqrt{2} - \sqrt{10})/2$ .      6.  $\sqrt[4]{2}(\sqrt{5} + \sqrt{3})$ .      7.  $\sqrt{a+b} + \sqrt{a-b}$ .  
 8.  $\sqrt{a} - \sqrt{b-a}$ .      9.  $1 + \sqrt{2}$ .      10.  $1 + 2\sqrt{3}$ .

**XL. Page 297**

1.  $7i$ .      2.  $3\sqrt{2}i$ .      3.  $-4\sqrt{6}$ .      4.  $2i$ .      5.  $-2$ .  
 6. 1.      7.  $i$ .      8.  $-i$ .      9.  $(x-y)i$ .  
 10.  $2 - \sqrt{6} + (2\sqrt{2} + \sqrt{3})i$ .      11.  $648\sqrt{6}$ .      12.  $-22$ .      13. 0.  
 14. 10.      15. 16.      16.  $-i$ .      17.  $\frac{a^2 - b^2}{a^2 + b^2} + \frac{2ab}{a^2 + b^2}i$ .  
 18.  $1 - \sqrt{2}i$ .      19.  $1 - \sqrt{3}i$ .      20.  $\sqrt{2}(1+i)$ .      23.  $x = 5/3, y = -4/3$ .  
 24.  $3 + 2i$ .      25.  $1 + i$ .      26.  $(a+b) + (a-b)i$ .

**§ 630. Page 299. Exs. 1, 2**

1. 2, -4.      2. 3,  $1/2$ .      3. 0, 5.      4.  $\pm\sqrt{6}/3$ .  
 1.  $6x^2 + 13x + 6 = 0$       2.  $x^2 - a^2 = 0$ .      3.  $4x^2 - x = 0$ .

**XLI. Page 301**

1. 5, -7.      2.  $3/2, -1/2$ .      3.  $5 \pm \sqrt{7}$ .  
 4.  $(-1 \pm 2i)/3$ .      5.  $(-3 \pm \sqrt{41})/4$ .      6.  $9/2, 1/2$ .  
 7. 12, -21.      8.  $17/6, -15/2$ .      9.  $23/4, 9/2$ .  
 10.  $32/5, -2/3$ .      11.  $1 + \sqrt{3}, 2 - \sqrt{3}$ .      12. 2,  $4 + i$ .  
 13. 4, 4.      14. 2, 2.      15.  $(1 \pm \sqrt{5})/2$ .  
 16. 3, -5.      17.  $3/2$ .      18. 5,  $16/7$ .  
 19.  $1/2$ .      20. 5,  $5/2$ .      21. 1,  $-58/91$ .  
 22.  $-2, 1/4$ .      23.  $1/3, -3a$ .      24.  $a + b, a - b$ .  
 25.  $b/c, -a/c$ .      26.  $2a + b, 2a - b$ .      27.  $a(3c \pm 2b)$ .  
 28.  $\frac{a+b}{a-b}, \frac{a-b}{a+b}$ .      29.  $\frac{a+b+c \pm \sqrt{a^2 + b^2 + c^2 - ab - ac - bc}}{3}$ .  
 30.  $\frac{a^2 + b^2}{2a}, \frac{a^2 + b^2}{2b}$ .

**XLII. Page 302**

- |   |   |                           |
|---|---|---------------------------|
| 1. 22, 23.  | 2. 15, 16.                              | 3. 5, 6.                  |
| 4. 13, 14, 15.  | 5. 86.                                  | 6. $7/5$ .                |
| 7. 42.  | 8. \$40.                                | 9. 5%.                    |
| 10. 4%.   | 11. 100.                                | 12. 6, $6\frac{2}{3}$ ft. |
| 13. 144, 112 sq. in.  | 14. $2(\sqrt{2} - 1)$ .                 | 15. 21 gal.               |
| 16. 40, 45 mi. per hour.  | 17. $1\frac{1}{2}$ hr., 4 mi. per hour. | 18. 4 hr. 20 min.         |
| 19. 6.  | 20. 2 hr.                               |                           |
| 21. (1) $1/4$ sec.; 1 sec.; no. (2) In $1/2$ sec. (3) In $3/2$ sec. |   |                           |

**XLIII. Page 308**

1. 2 and  $-1$ . 2.  $\infty, -2/3; \infty, \infty$ . 3.  $(x + 2y - 2)(3x - y + 1)$ . 4.  $\pm 1$ .  
 5.  $p^2 - 4q, p^4 - 4p^2q + 2q^2, (p^2 - 2q)/q$ . 6.  $-45/32, -7/2$ .  
 7.  $x^2 - x + 2 = 0, 2x^2 + x + 1 = 0, x^2 + 2x + 8 = 0, x^2 - x + 2 = 0$ .  
 8. 1. min. =  $-13$ . 2. min. =  $31/8$ . 3. max. = 5.  
 4. min. =  $-1/2$ , max. =  $1/2$ . 5. min. = 4.  
 6. max. =  $-(2 + \sqrt{2})/4$ , min. =  $-(2 - \sqrt{2})/4$ .  
 9. A square.  
 10. Toward a point  $2\frac{2}{3}$  miles from the nearest point.  
 11. 36 ft.,  $3/2$  sec.

**§ 643. Page 310. Ex. 3**

1.  $1/2, (2 \pm i\sqrt{2})/3$ . 2.  $-1, 4, 1 \pm \sqrt{2}$ .

**§ 644. Page 311. Ex. 7**

1.  $\pm 3, \pm \sqrt{6}/3$ . 2.  $(3 \pm \sqrt{5})/2, (3 \pm \sqrt{17})/2$ .  
 3.  $0, -a, -a(1 \pm \sqrt{57})/2$ . 4.  $-3, 2, -(1 \pm \sqrt{19})/3$ .

**§ 645. Page 313. Ex. 4**

1.  $1, (1 \pm i\sqrt{3})/2$ . 2.  $(3 \pm \sqrt{5})/2, (1 \pm i\sqrt{3})/2$ .  
 3.  $-1, (1 \pm i\sqrt{3})/2, (-1 \pm i\sqrt{3})/2$ .

**§ 646. Page 313. Ex. 3**

1.  $-2, 1 \pm i\sqrt{3}$ . 2.  $\sqrt{2}(1 \pm i)/2, -\sqrt{2}(1 \pm i)/2$ .  
 3.  $\pm i, (\sqrt{3} \pm i)/2, (-\sqrt{3} \pm i)/2$ .

## § 648. Pages 314, 315

2.  $1/4, -3/4$ .      4. 1, 6.      7. 3, 7.      9.  $(6 + \sqrt{6})/3$ .

## XLIV. Page 316

1.  $\pm 3/2, \pm \sqrt{2}$ .      2.  $7\sqrt[3]{63}/9$ .  
 3.  $\pm 2, \pm 3\sqrt{2}$ .      4.  $-1, -1, -1, 2/3, 2/3, 3/2$ .  
 5.  $1, -2, \pm \sqrt{3}i$ .      6.  $1, -1, 1 \pm i$ .  
 7.  $1, -1, 5/3, -1/3$ .      8.  $7, -1, 3 \pm \sqrt{5}$ .  
 9.  $(-1 \pm \sqrt{17})/4, (-1 \pm \sqrt{7}i)/4$ .      10.  $1, 1, \pm i$ .  
 11.  $\pm i, (-1 \pm i\sqrt{3})/2$ .      12.  $1, 2 \pm \sqrt{3}, 3 \pm 2\sqrt{2}$ .  
 13.  $3, 3(-1 \pm \sqrt{5} + i\sqrt{10 \pm 2\sqrt{5}})/4, 3(-1 \pm \sqrt{5} - i\sqrt{10 \pm 2\sqrt{5}})/4$ .  
 14.  $1, (1 \pm i\sqrt{3})/4$ .      15.  $0, \pm \sqrt{3}i$ .  
 16.  $5, -1, 2 \pm 3i$ .      17.  $-a, -b, -(a+b)/2$ .  
 18.  $(a+b)/2, [a+b \pm \sqrt{2-(a-b)^2}]/2$ .      19.  $0, -9/5, -(15 \pm \sqrt{401})/22$ .  
 20.  $\pm a, \pm 1/a$ .      21.  $1, -1/3, (1 \pm \sqrt{19})/3$ .  
 22.  $1, -1/2$ .      23. 2, 3.      24. 3, -2.      25.  $3/2$ .  
 26.  $16/25$ .      27. 1, -3.      28. 0, 16, 81.      29. 3.  
 30.  $(6 + \sqrt{6})/3$ .      31.  $1/5$ .      32.  $4/5$ .      33. 1.  
 34.  $\frac{27abc - (a+b+c)^3}{9(a^2 + b^2 + c^2 - ab - bc - ca)}$ .      35.  $6, -3, (3 \pm i\sqrt{71})/2$ .  
 36.  $0, -2a$ .      37.  $\pm \sqrt{182}/14, \pm 3\sqrt{7}/7$ .

## XLV. Page 320

1.  $2/3, -11/9; -4, -13/3$ .      2.  $-1, -1; 5/3, 3/5$ .  
 3.  $1/15, 2/15$ ; one infinite solution.      4.  $3, 1; -1/15, -41/5$ .  
 5.  $0, 0; 51/14, -17/14$ .      6.  $1, 1/2; 12/5, 32/15$ .  
 7. No finite solution.      8.  $5, 9; 333/28, 185/42$ .  
 9.  $1, 1/4; -12/13, -4/9$ .      10.  $1, -3; -1, 3; -1, 3; 2, -3$ .  
     11.  $2, -2; -2, 2; -1 \pm \sqrt{3}, 1 \pm \sqrt{3}$ .  
     12.  $0, 0; 15/8, 9/8$ ; two infinite solutions.  
     13.  $m = 1$  or  $-1$ .  
     14.  $m = 1, c = 1/3; m = -1/2, c = -1/3$ .  
     15.  $(2x - y + 4)(x + y + 3)$ .

## § 655. Page 321. Ex. 2

1.  $1, 0; -1, 0$ .      2.  $3, 1/5; 3, -1/5; -3, 1/5; -3, -1/5$ .

## § 656. Page 321. Ex. 3

1.  $2, 1; 3, -1$ ; two infinite solutions.  
 2.  $3i\sqrt{14}/14, i\sqrt{14}/7; 3i\sqrt{14}/14, -i\sqrt{14}/7;$   
 $-3i\sqrt{14}/14, i\sqrt{14}/7; -3i\sqrt{14}/14, -i\sqrt{14}/7$ .

## § 657. Page 323. Ex. 5

1. 0, 0; 0, 0; 5, -5; -3, -9.
2. -2, 3; -3, 2;  $(3 \pm \sqrt{17})/2$ ,  $(-3 \pm \sqrt{17})/2$ .

## § 658. Page 323. Ex. 2

- 1, 2; -1, 1;  $(1 \pm i\sqrt{23})/4$ ,  $(1 \pm i\sqrt{23})/4$ .

## XLVI. Page 324

1. 2, 3; 2, -3; -2, 3; -2, -3.
2.  $2, 1/3$ ;  $2, -1/3$ ;  $-2, 1/3$ ;  $-2, -1/3$ .
3. 7, -1; -5, 2; two infinite solutions.
4. -5, -1;  $15/2, 3/2$ ;  $2\sqrt{3}, 3\sqrt{3}$ ;  $-2\sqrt{3}, -3\sqrt{3}$ .
5. 2, -3;  $-2/3, 7/3$ ;  $(-6 \pm 2\sqrt{6})/3$ ,  $(3 \mp 4\sqrt{6})/3$ .
6. 1, 0; -1, 2; two infinite solutions.
7. 5, -1; 5, -21;  $-7, 19 \pm 2\sqrt{85}$ .
8. 5, 5; -5, -5;  $\sqrt{5}i, -\sqrt{5}i$ ;  $-\sqrt{5}i, \sqrt{5}i$ .
9. 3, 1; -3, -1;  $3\sqrt{5}i, -7\sqrt{5}i/5$ ;  $-3\sqrt{5}i, 7\sqrt{5}i/5$ .
10.  $1, 1/2$ ; -1, -1/2;  $\sqrt{91}/5, \sqrt{91}/16$ ;  $-\sqrt{91}/5, -\sqrt{91}/16$ ; 0, 0; 0, 0; 0, 0; 0, 0; eight infinite solutions.
11.  $3\sqrt{2}, 2\sqrt{2}$ ;  $-3\sqrt{2}, -2\sqrt{2}$ ;  $2\sqrt{2}, 3\sqrt{2}$ ;  $-2\sqrt{2}, -3\sqrt{2}$ .
12. 5, 4; -4, -5;  $10 \pm 2\sqrt{30}$ ,  $-10 \pm 2\sqrt{30}$ .
13. 2, 4; 3, -1;  $(-5 \pm \sqrt{13})/2$ ,  $(-3 \pm 5\sqrt{13})/2$ .

## XLVII. Page 325

1. 4, 1; -1, -4; one infinite solution.
2. 125, -27; -27, 125; one infinite solution.
3. 5, 3; -5, -3; 3, 5; -3, -5; four infinite solutions.
4. 2, 3;  $-1 + \sqrt{3}i$ ,  $3(-1 + \sqrt{3}i)/2$ ;  $-1 - \sqrt{3}i$ ,  $-3(1 + \sqrt{3}i)/2$ ; six infinite solutions.
5. -2, 2; -2, -2;  $(2 \pm 2\sqrt{7})/3$ ,  $(1 \pm \sqrt{7})/3$ ; two infinite solutions.
6. 0, 0; 6, 9;  $50/21$ ,  $-20/21$ ; one infinite solution.

## XLVIII. Page 328

1. 9, -4; -4, 9.
2. 14, -2; -2, 14
3. 17, 2; -17, -2; 2, 17; -2, -17.
4. 9, 2; -2, -9.
5. 8, 1; 1, 8; one infinite solution.
6. 7, 5; 5, 7;  $7\omega, 5\omega$ ;  $5\omega, 7\omega$ ;  $7\omega^2, 5\omega^2$ ;  $5\omega^2, 7\omega^2$ ,  
where  $\omega = (-1 + \sqrt{3}i)/2$ ; three infinite solutions.

7.  $4/3, 1/4; 1/3, 1$ ; one infinite solution.  
 8.  $3, 1; -1, -3; 1 \pm i\sqrt{10}, -1 \pm i\sqrt{10}$ .  
 9.  $2, 0; 0, 2; 1 \pm i\sqrt{3}, 1 \mp i\sqrt{3}$ ; one infinite solution.  
 10.  $4, -7/2; -7/2, 4$ .  
 11.  $5, -4; -4, 5; -10 + 3\sqrt{11}, -10 - 3\sqrt{11}; -10 - 3\sqrt{11}, -10 + 3\sqrt{11}$ ; two infinite solutions.  
 12.  $3, 1; 1, 3; -3, -1; -1, -3; \pm\sqrt{5}i, \pm\sqrt{6}i; \pm\sqrt{6}i, \pm\sqrt{5}i$ .  
 13.  $2, -5; -5, 2; (3 \pm \sqrt{31}i)/2, (3 \mp \sqrt{31}i)/2$ .  
 14.  $0, 2; 2, 0; (-23 \pm \sqrt{389})/12, (-23 \mp \sqrt{389})/12$ .  
 15.  $0, 0; \pm\sqrt{5}, \pm\sqrt{5}; \pm\sqrt{5}i, \mp\sqrt{5}i; \pm\sqrt{10}(1+i)/2, \mp\sqrt{10}(1-i)/2; \pm\sqrt{10}(1-i)/2, \mp\sqrt{10}(1+i)/2$ .

**XLIX. Page 329**

1.  $4, -1, 3; -2, 5, -3$ .  
 2. Misprint. Second equation should be  $y(z+x) = 6$ . The solutions are then  $4, 1, 2; -4, -1, -2$ .  
 3.  $-(a^2 \pm bc)/a, -(b^2 \pm ca)/b, -(c^2 \pm ab)/c$ .

**L. Page 330**

1.  $2/3, -11/9; -4, -13/3$ .  
 2. Misprint. Second equation should be  $x - y = 1$ . Solutions are then  $4, 3; -3, -4$ .  
 3.  $(b+a)/2, (b-a)/2; (a-b)/2, -(a+b)/2$ .  
 4.  $\sqrt{\frac{a-b}{a^2+b^2}}, \pm\sqrt{\frac{b-a}{a^2+b^2}}; -\sqrt{\frac{a-b}{a^2+b^2}}, \pm\sqrt{\frac{b-a}{a^2+b^2}}$ .  
 5.  $1, 1/4; -12/13, -4/9$ .      6.  $a, b; 2a-b, 2b-a$ .  
 7.  $5, 1/2; 1/2, 5$ .      8.  $a, b/2; -a, -b/2; a/2, b; -a/2, -b$ .  
 9.  $4, 1; -1, -4; 0, 0; 0, 0$ .      10.  $\frac{2ab}{b-a}, \frac{2ab}{b+a}; 0, 0$ .  
 11.  $3, -7; 35/2, 15/2; 0, 0; 0, 0$ .  
 12.  $5, 2; -5, 2; 2, -4/5; -2, -4/5$ .  
 13.  $3, 1; -3, -1; 6, 3; -6, -3$ .  
 14.  $6, 1; -27, -10; (7 \pm 3\sqrt{65})/2, (3 \pm \sqrt{65})/2$ .  
 15.  $7/2, 2; -2, -7/2; 136/65, 238/65; -238/65, -136/65$ .  
 16.  $\alpha, \beta; \alpha\omega, \beta\omega; \alpha\omega^2, \beta\omega^2$ ; where  $\alpha = \frac{\sqrt[3]{a^2b^2}}{b}, \beta = \frac{\sqrt[3]{a^2b^2}}{a}$ ,  
     and  $\omega = (-1 + i\sqrt{3})/2$ ; six infinite solutions.  
 17.  $\alpha, \beta; \alpha\omega, \beta\omega; \alpha\omega^2, \beta\omega^2$ ; where  $\alpha = \frac{\sqrt[3]{4(a^2-b^2)^2}}{2(a-b)}, \beta = \frac{\sqrt[3]{4(a^2-b^2)^2}}{2(a+b)}$ ,  
     and  $\omega = (-1 + i\sqrt{3})/2$ ; six infinite solutions.

18.  $a/(a^2 + b^2)$ ,  $b/(a^2 + b^2)$ ; 0, 0; two infinite solutions.  
 19.  $4a$ ,  $a$ ;  $-4a$ ,  $-a$ .  
 20.  $3/2$ ,  $1/2$ ;  $-1/2$ ,  $-3/2$ ; one infinite solution.  
 21.  $a$ , 0; 0,  $a$ ;  $a(1 \pm \sqrt{7}i)/2$ ,  $a(1 \mp \sqrt{7}i)/2$ .  
 22. 7, 3; 3, 7;  $(-17 \pm \sqrt{646})/5$ ,  $(-17 \mp \sqrt{646})/5$ .  
 23.  $(3 \pm \sqrt{3}i)/2$ ,  $(3 \mp \sqrt{3}i)/2$ ; 0, 0; 0, 0.  
 24.  $\pm a\sqrt{3}$ , 0;  $2a$ ,  $a$ ;  $-2a$ ,  $-a$ .  
 25. 1, 4; 4, 1; an infinite solution.  
 26. 3, 0; 0, 4; 0, 0; 0, 0.  
 27. 5, 0; 0, 5;  $\sqrt{10}$ ,  $-\sqrt{10}$ ;  $-\sqrt{10}$ ,  $\sqrt{10}$ .  
 28. 2, 4;  $-9$ , 15;  $-1$ ,  $-1$ ; 0, 0.  
 29.  $3/2$ ,  $1/2$ ;  $1/2$ ,  $3/2$ ;  $1 \pm i\sqrt{1155}/35$ ,  $1 \mp i\sqrt{1155}/35$ .  
 30.  $\frac{\sqrt{a^2+1}}{a}$ ,  $\sqrt{a^2+1}$ ;  $-\frac{\sqrt{a^2+1}}{a}$ ,  $-\sqrt{a^2+1}$ .  
 31. 2, 0;  $-3$ , 5; one infinite solution.  
 32.  $1$ ,  $5/3$ ; 2,  $-4/3$ ;  $(-9 \pm \sqrt{21})/6$ ,  $(-1 \pm 3\sqrt{21})/6$ .  
 33. 1, 3, 1;  $149/5$ ,  $-1/5$ ,  $-15$ .  
 34. 1, 2,  $-3$ , 0;  $-1$ ,  $-2$ , 3, 0.  
 35. 3, 1, 1;  $-3$ ,  $-1$ ,  $-1$ .  
 36. 1,  $-1$ , 2;  $-14/13$ ,  $23/13$ ,  $17/13$ ;  $(-29 \pm 3\sqrt{105})/26$ ,  $(-11 \mp 2\sqrt{105})/13$ ,  
 $(-1 \pm \sqrt{105})/26$ .

#### LI. Page 331

1. 7 and 5.                      2. 6 and 3.                      3. 5/6.  
 4. 8, 9, and 20.                5. 12 and 5 ft.                6. 15, 9, and 12 in.  
 7. 39, 36, and 15 in.        8. 16, 14, 9 ft.                9. 14 and 12 in.  
 10. A, \$4.50; B, \$5.            11. \$1200, 4%.  
 12. Two children, each of whom receives \$10,000; five grandchildren, each  
       of whom receives \$8000.  
 13. 4 and 2 mi. per hour.        14. A, 3 hr.; B, 4 hr.; C, 6 hr.  
 15. A, 24 in. per second; B, 20 in.  
 16. A, 8 in. per second; B, 2 in.    17. 18 mi.                      18. 96 mi.

#### LII. Page 339

17. Meets the  $x$ -axis at the points  $3 \pm \sqrt{8}$ , 0; touches the  $y$ -axis at the  
 point 0, 1.  
 20.  $m = \pm 1$ .            21.  $c = 1$  or  $4/3$ .            23.  $y = 2x + 1$  and  $x + 2y = 0$ .  
 24. When  $|\lambda| = 2$ , parabola; when  $|\lambda| < 2$ , ellipse; when  $|\lambda| > 2$ , hyperbola.

## LIII. Page 341

6.  $x < 30$ .                      7.  $x < -2$  or  $> 4$ .                      8.  $-1 < x < 3$ , or  $x > 6$ .

## LIV. Page 346

1.  $x = 20 - 17t$ ,  $y = 6 - 6t$ . The solutions corresponding to  $t = 0, -1, -2, \dots$  are positive.
2.  $x = 2 - 12t$ ,  $y = -6 - 43t$ . The solutions corresponding to  $t = -1, -2, \dots$  are positive.
3.  $x = -17 + 39t$ ,  $y = 7 - 16t$ . No positive solution.
4.  $x = -2 + 23t$ ,  $y = 43 - 72t$ . No positive solution.
5.  $x = 16 - 27t$ ,  $y = 28 - 49t$ . The solutions corresponding to  $t = 0, -1, -2, \dots$  are positive.
6.  $x = 54 - 97t$ ,  $y = 21 - 47t$ . The solutions corresponding to  $t = 0, -1, -2, -3, \dots$  are positive.
7.  $x = 2 + 21t$ ,  $y = 3 - 26t$ ,  $z = -1 - 11t$ . No positive solution.
8.  $x = 10 - 14t$ ,  $y = -4 + 35t$ ,  $z = -2 + 15t$ . No positive solution.
9.  $x = u$ ,  $y = 2v + 1$ ,  $z = 2u + 3v$ . The solutions corresponding to positive values of  $u$  and  $v$  are positive.
10.  $x = 7 - 3u - 2v$ ,  $y = 1 + 2u$ ,  $z = v$ . The positive solutions are 7, 1, 0; 5, 1, 1; 3, 1, 2; 1, 1, 3; 4, 3, 0; 2, 3, 1; 0, 3, 2; 1, 5, 0.
11. Fifty positive solutions.                      12.  $\frac{41}{35} = \frac{3}{5} + \frac{4}{7}$ .
13. 2 calves, 16 lambs; or 8 calves, 9 lambs; or 14 calves, 2 lambs.
14. 15, 2, 6; 12, 4, 7; 9, 6, 8; 3, 10, 10.                      15. 314.
16. The 167th and 169th divisions measured from either end.

## LV. Page 350

1. 32;  $192/5$ ;  $10a^{2b^2}$ ;  $\sqrt{30}$ .                      2.  $-3:2; 1:5$ .
3.  $x:y = -1:2$  or  $3:1$ .     $y:x = -2:1$  or  $1:3$ .
13. (1) 0, 0,  $3/2$ .    (2) 0, 0, 5,  $8/7$ .
14. 104, 156, 260.                      15. 8 gallons from A, 10 gallons from B.

## LVI. Page 353

1.  $-14/5$ .                      2.  $\pm 2\sqrt{3}/3$ .                      3.  $x^2 + 3y - 4 = 0$ .                      4.  $-72$ .
7. 3.                      8.  $15/8$ .                      9.  $\frac{a\sqrt{2}}{2}, \frac{a\sqrt[3]{4}}{2}$ .



## LVII. Page 356

1. 60, 630;  $25\frac{1}{2}$ , 225.
2.  $n(n+1)/2$ ;  $n^2$ ;  $n(n+1)$ .
3.  $n(3n-2)$ .
4.  $-1/3, 0, 1/3, 2/3, 1, 4/3, 5/3, 2, 7/3, 8/3$ .
5.  $-1/2, 0, 1/2, 1, 3/2$ .
6.  $l = 20, S = 160$ .
7.  $a = 3, S = -14$ .
8.  $d = 3, S = 178$ .
9.  $n = 52, d = -1/2$ .
10.  $d = 2, l = 12\frac{3}{4}$ .
11.  $n = 28, S = 187\frac{3}{4}$ .
12.  $a = 31, l = -1$ .
13.  $a = 25, d = -3$ .
14.  $n = 6, a = -72$ .
15.  $n = 5, l = 18$ .
16.  $1, 1\frac{1}{6}, 1\frac{1}{3}, 1\frac{1}{2}, 1\frac{2}{3}, 1\frac{5}{6}, 2, \dots$
17. 3, 5, 7.
18. 55, 350.
19. \$1716.
20. In 9 hours, midway between their starting points.

## LVIII. Page 360

1. 162, 122.
2.  $13\frac{1}{2}, 32\frac{1}{2}$ .
3. 8;  $2/3$ ;  $25/12$ .
4.  $\frac{341}{999}, \frac{78}{375}, 8\frac{516}{999}$ .
5.  $l = -3000, S = -3333.33$ .
6.  $r = 1/2, S = 95\frac{1}{4}$ .
7.  $n = 8, S = 255/16$ .
8.  $a = 1, S = 61$ .
9.  $n = 5, l = 2/3$ .
10.  $r = 5, l = -375$ .
11.  $n = 5, r = 4/3$ .
12.  $a = 45/8, l = -20/27$ .
13. Misprint. Should be  $83\frac{1}{8}$ . Then  $n = 6, a = 4$ .
14.  $a = 10, r = 3/5$ .
15.  $a = 3, r = 2$ .
16.  $ab$ .
17. 15, 45, 135.
18.  $3/16$ .
19. 8, 20, 50, 125.
20. 1, -3, 9.
21. 3, 12, 21 or 63, 12 -39.
22. -2, 2, 6, 18.
23. 75 feet.

## LIX. Page 363

1.  $3/11, 3/13$ .
2.  $30/23$ .
3.  $75/7, 150/13, 75/6, 150/11$ .
4. 2.
5. 3 and 5.
6. 2 and 8.

## LX. Page 369

1. 191 and 1350.
2. 6560 and 180,360.
3. (1) Second order; eighteenth term is 224.  
(2) Third order; twentieth term is 10,118.  
(3) Third order; twelfth term is -1.  
(4) Fourth order; tenth term is 10,100.
4. Third order;  $n$ th term is  $n(n+1)(n+2)$ . Sum of first  $n$  terms is  $n(n+1)(n+2)(n+3)/4$ .  
Fourth order;  $n$ th term is  $2n(n+1)^3$ .
5. 560, 105.
6. 1149.
7. 962.
8. 2024.
9. 220, 385.
10. 1274.
13. Second order; sum of first  $n$  terms is  $n(n^2+2)/3$ .  
Third order; sum of first  $n$  terms is  $n(n+1)(n+2)(n+3)/4!$ .

**LXI. Page 374**

1.  $y = 4 + 23x + 26x^2 + 7x^3$ ; when  $x = -5/2$ , then  $y = -3/8$ ;  
when  $x = -1/2$ , then  $y = -15/8$ .
2.  $f(x) = -90 + 73x - 16x^2 + x^3$ ;  $f(12) = 210$ .
3. 704.3716.      4. 110.592.      5. .04237.      6. 20.8734.
7.  $(2520 - 806x - 9x^2 - 13x^3)/840$ .

**LXII. Page 378**

1. 2,  $1/2$ , 6, 4, 6, -3, -6, -3,  $-2/3$ ,  $7/3$ ,  $3/2$ .
2. 1.0791, .6532, .1505, .2594.      3.  $(3 \log_a 2 + \log_a 3 + 2 \log_a 5)6$ .
4. (1)  $\frac{2}{3} \log_a b - \frac{1}{2} \log_a c - \frac{1}{3} \log_a d$ .      (2)  $\frac{1}{12} - \frac{1}{3} \log_a b$ .

**LXIII. Page 389**

The answers given below are obtained by the use of the four-place table, pp. 384, 385, and by following in every case the rule at the end of §751.

- |            |                              |                             |                              |
|------------|------------------------------|-----------------------------|------------------------------|
| 1. 36,460. | 2. 82.28.                    | 3. $1.210 \times 10^{13}$ . | 4. $-1.744 \times 10^{-5}$ . |
| 5. 22.13.  | 6. .1151.                    | 7. $-4.558 \times 10^7$ .   | 8. 16.38.                    |
| 9. -.4255. | 10. 77.                      | 11. 9.472.                  | 12. 137.                     |
| 13. 1.413. | 14. 8.218.                   | 15. -.676.                  | 16. 13.46.                   |
| 17. .01.   | 18. 5.461.                   | 19. 1.101.                  | 20. 5.34.                    |
| 21. 9108.  | 22. 632.8.                   | 23. -50.22.                 | 24. $2.453 \times 10^{-6}$ . |
| 25. -73.6. | 26. $2.647 \times 10^{13}$ . | 27. .5381.                  | 28. 96.56.                   |
| 29. 15.77. | 30. $2.652 \times 10^5$ .    |                             |                              |

**LXIV. Page 392**

The formulas at the top of p. 391 for interest compounded semiannually, quarterly, and so on should be  $A = P(1 + r/2)^{2n}$ ,  $A = P(1 + r/4)^{4n}$ , and so on.

1. 3.925, -1.578, .8361.
2. (1)  $x = 6$ .      (2)  $x = 1$  or 2.      (3)  $x = 2.152$ .
3. (1)  $x = 2$ , -5.      (2)  $x = 4.642$ .      (3)  $x = 3.051$ .      (4)  $x = 3.537$ .
4. \$41,410.      5. \$10,010.      7. \$694.80.      8. \$8030.
9. \$20,730, \$30,000.      10.  $b = 496.4$ , area = 77,500.      11. 5179.
12. Area of surface, 6998; volume, 55050.

**§ 763. Page 396**

- |          |                        |          |
|----------|------------------------|----------|
| 4. 65.   | 5. 380 single tickets. | 6. 144.  |
| 7. 1800. | 9. 325.                | 10. 3600 |

**§ 764. Page 397**

3. 144.

**§ 765. Page 397**

1. 5, 25, 125.                      2. 343.

**§ 766. Page 398**

4. 630, 630.                      5. 715.

**§ 769. Page 401**

6. 136, 252, 8855.                      7. 15.                      8. 8.                      9. 220.  
 10. 792, 330, 462.                      11. 120, 420, 252.                      12. 3,953,520.  
 13.  $10010 \cdot 7!$ .                      14.  $52!/(13!)^4$ ,  $52!/(13!)^4 \cdot 4!$ .                      15. 2250.

**§ 770. Page 402**

1. 15.                      3. 41.

**§ 771. Page 402**

$$C_6^{12} = 924, C_7^{15} = 6435.$$

**LXV. Page 405**

1. 24.                      2. 720.                      3. 336.                      4. 210, 5040.                      5. 161,700.  
 6. 30.                      7. 420.                      8. (1) 90,720. (2) 322,560, (3) 4320.  
 9. 60,480, 504, 84.                      10. 60.                      11. 1680.  
 12. 6.                      13. (1) 399. (2) 259.                      14. 360.  
 15. 1232, 224.                      16. 31.                      17. 3,783,780.  
 18. 462.                      19. 1446.                      20.  $5096520 \cdot 15!$ .  
 21. 825.                      22. (1) 60. (2) 325.                      23. 2970.  
 25. 63,063.                      26. 145,152.                      27. 300.  
 28.  $5^{15}$ ,  $15!/(3!)^5$ .                      29. 5760.                      30. 37,740.  
 32. 126.                      33. 252.                      34. 5040, 840, 7560.

**LXVI. Page 409**

1.  $\Sigma a^3 + 3 \Sigma a^2b + 6 \Sigma abc$ .  
 2.  $\Sigma a^5 + 5 \Sigma a^4b + 10 \Sigma a^3b^2 + 20 \Sigma a^3bc + 30 \Sigma a^2b^2c + 60 \Sigma a^2bcd$ .  
 3. 83160, 34,650, 16,632.                      4. 12,600.                      5. 15,120.                      6. 4455.                      7. 26,396.



17.  $-1, -1, 3, 3, 3$ .  
18.  $-2, 4, 5, 3/2$ .  
19.  $-2, 1/2, 3/2, \pm i$ .  
20.  $2, 2, -3, -4, -5$ .  
21.  $-2, -2, -1/2, (-1 \pm \sqrt{3} i)/2$ .  
22.  $6, 8, 1/2, 1/3$ .  
23.  $-1/2, -3/5, (-3 \pm \sqrt{5})/2$ .  
24.  $1/2, 1/3, 2/3, 3/2$ .  
25.  $-1/2, -2/3, -3/2, (1 \pm \sqrt{11} i)/2$ .  
26.  $1, -1, -2, -3/2, \pm \sqrt{2} i$ .  
27.  $1, -1, -2, -3/2, -1 \pm \sqrt{2} i$ .  
28.  $1, -1, 2, 2/5, (-1 \pm \sqrt{3} i)/2$ .

**LXXI. Page 435**

1.  $3/2$ .  
2. (1)  $1, -3/2, 9/4$ . (2)  $1 + 2\sqrt{2}i, -3, 1 - 2\sqrt{2}i$ .  
3. (1)  $-2 - \sqrt{5}, -2, -2 + \sqrt{5}$ . (2)  $1, 3, 5$ .  
5.  $r^2 - rp + q - 1 = 0$ . 6.  $-1 \pm \sqrt{2}i, -1 \pm \sqrt{2}i$ .  
7.  $-1, 3/2, 3/7$ . 8.  $-4, -6, 6, 5$ .  
9. (1)  $x^3 - px^2 + qx - r = 0$ .  
(2)  $x^3 + kpx^2 + k^2qx + k^3r = 0$ .  
(3)  $rx^3 + qx^2 + px + 1 = 0$ .  
(4)  $x^3 + (p - 3k)x^2 + (q - 2kp + 3k^2)x + (r - kq + pk^2 - k^3) = 0$ .  
(5)  $x^3 + (2q - p^2)x^2 + (q^2 - 2pr)x - r^2 = 0$ .  
(6)  $r^2x^3 + (q^2 - 2pr)x^2 - (2q - p^2)x + 1 = 0$ .  
10. (1)  $17/4$ . (2)  $-37/8$ . (3)  $1$ . (4)  $5/2$ .  
11. (1)  $2/3$ . (2)  $-11/3$ . (3)  $-1$ . (4)  $-31$ . (5)  $-7/3$ .

**LXXII. Page 443**

1.  $x^7 - 3x^4 + 2x^2 + 6x - 7 = 0$ .  
2. (1)  $x^4 - x^3 - 8x^2 + 24x + 64 = 0$ .  
(2)  $162x^4 + 27x^3 - 36x^2 - 18x + 8 = 0$ .  
3.  $10x^6 + 9x^5 + 3x^3 - x^2 + 5 = 0$ .  
4. (1)  $2x^5 + 21x^4 + 88x^3 + 181x^2 + 180x + 74 = 0$ .  
(2)  $2x^5 - 9x^4 + 16x^3 - 17x^2 + 12x + 2 = 0$ .  
5.  $x^4 - 10x^3 + 225x^2 + 1080x - 16,875 = 0$ .  
6.  $3x^4 - 162x^2 - 647x - 733 = 0$ .  
7. (1)  $x^3 - 3x^2 + 10 = 0$ , or  $x^3 + 3x^2 + 6 = 0$ .  
(2)  $x^3 + 2x^2 - 4 = 0$ , or  $27x^3 - 54x^2 - 76 = 0$ .  
8.  $x^4 - x^3 + 6x^2 - x + 4 = 0$ . 9.  $x^4 + 9x^3 + 29x^2 + 39x + 17 = 0$ .  
10. (1)  $rx^3 + (q^2 - 2pr)x^2 + r(p^2 - 2q)x + r^2 = 0$ .  
(2)  $(r - pq)x^3 + (p^3 - 2pq + 3r)x^2 + (3r - pq)x + r = 0$ .  
11. (1)  $x^3 + 4x^2 - 3x + 2 = 0$ . (2)  $x^3 - 6x^2 + 17x - 8 = 0$ .  
(3)  $4x^3 - 9x^2 + 18x - 27 = 0$ . (4)  $8x^3 + 16x^2 + 9x + 2 = 0$ .  
(5)  $16x^3 + 24x^2 + 27x - 8 = 0$ .  
12. (1)  $2$  and  $-6$ . (2)  $4$  and  $-1$ . (3)  $4$  and  $-7$ .  
(4)  $2$  and  $-5$ . (5)  $9$  and  $-2$ . (6)  $2$  and  $-1$ .

## LXXIII. Page 449

1.  $-1, -1/2, 1 \pm 2i$ .
2.  $1, 1/2, 2 \pm \sqrt{2}$ .
3.  $x^4 + 12x^3 + 44x^2 + 18x - 116 = 0$ .
4.  $x^4 - 2x^2 + 9 = 0$ .
5. (1) Four imaginary roots.  
 (2) At least *two* imaginary roots. Of the real roots not more than *one* can be positive nor more than *one* negative.  
 (3) No positive roots.  
 (4) No negative roots.  
 (5) At least *four* imaginary roots. Of the real roots not more than *two* can be positive nor more than *one* negative.  
 (6) At least *four* imaginary roots. Of the real roots not more than *one* can be positive nor more than *two* negative.  
 (7) At least *two* imaginary roots. Of the real roots not more than *two* can be positive nor more than *one* negative.  
 (8) When  $n$  is odd, at least  $3n - 3$  imaginary roots. Of the real roots not more than *two* can be positive nor more than *one* negative.  
 When  $n$  is even, at least  $3n - 6$  imaginary roots. Of the real roots not more than *two* can be positive nor more than *four* negative.
7. Two positive and three negative.
8. That  $x^{2n+1} + 1 = 0$  has one negative and  $2n$  imaginary roots; that  $x^{2n} - 1 = 0$  has at least  $2n - 2$  imaginary roots, not more than one positive nor more than one negative root; that  $x^{2n+1} - 1 = 0$  has one positive and  $2n$  imaginary roots.

## LXXIV. Page 453

1. Between 0 and 1, 2 and 3,  $-1$  and  $-2$ .
2. Between 1 and 2, 0 and  $-1$ ,  $-2$  and  $-3$ .
3. Between 1 and 2, 3 and 4,  $-1$  and  $-2$ .
4. Between 2 and 3,  $-1$  and  $-2$ ,  $-2$  and  $-3$ .
5. Between 1 and 2, 4 and 5,  $-1$  and  $-2$ .
6. Between  $-2$  and  $-3$ ,  $-4$  and  $-5$ ,  $-6$  and  $-7$ .
7. Between  $-2$  and  $-3$ .
8. Between 3 and 4,  $-3$  and  $-4$ .
9. Between 0 and 1, 2 and 3, 5 and 6, 0 and  $-1$ .
10. Between 1 and 2, 0 and  $-1$ ,  $-1$  and  $-2$ ,  $-4$  and  $-5$ .
11. Between 1 and 2, 4 and 5, 5 and 6, 0 and  $-1$ .
12. Between 1 and 2, 3 and 4, 0 and  $-1$ ,  $-2$  and  $-3$ ,  $-3$  and  $-4$ .

**LXXV. Page 459**

1. 1.213411.      2. 2.469545.      3. .179989.      4. 2.137811.
5. 2.768345.      6. -1.945341.      7. 1.903211.      8. -5.134578.
9. 3.236067.      10. -2.157451.      11. 2.356895 and 2.692021.
12. 1.602, 3.292, and -1.895.      13. 1.246, -.445, and -1.802.
14. .347, 1.532, and -1.879.      15. 1.558, -.578, -1.904, -4.075.
16. 2.5712.      17. 2.884 and 3.054.      18. 13.24.
19. Two roots between 0 and 1, one between 0 and -1.
20.  $2/3$ , -1, .254, 1.860, -2.114.

**LXXVI. Page 464**

1.  $10x^4 - 16x^3 + 2x - 20$ ,  $40x^3 - 48x^2 + 2$ ,  $120x^2 - 96x$ ,  $240x - 96$ , 240.
2.  $(x^4 - 2x^3 + 1) + 2(2x^3 - 3x^2)h + 6(x^2 - x)h^2 + 2(2x - 1)h^3 + h^4$ .
3. (1)  $3 - 6(x + 1) + 7(x + 1)^2 - 4(x + 1)^3 + (x + 1)^4$ .  
 (2)  $80(x - 2) + 80(x - 2)^2 + 40(x - 2)^3 + 10(x - 2)^4 + (x - 2)^5$ .  
 (3)  $\frac{2 + 3(x - 1) + 3(x - 1)^2 + (x - 1)^3}{2 + 2(x - 1) + (x - 1)^2}$ .
4. (1) -1, -1, 2.      (2)  $1/3, 1/3, -2$ .  
 (3)  $\pm\sqrt{6}i/2, \pm\sqrt{6}i/2$ .      (4)  $1 \pm \sqrt{3}, 1 \pm \sqrt{3}$ .  
 (5)  $3, 3, \pm\sqrt{2}i/2$ ,      (6)  $(-1 \pm \sqrt{3}i)/2, (-1 \pm \sqrt{3}i)/2, 2$ .  
 (7)  $2, 2, -1 \pm \sqrt{2}i$ .      (8)  $1, 1, 1, -1, -1$ .  
 (9)  $\pm i, \pm i, 2/3$ .
6.  $a = \pm 16$ .
7.  $a = 3, b = 1/9, x = -1/3$ ; or  $a = -3, b = -1/9, x = 1/3$ .
9.  $108p^5 = 3125r^3$ .      10.  $(ax + b)^n$ .  
 11.  $(-1 \pm \sqrt{3}i)/2, \pm i; (-1 \pm \sqrt{3}i)/2, \pm 1$ .  
 12.  $2 \pm 2\sqrt{2}, -4; 1 \pm \sqrt{2}, -1$ .

**LXXVII. Page 471**

1. Maximum value corresponds to  $x = 0$  and is 4; minimum value corresponds to  $x = 2$  and is 0.
2. (1) Minimum when  $x = 1/4$ .  
 (2) Maximum when  $x = (1 - \sqrt{3})/2$ , minimum when  $x = (1 + \sqrt{3})/2$ .  
 (3) Maximum when  $x = -2$ , minimum when  $x = 2$ .  
 (4) Maximum when  $x = 1/3$ , minimum when  $x = 3$ .  
 (5) Maximum when  $x = 0$ , minimum when  $x = 2$ .  
 (6) Maximum when  $x = 1/2$ , minimum when  $x = -1$  or 2.  
 (7) Has neither a maximum nor a minimum.  
 (8) Maximum when  $x = -1$ , minimum when  $x = -(2 \pm \sqrt{10})/2$ .

**LXXVIII. Page 477**

1. Two between 3 and 4, one between  $-1$  and  $-2$ .
2. Two between 3 and 4, one between  $-3$  and  $-4$ .
3. One between 0 and  $-1$ , two imaginary.
4. One between  $-1$  and  $-2$ , two imaginary.
5. Two between 2 and 3, one between  $-4$  and  $-5$ .
6. Two between 3 and 4, two between  $-1$  and  $-2$ .
7. One between 0 and 1, one between  $-1$  and  $-2$ , two imaginary.
8. Two between 0 and 1, two between 3 and 4.
9. Two between 0 and 1, one between 2 and 3, one between  $-3$  and  $-4$ .
10. One between 0 and 1, one between 1 and 2, two between  $-2$  and  $-3$ .
11. One real root.
12. No real root.
13. When  $n$  is even, no real root; when  $n$  is odd, one real root.
14. Two real roots.

**LXXIX. Page 482**

1.  $s_3 = -(a_1^3 - 3a_0a_1a_2 + 3a_0^2a_3)/a_0^3$ .  
 $s_4 = (a_1^4 - 4a_0a_1^2a_2 + 4a_0^2a_1a_3 + 2a_0^2a_2^2)/a_0^4$ .
2.  $\Sigma 1/\alpha^2 = (q^2 - 2pr)/r^2$ .  $\Sigma 1/\alpha^3 = (-q^3 + 3pqr - 3r^2)/r^3$ .  $\Sigma \alpha\beta^2 = 3r - pq$ .
3.  $x^3 + 7x^2 + 12x - 1 = 0$ .
4. (1)  $s_1 = 1$ ,  $s_2 = -5$ ,  $s_3 = -20$ ,  $s_4 = -9$ .  
 (2) 29. (3) 20. (4)  $-60$ . (5)  $449/256$ . (6)  $-111/4$ .

**§ 871. Page 485**

2.  $-1 + \sqrt[3]{A} + \sqrt[3]{B}$ ,  $-1 + \omega \sqrt[3]{A} + \omega^2 \sqrt[3]{B}$ ,  $-1 + \omega^2 \sqrt[3]{A} + \omega \sqrt[3]{B}$ ,  
 where  $A = (-1 + \sqrt{5})/2$  and  $B = (-1 - \sqrt{5})/2$ .

**§ 874. Page 486**

2.  $(1 \pm \sqrt{5})/2$ ,  $(3 \pm \sqrt{13})/2$ .

**§ 875. Page 488**

2.  $(1 \pm \sqrt{3}i)/2$ ,  $\sqrt{2}(1 \pm i)/2$ ,  $\sqrt{2}(-1 \pm i)/2$ .



## LXXX. Page 491

1.  $4, -2 \pm i\sqrt{3}$ .
2.  $8, (1 \pm i\sqrt{3})/2$ .
3.  $\sqrt[3]{A} + \sqrt[3]{B}, \omega \sqrt[3]{A} + \omega^2 \sqrt[3]{B}, \omega^2 \sqrt[3]{A} + \omega \sqrt[3]{B}$ ,  
where  $A = 2 + \sqrt{3}, B = 2 - \sqrt{3}$ .
4.  $A = 3/4 + \sqrt{1887/72}, B = 3/4 - \sqrt{1887/72}$ .
5.  $-1 + \sqrt[3]{A} + \sqrt[3]{B}, -1 + \omega \sqrt[3]{A} + \omega^2 \sqrt[3]{B}, -1 + \omega^2 \sqrt[3]{A} + \omega \sqrt[3]{B}$ ,  
where  $A = 4 + 2\sqrt{6}, B = 4 - 2\sqrt{6}$ .
6.  $1 + \sqrt[3]{A} + \sqrt[3]{B}, 1 + \omega \sqrt[3]{A} + \omega^2 \sqrt[3]{B}, 1 + \omega^2 \sqrt[3]{A} + \omega \sqrt[3]{B}$ ,  
where  $A = -5/2 + 5\sqrt{749/54}, B = -5/2 - 5\sqrt{749/54}$ .
7.  $(-\sqrt{3} \pm \sqrt{4\sqrt{3}-5})/2, (\sqrt{3} \pm i\sqrt{5+4\sqrt{3}})/2$ .
8.  $(1 \pm \sqrt{5})/2, (3 \pm \sqrt{13})/2$ .
9.  $-1 \pm \sqrt{2}, 1 \pm 2i$ .
10.  $(-5 \pm \sqrt{33})/2, (-3 \pm \sqrt{13})/2$ .
11.  $(1 \pm \sqrt{3}i)/2, (-1 \pm \sqrt{3}i)/2, (1 \pm 2\sqrt{2}i)/3$ .
12.  $\pm i, (1 \pm \sqrt{3}i)/2, (3 \pm \sqrt{5})/2, (1 \pm \sqrt{15}i)/4$ .
13.  $1, 1, -1, (-1 \pm \sqrt{15}i)/4, (-1 \pm \sqrt{35}i)/6$ .
14.  $z^3 + z^2 - 2z - 1 = 0$ .
15.  $(2a^2 - 3ab + c)^2/4 + (b - a^2)^3 \geq 0$ .
16.  $\cos \frac{2k\pi}{5} + i \sin \frac{2k\pi}{5}, k = 0, \dots, 4; \cos \frac{(2k+1)\pi}{6} + i \sin \frac{(2k+1)\pi}{6}$ ,  
 $k = 0, \dots, 5$ .
17. (1)  $2 \cos 20^\circ, 2 \cos 140^\circ, 2 \cos 260^\circ$ .  
(2)  $2\sqrt{2} \cos 15^\circ, 2\sqrt{2} \cos 135^\circ, 2\sqrt{2} \cos 255^\circ$ .
18. 3.
19. Radius of base is  $2\frac{1}{2}$ , altitude, 8.
20. The problem as it stands has no solution. This is indicated by the fact that if we attempt to solve for the altitude, we obtain a negative result. It can be shown that the volume of the greatest cylinder which can be inscribed in a right circular cone is four ninths that of the cone. Let the student prove that in the case of the cone described in the example the altitude of this greatest cylinder is 2.

## LXXXI. Page 497

1.  $p^3 - p(q^2 + r^2 + s^2) + 2qrs$ .
2.  $(b - c)x + (c - a)y + (a - b)z$
3.  $p(p^2 + q^2 + r^2 + s^2)$ .
4. 0.
5. 18.
6. 74.
7. -30.
10. (1)  $a_1b_2c_3d_4 - a_2b_1c_3d_4$ . (2)  $a_1b_2c_3d_4 - a_1b_3c_2d_4$ . (3)  $-a_2b_3c_4d_1$ .  
(4)  $a_1b_2c_3d_4 + a_1b_3c_4d_2 + a_1b_4c_2d_3 - a_1b_4c_3d_2 - a_1b_3c_2d_4 - a_1b_2c_4d_3$ .  
(5)  $a_1b_2c_3d_4 + a_2b_4c_3d_1 + a_4b_1c_3d_2 - a_4b_2c_3d_1 - a_2b_1c_3d_4 - a_1b_4c_3d_2$ .
11.  $a_2b_4c_3d_1e_5, -a_4b_2c_1d_3e_5, a_3b_4c_3d_2e_1, -c_1d_2a_3e_4b_5, -c_1b_2e_3a_4d_5, -d_3a_2e_4b_1c_5$ .

**LXXXII. Page 501**

1. (1)  $-22,680$ . (2)  $0$ . (3)  $4abcdef$ .

**LXXXIII. Page 507**

1.  $0$ .                      2.  $-4$ .                      3.  $0$ .                      4.  $-357,840$ .
5. 
$$\begin{vmatrix} 0 & bc - a^2 & b^2 - ac \\ b^2 - ac & 0 & bc - a^2 \\ bc - a^2 & b^2 - ac & 0 \end{vmatrix}$$
.
6. 
$$\begin{vmatrix} ap + cr & ap & cr \\ ap & ap + bq & bq \\ cr & bq & bq + cr \end{vmatrix}$$
.
7. 
$$\begin{vmatrix} a & -a & a^2 + ab & ac + ad \\ -b & b & ab + b^2 & bc + bd \\ c & c & -ac + bc & -c^2 + cd \\ d & d & ad - bd & cd - d^2 \end{vmatrix}$$
.
8. 
$$\begin{vmatrix} l^2 + m^2 + n^2 & lm + mn + nl & ln + ml + nm \\ ml + nm + ln & m^2 + n^2 + l^2 & mn + nl + lm \\ nl + lm + mn & nm + ln + ml & n^2 + l^2 + m^2 \end{vmatrix}$$
.
10. This exercise should read: "Prove that a determinant reduces to its leading term when all the elements at either side of the leading diagonal are zero."

**LXXXIV. Page 511**

1.  $x = 10/7, y = 1, z = 4/7$ .                      2.  $x = 1, y = 1/2, z = 1/3$ .
3.  $x = \frac{d(d-b)(d-c)}{a(x-b)(a-c)}, y = \frac{d(d-c)(d-a)}{b(b-c)(b-a)}, z = \frac{d(d-a)(d-b)}{c(c-a)(c-b)}$ .
4.  $x = 1, y = 1/2, z = 1/3, t = -1$ .                      5.  $x : y : z = -1 : 1 : 1$ .
6.  $x : y : z = k : l : -1$ , if  $a_1b_2 - a_2b_1 \neq 0$ .
7.  $\lambda = 0$  or  $-3 \pm 2\sqrt{21}$ .

**LXXXV. Page 519**

1. Common root is  $-3/2$ .
2. 
$$\begin{vmatrix} a_0 & a_1 & a_2 & 0 \\ 0 & a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 & 0 \\ 0 & b_0 & b_1 & b_2 \end{vmatrix}$$
.
3.  $(a+d)^3 + b^3 + c^3 - 3bc(a+d)$ .
4. (1)  $4p^3 + 27q^2$ . (2)  $c(4b^3 + 27a^2c)$ .                      5. The double root is  $-2$ .
6.  $x, y = 0, 0; 3, -1; -2, 2$ ; and one infinite solution.

## § 950. Page 527. Ex. 3

- (1)  $\frac{1}{2 \cdot 3} + \frac{3}{3 \cdot 4} + \frac{5}{4 \cdot 5} + \frac{7}{5 \cdot 6} + \cdots$ ; divergent.  
 (2)  $\frac{1}{2} + \frac{\sqrt{2}}{5} + \frac{\sqrt{3}}{10} + \frac{2}{17} + \cdots$ ; convergent.  
 (3)  $\frac{1}{9} + \frac{3}{35} + \frac{5}{91} + \frac{7}{189} + \cdots$ ; convergent.

## § 952. Page 528

5. When  $x < 1$ .6. When  $x > 1$ .

## LXXXVI. Page 530

1. Convergent.                    2. Convergent.                    3. Divergent.  
 4. Convergent.                    5. Divergent.                    6. Divergent.  
 7. Convergent.                    8. Convergent.                    9. Divergent.  
 10.  $\frac{2}{1 \cdot 3} + \frac{3}{2 \cdot 4} + \frac{4}{3 \cdot 5} + \frac{5}{4 \cdot 6}$ ; divergent.  
 11.  $\frac{1}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}}{\sqrt[3]{9}} + \frac{\sqrt[3]{3}}{\sqrt[3]{28}} + \frac{\sqrt[3]{4}}{\sqrt[3]{65}}$ ; divergent.  
 12.  $(\sqrt{2} - 1) + (\sqrt{5} - 2) + (\sqrt{10} - 3) + (\sqrt{17} - 4)$ ; divergent.  
 13. Convergent.                    14. Divergent.  
 15. When  $x < 1$ .                    16. When  $x < 1$ .

## LXXXVII. Page 534

1. (1) Convergent. (2) Divergent. (3) Convergent.  
 2. (1) Convergent for all real values of  $x$  except  $x = 0, 1, -1/2, 1/3, \dots, (-1)^{n-1}/n, \dots$ .  
 (2) Convergent when  $x$  is less than 1, otherwise it is divergent.  
 3. Should read: "If the series  $u_1 + u_2 + u_3 + \cdots$  is *absolutely* convergent, etc."

## LXXXVIII. Page 538

1.  $\infty$ .                                    2.  $1/2$ .                                    3.  $1/2$ .  
 4. For  $x = -1$  and for all values of  $x$  between  $-1$  and  $2$ .  
 5. For all real values of  $x$ .  
 6. For values of  $x$  which are greater than  $1/3$  and less than  $1/2$ .

**LXXXIX. Page 551**

4.  $a_1 = -1/2$ ,  $a_2 = 7/8$ ,  $a_3 = 7/16$ ,  $a_4 = -21/64$ .
5. (1)  $2 - x/4 - x^2/32 - 5x^3/768$ . (2)  $1 + 3x/2 - 9x^2/8 - 13x^3/16$ .
6. (1)  $2 - 3x - 3x^2 + 20x^3 + \dots$ . (2)  $x + 6x^2 + 4x^3 - x^4 + \dots$ .
7. (1)  $3x^2 - 2x^3 - x^4 + 3x^5 + \dots$ . (2)  $x^{-2} - 2x^{-1} + 1 + 9x + \dots$ .
8. (1)  $-11/6 + 5x/36 - 89x^2/216 + 65x^3/1296 - 761x^4/7776 + \dots$ ;  
converges when  $|x| < 2$ .
- (2)  $2 - 11x/3 + 46x^2/9 - 173x^3/27 + 616x^4/81 + \dots$ ; converges  
when  $|x| < 1$ .
9. (1)  $x^{-1} + x^{-2} + 7x^{-3} + 4x^{-4} + \dots$ ; converges when  $|x| > 3$ .
- (2)  $1 - x^{-1} + x^{-4} - x^{-6} + \dots$ .
10. (1)  $x = y - y^2 + y^3 - y^4 + \dots$ . (2)  $x = y + y^2/2 + y^3/6 + y^4/24 + \dots$ .
11.  $x = (y - 1) - (y - 1)^2/2 + (y - 1)^3/6 - (y - 1)^4/24 + \dots$ .
12.  $x = y^{1/2} - 3y/2 + 45y^{3/2}/8 - 27y^2 + \dots$ .
13. (1)  $y = 3x - 10x^2 + 60x^3 + \dots$ .
- (2)  $y = x^2 + x^5 + 3x^8 + \dots$ .  $y = x^{1/2} - x^2/2 - 3x^{3/2}/8 + \dots$ .  
 $y = -x^{1/2} - x^2/2 + 3x^{3/2}/8 + \dots$ .

**XC. Page 559**

1.  $\log_e 4 = 1.3862$ .  $\log_e 5 = 1.6093$ . 7.  $-7x^4/3^5 \cdot 2^{10}$ . 8.  $231x^3/2^{10}$
9. The first series converges when  $|x| < 3/2$ , the second when  $|x| < 3$ .
10.  $1 - 3x/4 + 45x^2/32 + 43x^3/128 - 333x^4/2048$ .
11.  $4/3 + 13x/27 + 53x^2/1296$ ; converges when  $|x| < 8/3$ .
12. (1)  $4/3$ . (2)  $\infty$ .
15.  $x + x^2/2 - 2x^3/3 + x^4/4 \dots$ ; converges when  $|x| < (\sqrt{5} - 1)/2$ .

**XCI. Page 563**

1.  $-19x^3 + 52x^4$ .
2. (1) Scale is  $a_n - a_{n-1} + a_{n-2} = 0$ ; terms are  $-2x^5 + x^6$ .
- (2) Scale is  $a_n + 2a_{n-1} + 3a_{n-2} = 0$ ; terms are  $31x^5 + 16x^6$ .
- (3) Scale is  $a_n + 3a_{n-1} + 3a_{n-2} + a_{n-3} = 0$ ; terms are  $28x^6 - 36x^7$ .
3. (1) Generating function is  $(2 - x)/(1 - x - 2x^2)$ ; general term is  
 $[2^n + (-1)^n]x^n$ .
- (2) Generating function is  $(3 - 8x)/(1 - 5x + 6x^2)$ ; general term is  
 $(2^{n+1} + 3^n)x^n$ .
5. Generating function is  $(1 + x + 4x^2)/(1 - x - 5x^2 - 3x^3)$ ; general  
term is  $[3^n + (-1)^n n]x^n$ .
6. Generating function is  $[a - (a - d)x]/(1 - 2x + x^2)$ .
8. Sum is  $2/(1 - 3x + 3x^2 - x^3)$ .

**XCII. Page 565**

- (1)  $x < 1$ . (2)  $x < \infty$ . (3)  $x < 3$ .

## XCIII. Page 575

1.  $\frac{3}{1}, \frac{13}{4}, \frac{16}{5}, \frac{93}{29}$ .
2.  $\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{4}{7}, \frac{41}{72}, \frac{496}{871}$ .
3.  $\frac{1}{1} + \frac{1}{5}$ .
4.  $8 + \frac{1}{6} + \frac{1}{4} + \frac{1}{2}$ .
5.  $\frac{1}{3} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{1} + \frac{1}{4}$ .
6.  $3 + \frac{1}{1} + \frac{1}{1} + \frac{1}{5} + \frac{1}{1} + \frac{1}{3}$ .
7.  $\frac{1}{6} + \frac{1}{1} + \frac{1}{6} + \frac{1}{3} + \frac{1}{9} + \frac{1}{7}$ .
8.  $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{4} + \frac{1}{2}$ ; fourth convergent is  $\frac{104}{79}$ ; error  $< \frac{1}{79^2}$ .  
(Exact error is  $1/79.177$ .)
9.  $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ ; fourth convergent is  $\frac{13}{30}$ ; error  $< \frac{1}{30^2}$ .
10.  $\frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{4} + \frac{1}{19} + \frac{1}{3} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{5}$ ; fourth convergent is  $\frac{3}{4}$ ; error  $< 1/4 \cdot 19$ .
11.  $4 + \frac{1}{8} + \frac{1}{8} + \dots$ ; fifth convergent is  $\frac{17684}{4289}$ ; error  $< \frac{1}{4289^2}$ .
12.  $5 + \frac{1}{10} + \frac{1}{10} + \dots$ ; fifth convergent is  $\frac{52525}{10301}$ ; error  $< \frac{1}{10301^2}$ .
13.  $2 + \frac{1}{2} + \frac{1}{4} + \dots$ ; fifth convergent is  $\frac{218}{89}$ ; error  $< \frac{1}{89^2}$ .
14.  $6 + \frac{1}{6} + \frac{1}{12} + \dots$ ; fifth convergent is  $\frac{33294}{5401}$ ; error  $< \frac{1}{5401^2}$ .
15.  $10 + \frac{1}{4} + \frac{1}{20} + \dots$ .
16.  $\frac{1}{4} + \frac{1}{1} + \frac{1}{3} + \frac{1}{1} + \frac{1}{8} + \dots$ .
17.  $4 + \frac{1}{2} + \frac{1}{1} + \frac{1}{3} + \frac{1}{1} + \frac{1}{2} + \frac{1}{8} + \dots$ .
18.  $8 + \frac{1}{2} + \frac{1}{2} + \frac{1}{1} + \frac{1}{7} + \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{16} + \dots$ .
19.  $5 + \frac{1}{5} + \frac{1}{10} + \dots$ .
20.  $\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \dots$ .
21.  $5 + \frac{1}{1} + \frac{1}{4} + \dots$ .
22.  $(\sqrt{37} - 4)/3$ .
23.  $(\sqrt{37} - 5)/3$ .
24.  $\frac{3\sqrt{35} + 50}{\sqrt{35} + 15}$ .
25.  $\frac{\sqrt{1806} + 36}{34}$ .
26.  $\frac{20 + \sqrt{10}}{43 + 2\sqrt{10}}$ .
30.  $\frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \dots$ .
33.  $577/408$ ; error  $< 1/408^2$ .
34.  $355/113$ .
35.  $87/32$ ; error  $< 1/32^2$ .
36.  $x = 546, y = 324$ .
37.  $x = 1350, y = -770$ .
38.  $x = 155 + 323t, y = 248 + 517t$ .

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