

REPRESENTATION THEORY
OF FINITE GROUPS
AND ASSOCIATIVE ALGEBRAS

CHARLES W. CURTIS
IRVING REINER

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American Mathematical Society • Providence, Rhode Island



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Preface

Representation theory is the study of concrete realizations of the axiomatic systems of abstract algebra. It originated in the study of permutation groups, and algebras of matrices. The theory of group representations was developed in an astonishingly complete and useful form by Frobenius in the last two decades of the nineteenth century. Both Frobenius and Burnside realized that group representations were sure to play an important part in the theory of abstract finite groups. The first book to give a systematic account of representation theory appeared in 1911 (Burnside [4]) and contained many results on abstract groups which were proved using group characters. Perhaps the most famous of these is Burnside's theorem that a finite group whose order has at most two distinct prime divisors must be solvable. Recently, a purely group-theoretic proof of Burnside's theorem has been obtained by Thompson. The new proof is of course important for the structure theory of groups, but it is at least as complicated as the original proof by group characters.

The second stage in the development of representation theory, initiated by E. Noether [1] in 1929, resulted in the absorption of the theory into the study of modules over rings and algebras. The representation theory of rings and algebras has led to new insights in the classical theory of semi-simple rings and to new investigations of rings with minimum condition centering around Nakayama's theory of Frobenius algebras and quasi-Frobenius rings.

Another major development in representation theory is R. Brauer's work on modular representations of finite groups. Like the original work of Frobenius, Brauer's theory has many significant applications to the theory of finite groups. At the same time it draws on the representation theory of algebras and suggests new problems on modules and rings with minimum condition. It also emphasizes the fundamental importance of number-theoretical questions in group theory and representation theory.

During the past decade there has been increased emphasis on integral representations of groups and rings, motivated to some extent by questions arising from homological algebra. This theory of integral representations has been a fruitful source of problems

and conjectures both in homological algebra and in the arithmetic of non-commutative rings.

The purpose of this book is to give, in as self-contained a manner as possible, an up-to-date account of the representation theory of finite groups and associative rings and algebras. This book is not intended to be encyclopedic in nature, nor is it a historical listing of the entire theory. We have instead concentrated on what seem to us to be the most important and fruitful results and have included as much preliminary material as necessary for their proofs.

In addition to the classical work given in Burnside's book [4], we have paid particular attention to the theory of induced characters and induced representations, quasi-Frobenius rings and Frobenius algebras, integral representations, and the theory of modular representations. Much of this material has heretofore been available only in research articles. We have concentrated here on general methods and have built the theory solidly on the study of modules over rings with minimum condition. Enough examples and problems have been included, however, to help the research worker who needs to compute explicit representations for particular groups. We have included some applications of group representations to the structure theory of finite groups, but a definitive account of these applications lies outside the scope of this book. In Section 92 we have given a survey of the present literature dealing with these applications and have included in this book all the representation-theoretic prerequisites needed for reading this literature, though not all the purely group-theoretic background which might be necessary.

No attempt has been made to orient the reader toward physical applications. For these we may refer the reader to recent books and articles dealing with that part of group theory relevant to physics, and in particular to Wigner [1], Gelfand-Sapiro [1], Lomont [1], and Boerner [1].

It has also been necessary to omit the vast literature on representations of the symmetric group. Fortunately the reader is now able to consult the excellent book on this topic by Robinson [1].

Many of the results on group representations have been generalized to infinite groups and also to infinite-dimensional representations of topological groups. We have felt that these generalizations do not properly fall within the scope of this book and, in fact, would require a lengthy separate presentation.

The book has been written in the form of a textbook; a preliminary

version has been used in several courses. We have assumed that the reader is familiar with the following topics, which are usually treated in a “standard” first-year graduate course in algebra: elementary group theory, commutative rings, elementary number theory, rudiments of Galois theory, vector spaces, and linear transformations.

We are confident that the expert as well as the student will find something of interest in this book. We offer no apology, however, for writing to be understood by a reader unfamiliar with the subject. In keeping with this objective, we have not always presented results in their greatest generality, and we have included details which will sometimes seem tedious to the experienced reader. After serious deliberation, we decided not to introduce the full machinery of homological algebra. Although it would have simplified several sections of the book, we felt that many readers were not likely to be well-grounded in homological algebra, and this book was not intended to be a first course in the subject.

The first three chapters are written at the level of a first-year graduate course and include introductory material as well as background for later chapters. Much of this material may be skimmed rapidly or omitted entirely at a first reading, though Sections 9–13 should be read with care.

Chapters IV–VII form a unit containing the structure theory of semi-simple rings with minimum condition, and the applications of this theory to group representations and characters.

Chapters IV, VIII, IX, and X form a unit on rings with minimum condition and finite-dimensional algebras. Chapter IV develops the theory of the radical and semi-simplicity by the perhaps old-fashioned method of calculations with idempotents, because idempotents furnish the main tool in the study of non-semi-simple rings and algebras in Chapters VIII and IX.

Chapters III and XI form a more or less self-contained account of algebraic number theory and integral representations of groups. Some knowledge of earlier chapters is needed, especially in Sections 77–78.

Chapter XII is devoted to the theory of modular representations and requires a knowledge of parts of all the preceding chapters. The exact prerequisites for reading Chapter XII are given at the beginning of the chapter.

For the reader whose main interest is in representations of finite groups, we may suggest the following sections for a first brief reading: 9–13, 23–27, 30–34, 38–40, 43–46, 49–50, 54–55, 61, 82–92.

These sections are to some extent self-contained, provided that the reader is willing to postpone to the second reading the proofs of some of the results needed from other sections.

Exercises are included at the end of almost every section. Some provide easy checks on the reader's comprehension of the text; others are intended to challenge his abilities. Many are important results in their own right and may occasionally be referred to when needed in later sections.

Sections are numbered consecutively throughout the book. A cross reference to (a.b) refers to Section a and to the bth numbered item in that section.

There is a fairly large bibliography of works which are either directly relevant to the text or offer supplementary material of interest. An attempt has been made to give credit for some of the major methods and theorems, but we have stopped far short of trying to trace each theorem to its source.

We are indebted to many persons and organizations for assisting us with this work. Our students, friends, colleagues, and families have listened to us lecture on these subjects, read portions of the manuscript and proof sheets, made suggestions and corrections, and given us encouragement. We are deeply appreciative of their kind help. Our interest in this subject was stimulated by a seminar conducted at the Institute for Advanced Study in 1954-1955. We are indebted to the participants in that seminar for their help and to the Institute for making possible the preparation of mimeographed seminar notes. It is a pleasure to acknowledge the generous support we have received for the work on this book from the Office of Naval Research. Finally we are grateful to Interscience Publishers for publishing it and giving us their patient and friendly cooperation.

Charles W. Curtis
Irving Reiner

June 1962

Contents

Notation	xiii
I. Background from Group Theory	1
1. Permutation Groups and Orbits	1
2. Subgroups and Factor Groups	3
3. Conjugate Classes	8
4. Abelian Groups	10
5. Solvable and Nilpotent Groups	14
6. Sylow Subgroups.....	17
7. Semi-direct Products	21
II. Representations and Modules	25
8. Linear Transformations	26
9. Definitions and Examples of Representations	30
10. Representations of Groups and Algebras	38
11. Modules	50
12. Tensor Products	59
13. Composition Series.....	76
14. Indecomposable Modules	81
15. Completely Reducible Modules	86
III. Algebraic Number Theory	91
16. Modules over Principal Ideal Domains	91
17. Algebraic Integers	102
18. Ideals	107
19. Valuations; P -adic Numbers	115
20. Norms of Ideals; Ideal Classes	123
21. Cyclotomic Fields	135
22. Modules over Dedekind Domains	144
IV. Semi-simple Rings and Group Algebras	157
23. Preliminary Remarks	157
24. The Radical of a Ring with Minimum Condition	159
25. Semi-simple Rings and Completely Reducible Modules	163
26. The Structure of Simple Rings	173
27. Theorems of Burnside, Frobenius, and Schur	179
28. Irreducible Representations of the Symmetric Group ..	190

29.	Extension of the Ground Field	198
V.	Group Characters	207
30.	Introduction	207
31.	Orthogonality Relations	217
32.	Simple Applications of the Orthogonality Relations....	224
33.	Central Idempotents	233
34.	Burnside's Criterion for Solvable Groups	239
35.	The Frobenius-Wielandt theorem on the Existence of Normal Subgroups in a Group	241
36.	Theorems of Jordan, Burnside, and Schur on Linear Groups.....	250
37.	Units in a Group Ring.....	262
VI.	Induced Characters	265
38.	Introduction	265
39.	Rational Characters	279
40.	Brauer's Theorem on Induced Characters	283
41.	Applications	292
42.	The Generalized Induction Theorem	301
VII.	Induced Representations	313
43.	Induced Representations and Modules	314
44.	The Tensor Product Theorem and the Intertwining Number Theorem	323
45.	Irreducibility and Equivalence of Induced Modules ...	328
46.	Examples: The Tetrahedral and Octahedral Groups ..	329
47.	Applications: Representations of Metacyclic Groups ..	333
48.	A Second Application: Multiplicity-free Representations	340
49.	The Restriction of Irreducible Modules to Normal Subgroups	342
50.	Imprimitive Modules.....	346
51.	Projective Representations	348
52.	Applications	355
53.	Schur's Theory of Projective Representations	358
VIII.	Non-Semi-Simple Rings	367
54.	Principal Indecomposable Modules	367
55.	The Classification of the Principal Indecomposable Modules into Blocks	377
56.	Projective Modules.....	380

57. Injective Modules	384
58. Quasi-Frobenius Rings	393
59. Modules over Quasi-Frobenius Rings	403
IX. Frobenius Algebras	409
60. Injective Modules for a Finite-Dimensional Algebra ..	409
61. Frobenius and Quasi-Frobenius Algebras	413
62. Projective and Injective Modules for a Frobenius Algebra	420
63. Relative Projective and Injective Modules	426
64. Group Algebras of Finite Representation Type.....	431
65. The Vertex and Source of an Indecomposable Module	435
66. Centralizers of Modules over Symmetric Algebras ...	440
67. Irreducible Tensor Representations of $GL(V)$	449
X. Splitting Fields and Separable Algebras	453
68. Splitting Fields for Simple Algebras and Division Algebras	453
69. Separable Extensions of the Base Field	459
70. The Schur Index.....	463
71. Separable Algebras.....	480
72. The Wedderburn-Malcev Theorem	485
XI. Integral Representations.....	493
73. Introduction	494
74. The Cyclic Group of Prime Order	506
75. Modules over Orders.....	515
76. P -Integral Equivalence	531
77. Projective Modules: Local Theory.....	542
78. Projective Modules: Global Theory	550
79. The Jordan-Zassenhaus Theorem	558
80. Order Ideals	563
81. Genus	567
XII. Modular Representations	583
82. Introduction	584
83. Cartan Invariants and Decomposition Numbers	590
84. Orthogonality Relations	598
85. Blocks	604
86. The Defect of a Block	611
87. Defect Groups	618
88. Block Theory for Groups with Normal P -Subgroups ..	627

89.	Block Distribution of Classes.....	635
90.	Miscellaneous Topics.....	638
	A. Generalized Decomposition Numbers	638
	B. Conjugate Characters	641
	C. The Number of Characters Belonging to a Block..	643
	D. Numerical Bounds	645
91.	Examples	646
92.	Literature on Applications to Group Theory	650
	A. Groups of a Given Order	651
	B. Characterizations of Simple Groups	652
	C. Criteria for Existence of Normal Subgroups	654
	Bibliography	655
	Index	673

Notation

Z	= ring of rational integers
Q	= rational field
$a b$: a divides b where $a, b \in Z$
$a \nmid b$: a does not divide b , ($a, b \in Z$)
$p^n a$: $p^n a$ but $p^{n+1} \nmid a$ where p is prime and $a \in Z$
G.C.D.	: greatest common divisor
L.C.M.	: least common multiple

Group theory notation

$[G : 1]$	= number of elements in G
$[G : H]$	= number of distinct left cosets of H in G
$H \triangleleft G$: H is a normal subgroup of G
$[x]$	= cyclic group generated by x
S_n	= symmetric group on n symbols
$P(X)$	= group of all permutations of a set X
$C(G)$	= center of the group G
$C(x)$	= centralizer of x in G
$C_G(H)$	= centralizer of H in G
$N(H)$	= normalizer H in G
$A(G)$	= group of automorphisms of G
$I(G)$	= group of inner automorphisms of G
$G_1 \times G_2$	= direct product of G_1 and G_2
a_L	: denotes the left multiplication: $x \rightarrow ax$
$G_1 \cong G_2$: G_1 is isomorphic to G_2
$[G, G]$	= commutator subgroup of G

Notations from field theory

char K	= characteristic of the field K
Irr (α, K)	= minimal polynomial of α over K
alg. int. $\{K\}$	= ring of all algebraic integers in K

Notations from module theory and linear algebra

${}_R R$	= left regular R -module
$M \dot{+} N$: external direct sum

$M \oplus N$: internal direct sum
$\Sigma \oplus M_i$: direct sum
A.C.C.	: ascending chain condition
D.C.C.	: descending chain condition
$f N$: restriction of f to the subset N of the domain of f
$\text{Hom}_R(M, N)$	= additive group of R -homomorphisms of M into N
D_n	= ring of all $n \times n$ matrices with entries in D
$(M:K)$	= dimension of M over K
tX	= transpose of the matrix X
$ X $	= $\det X$ = determinant of the matrix X
$\mathbf{0}$	= zero matrix
I	= identity matrix
$\text{diag}\{a_1, \dots, a_n\}$	= diagonal matrix with diagonal entries a_1, \dots, a_n
$GL_n(K)$	= group of invertible matrices in K_n , K = field
$GL(M)$	= group of invertible elements in $\text{Hom}_K(M, M)$, M = vector space over field K
$M \otimes_R N$	= tensor product of right R -module M and left R -module N
$T \times U$	= Kronecker product of matrices
M^g	= induced module
T^g	= induced representation

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Index

A

Abelian groups, 10-13
Abelian group of ideal classes, 112
Abelian normal subgroup of G , 236
Absolute irreducibility, 202
Absolutely irreducible KG -modules, 214-215, 292
Algebra over a field, 43
 tensor products of, 71-73
Algebraic integers, 102-107
 definition of, 103
Algebraic number, norm of, 131-133
Algebraic number theory, 91-154
Algebras, over algebraically closed fields, 203
 Frobenius, 409-452
 representations of, 38-49
Algorithm used in S_n , 197
Annihilator, 97, 395
Artin theorem of induced characters, 279-282, 550
Ascending central series, 15
Ascending chain condition, 55
Associative non-degenerate bilinear form, 415, 424
Associativity of the tensor product, 67
Automorphism, 5
Averaging over a finite group, 42, 420

B

Balanced map, 60-61
Base field, separable extensions of, 459-463
Basis, 52, 381

B -equivalent modules, 568-570
Bilinear mapping, definition of, 59
Binding function, 504, 520
 definition of, 500
Binding system, 500
Blichfeldt's theorem, 348, 356
Block, 378, 604-611
 belonging to the same, 379-380
 defect of, 611-618
 of defect 0, 611-613
 direct sum of, 379
 of KG , 604
 number of characters belonging to, 643-645
Block distribution of classes, 635-638
Block idempotents, 604
Block theory for groups with normal p -subgroups, 627-635
Bounded representation type, definition of, 431
Brauer character, 588
 of T , 589
Brauer-Feit theorem, 643-645
Brauer-Nesbitt theorem, 215-216, 585-586, 611-614
Brauer-Suzuki theorem, 276-278
Brauer theorem, 591-592, 602, 642-643
Brauer theorem on induced characters, 283-292, 294
Brauer theorem on splitting fields, 292-295
 B -source, definition of, 438
Burnside's criterion for solvable groups, 239-241
Burnside theorem, 182, 232-233
 analogue of, 233
Burnside theorem of linear groups, 251-252

C

- Canonical homomorphism, 6
- Canonical invariants determined uniquely by M , 513
- Cartan invariants, 590-598
- Cartan matrix, 593
- Cauchy sequence, 119
- Center of the group, 6
- Central idempotents, 172, 233-239
- Centralizer, 8
- Centralizer of H in G , 10
- Centralizer of an R -module, 53
- Centralizers of modules over symmetric algebras, 440-448
- Central series, ascending, 15
- Character,
 - afforded by a module, 209-210
 - of a direct sum, 211
 - of G constant on S , 243
- Characteristic polynomial,
 - of A , 207-209
 - of a matrix, 207
- Characteristic roots, 207
- Characteristic subgroup, 7-8
- Characters, exceptional, 275
- Characters, generalized, 272
 - linear, 272
 - one-dimensional, 272
- Characters, induced, 265-311
- Characters, product of, 211
- Characters, rational, 279-283
 - definition of, 279
- Character table of a group, as determined by constants, 237-238
- Character tables, 224-228
 - of A_4 , 226
 - of A_5 , 238
 - of S_3 , 225-226
 - of S_4 , 226-228
- Chinese remainder theorem, 112-113, 303
- Class equation, 8
- Classes of finite-dimensional algebras over K , 440
- Class function, 210, 241, 266
- Class number of a field, 126
- Clifford's theorem, 343-344
- Coboundaries, definition of, 521
- Cocycles, definition of, 521
- Cohomology group, definition of, 521
- Cohomology theory of associative algebras, 486
- Column permutations, 191, 193-194, 197
- Commutative law for finite skewfields, 458-459
- Commutator subgroup, 15
- Commuting unitary matrices, 256, 257-259
- Complete field with respect to a valuation, 120
- Completely primary ring, 371-372
- Completely reducible matrix representation, 40
- Completely reducible modules, 86-90, 163-173
 - definition of, 86
- Completely reducible representation, 39
- Completely reducible set of linear transformations, 254-255
- Component of M , definition of, 409
- Composition factors, 15, 77, 375, 378-379
- Composition factors of M , sum of the characters of, 211
- Composition series, 14-15, 76-81, 368, 373-374
 - definition of, 77
 - equivalent, 77
 - length, 77

- Conjugate character, 304, 630, 641-643
 p -, 641
 Conjugate classes, 8-10, 190
 of G , 187
 in S_n , 10
 K -Conjugate elements, 306
 Conjugate ideal, 125
 Conjugate representation, 471
 Content of a polynomial, 106-107, 118
 Contragredient module, 318
 Contragredient representation, 318
 Convergent sequence, 120
 Coordinate functions, 182
 Cyclic group, 34-35
 of prime order, 506-515
 Cyclic module, 52, 97-99
 Cyclotomic fields, 135, 136, 137, 138-144, 265
 Cyclotomic polynomial, 137
 Cyclic p -Sylow subgroup, 431
- D**
- Decomposable G -module, 519
 Decomposable module, definition of, 81, 496-497
 Decomposition matrix, definition of, 591
 Decomposition numbers, 590, 591, 592, 593, 594-598
 definition of, 591
 Dedekind domain, 108
 modules over, 144-155
 Defect of a block, 611-618
 Defect of a class, 618
 Defect group of a block, 618, 623
 Defect group of a class, 618
 Defect groups, 618-634
 Degree of μ , 272
 Degree of a representation, 30
 Degrees of the irreducible projective representations of G , 364
 Degrees of the irreducible representations of a finite group G , 236
 de Leeuw-Reiner theorem, 529-530
 Derived series, 16
 Descending chain condition, 55
 Determinantal divisor, 96
 Determinant ideal, 566
 Determinant of a linear transformation, 29
 Deuring-Noether theorem, 200, 538
 Diagonal subgroup, 315
 Diagram, 191, 197
 Dickson corollary, 439-440, 601
 Diederichsen-Reiner theorem, 508, 509, 510-514
 Dihedral group, 22, 334, 339, 342
 D_6 of order 12, 646-648
 of order 10, 238
 Direct product
 external, 11
 internal, 11
 of matrices, 69
 Discrete valuation, 116
 Discriminant of a field, 143
 Disjoint cycles, 2-3
 Disjoint modules, 328
 Disjoint orbits, 337
 Divisible module, 386-387
 Division algebra, 180
 Double centralizer property, 175-177, 403, 405
 Double centralizer theorem, 405-406
 Double cosets, 4-5
 Doubly transitive, 7
 Doubly transitive permutation group, 230-231
 Dual bases, 423

Dual module, 317, 394
 Dual space, 316-317

E

Eckmann-Schöpf theorem, 390-391
 Eisenstein criterion, 140
 Elementary divisors, 599
 of G , 12
 of a matrix, 101-102
 Elementary divisor theorem, 12-13,
 264, 403
 Elementary subgroup, 284, 302, 476
 Enveloping algebra, 464
 definition of, 43
 Equivalence
 of induced modules, 328-329
 integral, 494, 515
 P -integral, 531-542
 of a representation, 30
 Equivalent composition series, 77
 Equivalent ideals, 125-126
 Equivalent matrices, 94
 Equivalent representations, 515
 definition of, 494
 Equivalent valuations, 117
 Exact sequence, 381
 Exceptional characters, 275
 Existence of normal subgroups in a
 group, 241-250
 Exponent of a group, 37, 292
 Extension, 381
 of an algebra, 488
 of a ground field, 198-206
 of a group, 23
 of an ideal, 133-134
 of a module, 486
 External direct product, 11
 External direct sum, 11, 52

F

Factor commutator group, 16
 Factor groups, 3-8
 Factor module, 53
 Factor set, 349, 488
 Factors of the normal series, 14
 Faithful irreducible representation,
 232
 Faithful module, 178
 Faithful representation, 279
 Field, class number of, 126
 Field of P -adic numbers, 121
 Field of scalars of a vector space,
 extension of, 70-71
 Finite abelian groups 10-13, 36-38
 Finite-dimensional algebra, 188
 over an algebraically closed field,
 181-182
 injective modules for, 409-413
 Finite dimensional over K , 157-158
 Finite group, representation of, 30
 Finitely generated module, 52, 381
 Finitely generated periodic subgroup,
 252-254
 Finite nilpotent group, 356
 Finite periodic subgroup, 251-252
 Finite representation type
 definition of, 431
 group algebras of, 431-435
 Finite subgroups of rotation group O_3
 in three-dimensional Euclidean
 space, 329-330
 First regular representation, 413
 Fractional ideal, 107, 125-126, 146, 150
 Free (left) R -module, 52
 Frobenius algebras, 367, 409-452, 459-
 463
 definition of, 413
 dual bases of, 423

- generalization of the orthogonality relations, 419
 injective and projective modules for, 420-426
 multiplicity relations for, 418, 419, 420
 Frobenius algebras over perfect fields, general multiplicity theorem for, 419
 Frobenius reciprocity theorem, 271, 279, 327, 333, 477
 Frobenius-Schur theorem, 183, 184, 185, 186
 Frobenius theorem, 179-190, 242, 295-301
 Frobenius-Wielandt theorem, existence of normal subgroups in a group, 241-250
F-trace, 477
- G**
- Gaschütz-Ikeda-Higman theory, 453, 461
 Gaschütz-Ikeda operator, 481
 Gaschütz's theorem, 421
 generalization of, 426, 427, 428, 429, 430
 Ikeda's generalization of, 425-426
 Generalized character, 272
 lying in K , 311
 ring of, 285
 Generalized decomposition numbers, 638-641
 definition of, 638
 Generalized derivation, definition of, 487
 Generalized induction theorem, 301-311
- Generalized quaternion groups, 23, 334, 339-340, 342
 General linear group, 28
 Generators, set of, 381
 Genus, 567-578
 definition of, 570
G-equivalence, 1
 (G, H)-injective, 427
 Global theory of projective modules, 550-558
G-module, definition of, 559, 567
 Grothendieck rings, 550
 Ground field, extension of, 198, 199, 200-206
 Group algebra, 43-44
 of finite representation type, 431-435
 Group character
 definition of, 210
 theory of, 207-264
 Group of a given order, 651-652
 Group representations, 38-49
 Group of rotations of the cube, 331-333
 Group ring, 44
 units in, 262-264
 Group whose Sylow groups are cyclic, 334
- H**
- Hall theorem, 301
 Height, 644
 Heller theorem, 542
 Hermitian form, 49, 254
 Holomorph, 22
 Homogeneous components, 345, 347-348
 Homomorphism, 6-7
 Hyper-elementary subgroup, 302

I

- Ideal in an algebraic number field, 107
- Ideal classes, 123-135, 130, 146
- Ideals, 107-115
 extensions of, 133-134
 fractional, 107
 inverse, 109
 maximal, 107
 norm of, 123-124
 prime, 107-108, 110-112
 product of, 109
 proper, 108, 110
 sum of, 109
- Idempotent element, 164
- Idempotents, 160-161
 orthogonal set of, 165
- Ikeda's generalization of Gaschütz's theorem, 425-426
- Imprimitive modules, 346-348
 definition of, 346
- Imprimitivity system, 346
- Indecomposability, 12, 81
- Indecomposable cyclic submodule, 98
- Indecomposable G -module, 519
- Indecomposable integral representations of a cyclic group of square-free order, 581
- Indecomposable integral representations of the dihedral group of order $2p$, 581
- Indecomposable modules, 81-86, 578-581
 definition of, 81, 497
 vertex and source of, 435-440
- Indecomposable two-sided ideal, 378-379
- Index of D , definition of, 458
- Index of H in G , 4
- Induced characters, 265-311
 Artin's theorem of, 279-282
 Brauer's theorem of, 283-292
 definition of μ^g , 265-266
- Induced matrix representation, 314
- Induced modules, 73-75, 314-323
 equivalence of, 328-329
 irreducibility of, 328-329
 definition of M^g , 73
- Induced monomial representation, 314, 329
- Induced representations, 73-75, 313-365
- Induction, transitivity of, 267
- Induction theorem, generalized, 301-311
- Inertia group, 346, 631
- Injective hull, definition of, 389
- Injective modules, 384-393, 410-411
 definition of, 385-386
 for a finite-dimensional algebra, 409-413
 for a Frobenius algebra, 420-426
 relative, 426-430
- Inner binding function, 504, 520
 definition of, 501
- Inner generalized derivation, definition of, 487
- Inner product of characters, 222-223, 270
- Inner tensor product, 315
- Integers, algebraic, 102-107
- Integral domain, 91
- Integral ideal, 107, 111-114, 153-154
- Integrally closed, 106
- Integral representations, theory of, 493-578
- Internal direct product, 11
- Internal direct sum, 51
- Intertwine, 198-199
- Intertwining number, 319-320, 375

- Intertwining number theorem, 323-328, 340-342
 proof of, 327
- Invariance, 2
- Invariant, 319-320
- Invariant factors, 94, 151
- Invariant factor theorem, 12-13, 150-153, 403
 for matrices, 94-97
 for modules, 97-98
- Inverse different, definition of, 525
- Inverse ideal, 109
- Invertible element, 28
- Irreducibility of induced modules, 328-329
- Irreducible G -module, 519
- Irreducible submodules of the tensor space V , 449
- Irreducible matrix representation, 40
- Irreducible modules, 76
 absolute irreducibility of, 202
 for particular groups, 313
 restriction to normal subgroups, 342-346
- Irreducible monomial representation, 347
- Irreducible representation, 39
 of a direct product, 189-190
 of a nilpotent group, 355-358
 products of, 231-233
 projective representation, 349
 of the symmetric group, 190, 191, 192-198
- Irreducible tensor representations of $GL(V)$, 449-452
- Ito's theorem, 365
- Jordan-Hölder theorem, 15, 79, 506
- Jordan theorem on linear groups, 250-262
 proof of, 258-262
- Jordan-Zassenhaus theorem, 494, 558-563
 for the case when M is absolutely irreducible, 573
 proof of, 559-563

K

- Kasch-Kneser-Kupisch theorem, 433-434
- K -character of G , 476
- K -conjugate character, 304
- K -conjugate elements, 306
- K -elementary subgroup, 302, 476
- K -equivalent representations, 515
- Kernel of an extension of a module, 486
- K -homomorphisms, 26
- K -linear transformations, 26
- Kronecker or direct product of matrices, 69
- Krull-Schmidt theorem, 83-85, 201-202, 373, 383, 403-404, 414, 432, 437
 applied to G -modules, 542
 applied to G^* -modules, 540-541, 548

L

- J**
- Jones theorem, 579-580
- Left A -module, 47
- Left annihilator, 395
- Length of the composition series, 77
- Left cosets, 3-4

- Left G -module, 46
 Left ideals, 50
 Left minimum condition, 157
 Left multiplication, 3
 Left noetherian ring, 55, 56
 Left regular module, 48, 50
 Left R -modules, definition of, 50
 Left socle, definition of, 394
 Levi-Malcev theorem for Lie algebras, 486
 Lifting idempotents, 545
 Linear character, 272, 362
 on \bar{A} , 606
 Linear groups, Jordan, Burnside, and Schur theorems, 250-262
 Linearly dependent set of functions, 182
 Linear transformations, 26-30
 definition of, 26
 Linked, 378
 Local theory of projective modules, 542-550
- M**
- Mackey theorem, 323-328, 341-342, 353-355
 Maranda's theorem, 539-540, 568-570
 Maschke's theorem, 41, 88, 355, 423, 501
 matrix analogue of, 501-502
 Matrices, equivalent, 94
 invariant factor theorem, 94-97
 Matrices over a field, 27
 Matrix, characteristic polynomial, 207
 Matrix, trace of, 207
 Matrix representation, 30
 afforded by an A -module, 47
 afforded by a representation space, 31
- Matrix with respect to a basis, 27
 Maximal ideal, 107
 Maximal related extension, definition of, 389-390
 Maximal submodule, 77
 Maximum principle, 56, 88-89
 Metabelian groups, 357
 Metacyclic group G , definition of, 333-334
 Metacyclic groups, representations of, 333-340
 M -group, 357-358
 Minimal left ideal, definition of, 163
 Minimal submodule, 77
 Minimum polynomial of a matrix, 179
 Möbius inversion formulas, 136-137
 Möbius μ -function, 136
 Möbius transform, 136
 Modular character, 588
 Modular representation, 532, 583-654
 associated with T , 584
 Modules,
 over Dedekind domains, 144-155
 indecomposable, 81-86
 injective, 384-393
 invariant factor theorem for, 97-98
 over orders, 515-531
 over principal ideal domains, 91-102
 projective, 380-384
 over quasi-Frobenius rings, 403-408
 tensor product of, 211
 Modules, principal indecomposable, 367-377
 classification into blocks, 377-380
 definition of, 369
 Monic polynomial, 102-103
 Monomial representation, 314, 321, 355-358
 Morita-Tachikawa theorem, 397-398
 Multiplicity-free representations, 340-342

- Multiplicity relations for Frobenius algebras, 418-420
- Multiplicity theorem for Frobenius algebras over perfect fields, 419
- Multiplier, 359
- Nucleus, 442
- Numerical bounds, 645-646

O

- N**
- Nagao-Nakayama theorem, 412-413
 - Natural homomorphism, 6
 - Natural mapping, 53
 - n -dimensional right vector space, 173-175
 - Nilpotent element, 159-160
 - Nilpotent groups, 14-17
 - irreducible representations of, 355-358
 - Nilpotent ideal, 160-162
 - Noether-Deuring theorem, 200-202
 - Noetherian module, 55
 - Noetherian ring, 55, 106
 - Non-Archimedean valuation, 116
 - Non-degenerate bilinear form, associative, 415, 424
 - Non-degenerate pairing, 397
 - Non-semi-simple rings, 367-408
 - Non-singularity of the Cartan matrix, 550, 602
 - Norm, 127
 - Norm of an algebraic number, 131-133
 - Normalized factor set, 361
 - Normalizer of H in G , 10
 - Normal series, 14
 - factors of, 14
 - Normal subgroup, 5
 - criteria for existence of, 654, 241-250
 - restriction of irreducible modules, 342-346
 - Norms of ideals, 123-135
 - Octahedral groups, 329-333
 - One-character, 222
 - One-dimensional characters, 272
 - One-dimensional representations, 36-38
 - One-representation, 222
 - Orbits, 1-3
 - Order, 97, 516
 - Order of cyclotomic polynomial, 137
 - Order ideal, 96, 563-567
 - of M , definition of, 564
 - Orders, modules over, 515-531
 - Orthogonal idempotents, 165, 369
 - Orthogonality relations, 217-224, 300, 304, 598-604
 - simple applications of, 224-233
 - Orthogonality relations for Frobenius algebras, 419
 - Orthogonal set of idempotents, 165
 - Osima theorem, 610
 - Outer tensor product, 315
- P**
- P -adic integers, 117
 - P -adic number field, 121
 - P -adic numbers, valuations, 115-123
 - P -adic valuation, definition of, 116-117
 - Pairing, 396-397
 - Pairwise relatively prime, 112
 - Pairwise relatively prime integral ideals, 112-113
 - Partially ordered set, 89, 385
 - Partition of n , 9-10, 190

- p -elementary divisors, 599
 p -elementary subgroup, 284-285
 Perfect field, 376-377
 definition of, 464
 Periodic of bounded period, 250
 Periodic group, 250
 Permutation groups, 1-3, 228-231
 Permutation matrix, 31-34
 Permutation representations, 31, 314
 p -group, 8-9
 P -integral elements of a field, 117
 P -integral equivalence, 531-542
 p -irregular, 584
 P^k -modular representation of G of
 degree m , definition of, 532
 Polynomial, content of, 106-107, 118
 Positive definite hermitian form, 49,
 254
 p -part, 477
 p -rank for a matrix X over Z , 591
 p -regular, 283-284, 584
 p -regular classes, 288
 p -regular components, 284
 Primary component, 11
 Prime ideal, 107-108, 110-112, 124-
 125
 Primitive idempotent, 172, 369
 Primitive n th root of 1, 136-138
 Primitive polynomial, 103
 Principal character, 222
 Principal ideal, 113-114
 Principal ideal domain, 91
 modules over, 91-102
 Principal indecomposable modules,
 367-377, 410
 classification into blocks, 377-380
 definition of, 369
 Product of characters, 211
 Product of ideals, 109
 Product modules, 267-269
 Product of modules, 211
 Products of irreducible representa-
 tions, 231-233
 Projection, 41-42
 Projections associated with a given
 direct sum decomposition, 54
 Projective class group, 557
 Projective G -module, as determined by
 its behavior mod p , 543-547
 Projective module, 148-149, 380-384,
 409
 for a Frobenius algebra, 420-426
 global theory, 550-558
 local theory, 542-550
 relative, 426-430
 Projective relative to H , 427
 Projective representations, 348-355
 definition of, 349
 irreducible, 349
 Schur's theory of, 358-365
 Proper ideal, 108, 110
 p -singular, 284, 288, 584
 p -singular component, 284
 p -Sylow subgroup, 17-20
 of G , 239-241
 Pure submodule, 100
- ## Q
- Quasi-Frobenius algebras, 413-420
 definition of, 413
 Quasi-Frobenius rings, 367, 393-403
 definition of, 395-396
 modules over, 403
 Q -elementary subgroups, 302
 Q -equivalent, definition of, 494
- ## R
- Radical, definition of, 162

- Radical of a ring with minimum condition, 159-163
- Rank, 93
- Rank of a module, 145
- Rational characters, 279-283
definition, 279
- R -basis, 52
- Realizable matrix representation, 293
- Realizable in a subfield, 464
- Reduced regular module, 404
- Reducible, 519
- Reducible matrix representation, 40
- Reducible module, 77
- Reducible representation, 39
- Regular matrix representation, 32-34
- Regular module with respect to the pairing τ , 442
- Regular representation, 32
- Reiner-Nakayama theorem, 542-543, 550-551
- Reiner theorem, 574-578
- Related extension, definition of, 389-390
- Relative homological algebra, 420
- Relatively prime, 112
- R -endomorphisms, 53
- Representation-group, definition of, 361
- Representations
of an algebra, 45
of G by matrices with entries in a ring R , 493
induced, 313-365
of metacyclic groups, 333-340
multiplicity free, 340-342
- Representations, projective, 348-355
definition of, 349
irreducible, 349
Schur's theory of, 358-365
- Representation space, 30
- R -equivalent representations, 515
- Restriction to normal subgroups of irreducible modules, 342-346
- R -free, 52
- R -homomorphism, 53
- Right annihilator, 395
- Right cosets, 3-4
- Right ideals, 50
- Right minimum condition, 157
- Right regular module, 50
- Right socle, definition of, 394
- Ring of generalized characters, 272, 285
- Rings, non-semi-simple, 367-408
- Ring with minimum condition
definition of, 157
radical of, 159-163
- R -linear combination, 52
- Roquette's simplification of proof of Brauer's theorem, 301
- R -order, definition of, 516
- Rotations of the cube, group of, 331-333
- Row permutations, 191, 197
- R -reducible, 519
- R -torsion-free, 52

S

- Schur index, 292-295, 453, 463-479
computation of, 470-471
definition of, 293
- Schur index of U with respect to k ,
definition of, 466
- Schur's lemma, 80, 181, 350, 475
converse of, 189
- Schur theorem, 24, 249, 361-362
- Schur theorem of linear groups, 250-262
proof of, 252-258
- Schur's theory of projective representations, 358-365
- Second regular representation, 413

- Semi-direct products, 21-23
 Semi-simple, 162
 Semi-simple ring R , simple components of, 170
 Semi-simple rings, 163-173
 Separability, intrinsic criterion for, 481-485
 Separable algebras, 453-492
 definition of, 480
 Separable extensions of the base field, 459-463
 Sequence, exact, 381
 Series, composition, 76-81
 Set of generators, 381
 Simple components, 170
 Simple groups, characterizations of, 652-654
 Simple ring, 168-169
 structure of, 173-179
 Skewfield, 167-168, 173-175
 Skew symmetric tensors, 452
 Socle, definition of, 394
 Solvable groups, 14-17
 Solvable groups, Burnside's criterion for, 239-241
 Source, definition of, 438
 Source and vertex of an indecomposable module, 435-440
 Split extension, 23-24, 381, 486, 488
 Split factor set, definition of, 489
 Splitting fields, 265, 453-492
 for an algebra, 202, 455
 Brauer's theorem on, 292-295
 cyclotomic, 135-144
 for division algebras, 453-459
 for a group, 203
 for simple algebras, 453-459
 Splitting field theorem for characteristic p , proof of, 474-475
 Square-free order group, 334
 (S, R) -bimodule over the rings R and S , 66
 Subalgebra, 43
 Subgroups, 3-8
 Subgroup theorem, 324-325, 437
 Submodule, 50
 Subordinates, 622, 628
 Subsets of a ring, 158-159
 Sum of ideals, 109
 Swan theorem, 543-550, 552-558
 Sylow subgroups, 17-21
 Symmetric algebra, 401
 definition of, 440
 Symmetric algebras, centralizers of modules over, 440-448
 Symmetric group
 irreducible representations of, 190-198
 S_4 , 648-650
 Symmetric tensors, 452
 Symmetry operator, 450
 System of imprimitivity, 346
- ## T
- Table, 191, 197
 Tensor product, 59-76
 of algebras over a field, 71-73
 of modules, 62, 313
 representation, 69
 of two induced modules, 323, 325
 of vector spaces, 67-70
 Tensor product theorem, 323-328
 proof of, 325-326
 Tensor space V , 449
 Tetrahedral groups, 329-333
 Torsion-free, 92
 Torsion-free element, 97
 Torsion-free element of R -module, definition of, 564
 Torsion-free module, 97
 Torsion module, 97, 564

- Totally ordered set, definition of, 385
- Trace of a matrix, 29, 207
- Transitive, 2
- Transitive imprimitive module, 346-347
- Transitive permutation representation, 321
- Transitivity of induction, 267
- Transpose, 317
- Transpose matrix, 317
- Trivial orbit, 2
- Two-sided ideal, indecomposable, 378
- U**
- Unimodular matrix, 93
definition of, 494
over R , 515
- Unique factorization domain, 91
- Unit, 28
- Unitary basis, 254
- Unitary group, 49, 254
- Unitary matrix, 255-256
- Unitary transformation, 49, 256-258
- Units in a group ring, 262-264
- Upper bound of a subset, definition of, 385
- V**
- Valuation, 116
 P -adic numbers, 115-123
- Valuation ring, 117
- Value group of a valuation, 116
- Van der Monde determinant, 34, 105
- Vertex, definition of, 438
- Vertex and source of an indecomposable module, 435-440
- W**
- Wedderburn-Malcev theorem, 377, 485-492
proof of, 491-492
- Wedderburn's theorem, 403, 453, 458, 475
- Wedderburn structure theorem, 157, 175-178, 188-189
- Wielandt-Frobenius theorem, permutation groups, 246-247
- Wielandt theorem, 242, 247-248
- Witt-Bermann induction theorem, 302-311, 475
- Z**
- Zassenhaus-Reiner theorem, 538-539
- Zassenhaus theorem, 563
- Z -composition factors, 498
- Z -composition series, 498
- Z -equivalent, definition of, 494
- ZG -module, definition of, 494
- Z -irreducible, definition, 497
- Zorn's lemma, 385
- Z -reducible, definition of, 497

