Foundations of Mechanics

Second Edition

Ralph Abraham
Jerrold E. Marsden

AMS CHELSEA PUBLISHING
American Mathematical Society • Providence, Rhode Island

FOUNDATIONS OF MECHANICS

SECOND EDITION

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Ralph Abraham
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with the assistance of Tudor Ratiu
and Richard Cushman

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IN MEMORIAM

RUFUS BOWEN
1947–1978

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1930–1978

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Preface to the AMS Chelsea Edition

This book is the American Mathematical Society printing of *Foundations of Mechanics*, which was first published in 1967 by W. A. Benjamin and whose second edition was published by Benjamin Cummings in 1978, with significant improvements in subsequent printings. The book was also distributed by Perseus Press for the last decade or so. It is the Updated 1985 (Fifth) Printing that is reproduced here.

We have compiled a list of errata and updates, which appears at the end of the book. This can also be found on the book's websites, which will be maintained as additional updates are needed: `http://www.cds.caltech.edu/~marsden/books/Foundations_of_Mechanics.html` and `http://www.ams.org/bookpages/chel-364`.

Because of issues involving permissions for the printed form, we have not reproduced the "Museum" for the book, the gallery of photographs of some of the historical giants of mechanics. However, this museum can be found on the book's websites.

We are grateful to many readers who helped us gather errata and updates for the AMS printing of the book. We are especially indebted to Tudor Ratiu for his diligent work in this regard.

Ralph Abraham
Jerrold E. Marsden
Spring, 2008

Preface to the Second Edition

Since the first edition of this book appeared in 1967, there has been a great deal of activity in the field of symplectic geometry and Hamiltonian systems. In addition to the recent textbooks of Arnold, Arnold–Avez, Godbillon, Guillemin–Sternberg, Siegel–Moser, and Souriau, there have been many research articles published. Two good collections are "Symposia Mathematica," vol. XIV, and "Géométrie Symplectique et Physique Mathématique," CNRS, Colloque Internationaux, no. 237. There are also important survey articles, such as Weinstein [1977b]. The text and bibliography contain many of the important new references we are aware of. We have continued to find the classic works, especially Whittaker [1959], invaluable.

The basic audience for the book remains the same: mathematicians, physicists, and engineers interested in geometrical methods in mechanics, assuming a background in calculus, linear algebra, some classical analysis, and point set topology. We include most of the basic results in manifold theory, as well as some key facts from point set topology and Lie group theory. Other things used without proof are clearly noted.

We have updated the material on symmetry groups and qualitative theory, added new sections on the rigid body, topology and mechanics, and quantization, and other topics, and have made numerous corrections and additions. In fact, some of the results in this edition are new.

We have made two major changes in notation: we now use f^* for pull-back (the first edition used f_*), in accordance with standard usage, and have adopted the "Bourbaki" convention for wedge product. The latter eliminates many annoying factors of 2.

A. N. Kolmogorov's address at the 1954 International Congress of Mathematicians marked an important historical point in the development of the theory, and is reproduced as an appendix. The work of Kolmogorov, Arnold, and Moser and its application to Laplace's question of stability of the solar system remains one of the goals of the exposition. For complete details of all the theorems needed in this direction, outside references will have to be consulted, such as Siegel–Moser [1971] and Moser [1973a].

We are pleased to acknowledge valuable assistance from

Paul Chernoff Wlodek Tulczyjew
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Further, we express our gratitude to Chris Shaw, who made exceptional efforts to transform our sketches into the graphics which illustrate the text, to Peter Coha for his assistance in organizing the Museum and Bibliography, and to Ruthie Cephas, Jody Hilbun, Marnie McElhiney, Ruth (Bionic Fingers) Suzuki, and Ikuko Workman for their superb typing job.

Theoretical mechanics is an ever-expanding subject. We will appreciate comments from readers regarding new results and shortcomings in this edition.

RALPH ABRAHAM
JERROLD E. MARSDEN

Preface to the First Edition

In the Spring of 1966, I gave a series of lectures in the Princeton University Department of Physics, aimed at recent mathematical results in mechanics, especially the work of Kolmogorov, Arnold, and Moser and its application to Laplace's question of the stability of the solar system. Mr. Marsden's notes of the lectures, with some revision and expansion by both of us, became this book.

Although the lectures were attended equally by mathematicians and physicists, our goal was to make the subject available to the nonspecialists. Therefore, the mathematical background assumed was dictated by the physics graduate students in the audience. Hoping this would be typical of the people interested in this subject, I have made the same assumptions in the book.

Thus, we take for granted basic undergraduate calculus and linear algebra, and a limited amount of classical analysis, point set topology, and elementary mechanics. Then we begin with modern advanced calculus, and go on to a complete and self-contained treatment of graduate level classical mechanics. The later chapters, dealing with the recent results, require an ever-increasing adeptness in general topology, and we have collected the topological topics required in Appendix A.

To further aid the nonmathematician, the proofs are unusually detailed, and the text is replete with cross-references to earlier definitions and propositions, all of which are numbered for this purpose. The extent of these is testimony of Mr. Marsden's patience.

As our goal is to make a concise exposition, we prove propositions only if the proofs are easy, or are not to be found readily in the literature. This

results in an irregular collection of proofs—in the first four chapters nearly everything is proved, being easy, and in the last three chapters there are several longer proofs included and many omitted. Some of those included are necessary because the propositions are original, and can be omitted in a first reading or an elementary course.

For the mathematical reader, the proofs we have omitted can easily be found in books or journals, and we give complete references for each (References in square brackets refer to the Bibliography.) For this reason, the book, although not self-contained, gives a complete exposition.

In this connection we are grateful to Al Kelley for the opportunity of publishing two research articles of his, as Appendixes B and C, which have not appeared elsewhere. In each of these he proves an original theorem that is important to our development of the subject. As Kolmogorov's address at the 1954 International Congress of Mathematicians (in Russian), which inspired the most important of the recent results, has not been available in English, we include a translation of it in Appendix D. The exercises at the end of each section are nearly all used in a later section, and may be read as part of the text.

I am indebted to Arthur Wightman for his enthusiasm in making arrangements for my lectures and the publication of the book, to René Thom for discussions on structural stability and a preliminary manuscript of part of his book on that subject, to Jerrold Marsden for his energetic collaboration in the writing of this book, and to many colleagues for valuable suggestions. Some of these are acknowledged in the Notes at the end of each chapter, which also give general historical and bibliographical information.

We are both happy to express our gratitude to June Clausen for editing and typing the bulk of the manuscript, and to Patricia Clark, Bonnie Kearns, Elizabeth Epstein, Elizabeth Margosches, and Jerilynn Christiansen for their valuable assistance.

RALPH ABRAHAM

Princeton, New Jersey
June 1966

Introduction

Mechanics begins with a long tradition of qualitative investigation culminating with KEPLER and GALILEO. Following this is the period of quantitative theory (1687–1889) characterized by concomitant developments in mechanics, mathematics, and the philosophy of science that are epitomized by the works of NEWTON, EULER, LAGRANGE, LAPLACE, HAMILTON, and JACOBI. Both of these periods are thoroughly described in DÜGAS [1955].

Throughout these periods, the distinguished special case of *celestial mechanics* had a dominant role (see MOULTON [1902] for additional historical details). Formalized in the quantitative period as the *n-body problem*, it recurs in the writings of all of the great figures of the time. The question of *stability* was one of main concerns, and was analyzed with series expansion techniques by LAPLACE (1773), LAGRANGE (1776), POISSON (1809), DIRICHLET (1858), and HARETU (1878), all of whom claimed to have proved that the solar system was stable.

As DIRICHLET died before writing down this proof, KING OSCAR of Sweden offered a prize for its discovery, which was given to POINCARÉ in 1889. The results of POINCARÉ, suggesting that the series expansions of LAPLACE et al. diverged, and the discovery by BRUNS [1887] that no quantitative methods other than series expansions could resolve the n-body problem brought the quantitative period to an end. (See MOSER [1973a] for additional historical information.) For celestial mechanics this situation represented a great dilemma, comparable to the crises associated with relativity and quantum theory in other aspects of mechanics. The resolution we owe to the

genius of POINCARÉ, who resurrected the qualitative point of view, accompanied by completely new mathematical methods. The inventions of POINCARÉ, culminating in modern differential geometry and topology, constitute a recent and lesser known example of concomitant development of mathematics and mechanics, comparable to calculus, differential equations, and variational theory.

The neoqualitative period in mechanics, that is, from POINCARÉ to the present, consists primarily in the amplification of the qualitative, geometric methods of POINCARÉ, the application of these methods to the qualitative questions of the previous period—for example, stability in the n-body problem—and the consideration of new qualitative questions that could not previously be asked.

POINCARÉ's methods are characterized first of all by the global geometric point of view. He visualized a dynamical system as a field of vectors on phase space, in which a solution is a smooth curve tangent at each of its points to the vector based at that point. The qualitative theory is based on geometrical properties of the *phase portrait*: the family of solution curves, which fill up the entire phase space. For questions such as stability, it is necessary to study the entire phase portrait, including the behavior of solutions for all values of the time parameter. Thus it was essential to consider the entire phase space at once as a geometric object. Doing so, POINCARÉ found the prevailing mathematical model for mechanics inadequate, for its underlying space was Euclidean, or a domain of several real variables, whereas for a mechanical problem with angular variables or constraints, the phase space might be a more general, nonlinear space, such as a generalized cylinder. Thus the global view in the qualitative theory led POINCARÉ to the notion of a *differentiable manifold* as the phase space in mechanics. In mechanical systems, this manifold always has a special geometric structure pertaining to the occurrence of phase variables in canonically conjugate pairs, called a *symplectic structure*. Thus the new mathematical model for mechanics consists of a *symplectic manifold*, together with a *Hamiltonian vector field*, or global system of first-order differential equations preserving the symplectic structure.

This model offers no natural system of coordinates. Indeed a manifold admits a coordinate system only locally, so it is most efficient to use the intrinsic calculus of CARTAN rather than the conventional calculus of NEWTON in the analysis of this model. The complete description of this model for mechanics comes quite a bit after POINCARÉ, as the intrinsic calculus was not fully developed until the 1940s. One advantage of this model is that by suppressing unnecessary coordinates the full generality of the theory becomes evident.

The second characteristic of the qualitative theory is the replacement of analytical methods by differential-topological ones in the study of the phase portrait. For many questions, for example the stability of the solar system, one is interested finally in qualitative information about the phase portrait. In earlier times, the only techniques available were analytical. By obtaining a

complete or approximate quantitative solution, qualitative or geometric properties could be deduced. It was POINCARÉS idea to proceed directly to qualitative information by qualitative, that is, geometric methods. Thus POINCARÉ, BIRKHOFF, KOLMOGOROV, ARNOLD, and MOSER show the existence of periodic solutions in the three-body problem by applying differential-topological theorems to the phase portraits in addition to analytical methods. No analytical description of these orbits has been given. In some cases the orbits have been plotted approximately by computers, but of course the computer cannot prove that these solutions are periodic.

A third aspect of the qualitative point of view is a new question that emerges in it—the problem of *structural stability*, the most comprehensive of many different notions of stability. This problem, first posed in 1937 by Andronov–Pontriagin, asks: If a dynamical system X has a known phase portrait P, and is then perturbed to a slightly different system X' (for example, changing the coefficients in its differential equation slightly), then is the new phase portrait P' close to P in some topological sense? This problem is of obvious importance, since in practice the qualitative information obtained for P is to be applied not to X, but to some nearby system X', because the coefficients of the equation may be determined experimentally or by an approximate model and therefore approximately.

The traditional mutuality of mechanics and philosophy has declined in recent years, perhaps because of the justifiable interest in the problems posed by relativity and quantum theory. But current problems in mechanics give new insight into the structure of physical theories.

At the turn of this century a simple description of physical theory evolved, especially among continental physicists—DUHEM, POINCARÉ, MACH, EINSTEIN, HADAMARD, HILBERT—which may still be quite close to the views of many mathematical physicists. This description—most clearly enunciated by DUHEM [1954]—consisted of an *experimental domain*, a *mathematical model*, and a *conventional interpretation*. The model, being a mathematical system, embodies the logic, or axiomatization, of the theory. The interpretation is an agreement connecting the parameters and therefore the conclusions of the model and the observables in the domain.

Traditionally, the philosopher-scientists judge the usefulness of a theory by the criterion of *adequacy*, that is, the verifiability of the predictions, or the quality of the agreement between the interpreted conclusions of the model and the data of the experimental domain. To this DUHEM adds, in a brief example [1954, pp. 138 ff.], the criterion of *stability*.

This criterion, suggested to him by the earliest results of qualitative mechanics (HADAMARD), refers to the stability or continuity of the predictions, or their adequacy, when the model is slightly perturbed. The general applicability of this type of criterion has been suggested by RENÉ THOM [1975].

This stability concerns variation of the model only, the interpretation and domain being fixed. Therefore, it concerns mainly the model, and is primarily

a mathematical or logical question. It has been studied to some extent in a general logical setting by the physicologicians BOULIGAND [1935] and DESTOUCHES [1935], but probably it is safe to say that a clear enunciation of this criterion in the correct generality has not yet been made. Certainly all of the various notions of stability in qualitative mechanics and ordinary differential equations are special cases of this notion, including LAPLACE's problem of the stability of the solar system and structural stability, as well as THOM's stability of biological systems.

Also, although this criterion has not been discussed very explicitly by physicists, it has functioned as a tacit assumption, which may be called the *dogma of stability*. For example, in a model with differential equations, in which stability may mean structural stability, the model depends on parameters, namely the coefficients of the equation, each value of which corresponds to a different model. As these parameters can be determined only approximately, the theory is useful only if the equations are structurally stable, which cannot be proved at present in many important cases. Probably the physicist must rely on faith at this point, analogous to the faith of a mathematician in the consistency of set theory.

An alternative to the dogma of stability has been offered by THOM [1975]. He suggests that stability, precisely formulated in a specific theory, be added to the model as an additional hypothesis. This formalization, despite the risk of an inconsistent axiomatic system, reduces the criterion of stability to an aspect of the criterion of adequacy, and in addition may admit additional theorems or predictions in the model. As yet no implications of this axiom are known for celestial mechanics, but THOM has described some conclusions in his model for biological systems.

A careful statement of this notion of stability in the general context of physical theory and epistemology could be quite useful in technical applications of mechanics as well as in the formulation of new qualitative theories in physics, biology, and the social sciences.

Most of this book is devoted to a precise statement of mathematical models for mechanical systems and to precise definitions of various types of stability in this narrow context. These are illustrated by a number of examples, but by one example in depth, namely, the restricted three-body problem in Chapter 10.

Preview

To motivate the introduction of symplectic geometry in mechanics, we briefly consider Hamilton's equations. The starting point is Newton's second law, which states that a particle of mass $m>0$ moving in a potential $V(q)$, $q=(q^1,q^2,q^3)\in \boldsymbol{R}^3$, moves along a curve $q(t)$ in $\boldsymbol{R}^3$ in such a way that $m\ddot{q}=-\,grad\,V(q)$. If we introduce the momentum $p_i=m\dot{q}^i$ and the energy $H(q,p)=(1/2m)\|p\|^2+V(q)$, then Newton's law is equivalent to Hamilton's equations:

$$\begin{cases} \dot{q}^i=\partial H/\partial p_i \\ \dot{p}_i=-\partial H/\partial q^i, \qquad i=1,2,3 \end{cases}$$

One proceeds to study this system of first-order equations for a general $H(q,p)$. To do this, we introduce the matrix $J=\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, where I is the 3×3 identity, and note that the equations become $\dot{\xi}=J\cdot grad\,H(\xi)$, where $\xi=(q,p)$. (In complex notation, setting $z=q+ip$, they may be written as $\dot{z}=-2i\,\partial H/\partial\bar{z}$.)

Set $X_H=J\cdot grad\,H$. Then $\xi(t)$ satisfies Hamilton's equations iff $\xi(t)$ is an integral curve of X_H, that is, $\dot{\xi}(t)=X_H(\xi(t))$. The relationship between X_H and H can be rewritten as follows: introduce the skew-symmetric bilinear form ω

on $\boldsymbol{R}^3 \times \boldsymbol{R}^3$ defined by

$$\omega(v_1, v_2) = v_1 \cdot J \cdot v_2$$
$$v_1, v_2 \in \boldsymbol{R}^3 \times \boldsymbol{R}^3$$
$$v_1 = (x_1, y_1)$$
$$v_2 = (x_2, y_2)$$

[In complex notation on $C^3 \cong \boldsymbol{R}^3 \times \boldsymbol{R}^3$, $\omega(v_1, v_2) = -Im\langle v_1, v_2 \rangle$, where $v_1 = x_1 + iy_1$, $v_2 = x_2 + iy_2$, and $\langle\ ,\ \rangle$ is the Hermitian inner product.]

Then we have, for all $\xi \in \boldsymbol{R}^3 \times \boldsymbol{R}^3$ and $v \in \boldsymbol{R}^3 \times \boldsymbol{R}^3$,

$$\omega(X_H(\xi), v) = dH(\xi) \cdot v$$

where $dH(q,p) = (\partial H/\partial q^i, \partial H/\partial p^i)$, a row vector in $\boldsymbol{R}^3 \times \boldsymbol{R}^3$, as is easily checked. One calls ω the *symplectic form* on $\boldsymbol{R}^3 \times \boldsymbol{R}^3$, and X_H the *Hamiltonian vector field* with energy H.

Suppose we make a change of coordinates $\eta = f(\xi)$, where $f: \boldsymbol{R}^3 \times \boldsymbol{R}^3 \to \boldsymbol{R}^3 \times \boldsymbol{R}^3$ is smooth. If $\xi(t)$ satisfies Hamilton's equations, the equations satisfied by $\eta(t) = f(\xi(t))$ are $\dot{\eta} = A\dot{\xi} = AJ\,grad_\xi H(\xi) = AJA^*\,grad_\eta H(\xi(\eta))$, where $(A)^{ij} = (\partial\eta^i/\partial\xi^j)$ is the Jacobian of f, and A^* is the transpose of A. The equations for η will be Hamiltonian with energy $K(\eta) = H(\xi(\eta))$ if and only if $AJA^* = J$. A transformation satisfying this condition is called *canonical* or *symplectic*, (or a symplectomorphism). In terms of the symplectic form ω, this condition, denoted $f^*\omega = \omega$, says the transformation f leaves ω unchanged.

The space $\boldsymbol{R}^3 \times \boldsymbol{R}^3$ of the ξ's is called the *phase space*. For a system of N particles we would use $\boldsymbol{R}^{3N} \times \boldsymbol{R}^{3N}$.

For many fundamental physical systems, the phase space is a manifold rather than Euclidean space. For instance, manifolds often arise when constraints are present. For example, the phase space for the motion of the rigid body is the tangent bundle of the group $SO(3)$ of 3×3 orthogonal matrices with determinant $+1$. (See Sect. 4.4 for details.) Not only are manifolds important in these examples, but their terminology and notation lead to a clearer understanding of the basic structure of mechanics.

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GLOSSARY OF SYMBOLS

$\boldsymbol{E}, \boldsymbol{F}, \ldots$	finite-dimensional real vector spaces
$\|\boldsymbol{x}\|$	norm of $\boldsymbol{x}$
$L(\boldsymbol{E}, \boldsymbol{F})$	continuous linear mapping of $\boldsymbol{E}$ to $\boldsymbol{F}$
A^t or $A^* \in L(\boldsymbol{F}^*, \boldsymbol{E}^*)$	transpose of $A \in L(\boldsymbol{E}, \boldsymbol{F})$
$L^k(\boldsymbol{E}, \boldsymbol{F})$	multilinear mappings
$L_a^k(\boldsymbol{E}, \boldsymbol{F}) \subset L^k(\boldsymbol{E}, \boldsymbol{F})$	skew-symmetric mappings
$L_s^k(\boldsymbol{E}, \boldsymbol{F}) \subset L^k(\boldsymbol{E}, \boldsymbol{F})$	symmetric mappings
$U \subset \boldsymbol{E}$	open subset
$f\colon U \subset \boldsymbol{E} \to \boldsymbol{F}$	smooth (C^∞) mapping
$x \mapsto f(x)$	effect of f on x
$D^k f\colon U \subset \boldsymbol{E} \to L_s^k(\boldsymbol{E}, \boldsymbol{F})$	derivatives of f
$D_i f\colon U \subset \boldsymbol{E} \to L_s^k(\boldsymbol{E}_i, \boldsymbol{F})$	partial derivatives of f
$c'(t) = Dc(t) \cdot 1$	tangent to a curve
$M, N, \ldots$	C^∞ manifold
$\pi\colon E \to B$	vector bundle
$\Gamma^\infty(\pi)$	C^∞ sections of π
$T_m M$	tangent space at $m \in M$
$T_m f$ or $Tf(m)$	tangent of f at m
$\tau_M\colon TM \to M$	tangent bundle
$\tau_M^*\colon T^*M \to M$	cotangent bundle
$(\tau_M)_s^r\colon T_s^r(M) \to M$	tensor bundles
$\omega_M^k\colon \omega^k(M) \to M$	exterior form bundles
$f \in \mathcal{F}(M)$	C^∞ real-valued functions
$X \in \mathcal{X}(M) = \Gamma^\infty(\tau_M)$	vector fields
$\alpha \in \mathcal{X}^*(M) = \Gamma^\infty(\tau_M^*)$	one-forms
$t \in \mathcal{T}_s^r(M) = \Gamma^\infty\big((\tau_M)_s^r\big)$	tensor fields
$\otimes$	tensor product
$\omega \in \Omega^k(M) = \Gamma^\infty(\omega_M^k)$	k-forms
$\wedge$	exterior product
$f\colon M \to N$	mapping of manifolds
$f^*\colon \Omega^k(N) \to \Omega^k(M)$	pullback of forms
$\varphi\colon M \to N$	diffeomorphism of manifolds
$\varphi_s^r\colon T_s^r(M) \to T_s^r(N)$	induced tensor bundle isomorphism
$\varphi_*\colon \mathcal{T}_s^r(M) \to \mathcal{T}_s^r(M)$	induced tensor field isomorphism
$U \subset M$	open submanifold
$(U, \varphi), \varphi\colon U \to U' \subset E$	local chart
$\boldsymbol{e}_1, \ldots, \boldsymbol{e}_n$	basis of $\boldsymbol{E}$
$\boldsymbol{\alpha}^1, \ldots, \boldsymbol{\alpha}^n$	dual basis of $\boldsymbol{E}^* = L(\boldsymbol{E}, \boldsymbol{R})$
$\underline{e}_1, \ldots, \underline{e}_n$	induced generators of $\mathcal{X}(U)$
$dx^1, \ldots, dx^n$	induced dual generators of $\mathcal{X}^*(U)$
$F_X\colon \mathcal{D}_X \subset M \times \boldsymbol{R} \to M$	integral of vector field
$\boldsymbol{L}_X$	Lie derivative
$[X, Y]$	Lie bracket

$\boldsymbol{d}$	exterior derivative
$\boldsymbol{i}_X$	inner product
Ω	volume n form
μ_Ω	measure of Ω
$div_\Omega X$	divergence of a vector field
$det_\Omega f$	determinant of a mapping
ω	symplectic form
$X^b = \omega_b(X)$	lowering action
$\alpha^\# = \omega_\#(\alpha)$	raising action
$X_H = (dH)^\#$	Hamiltonian vector field
$Sp(E,\omega)$	symplectic group
$-\theta_0$	canonical one-form on T^*M
$\omega_0 = -d\theta_0$	canonical two-form on T^*M
$\{f,g\}$	Poisson bracket of functions
$\{\alpha,\beta\}$	Poisson bracket of one-forms
$\mathcal{X}_{\mathcal{LH}}$	locally Hamiltonian vector fields
$\mathcal{X}_{\mathcal{H}}$	globally Hamiltonian vector fields
Σ_e	energy surface
$\boldsymbol{FL}$	Legendre transformation
ω_L	pullback of ω_0 by FL
$\Omega = -\boldsymbol{d}\theta$	symplectic form determined by a metric
(M,ω) or (P,ω)	symplectic manifold
$\Phi: G \times P \to P$	action of a Lie group G on P
$\mathfrak{g}$ or $\mathcal{L}(G)$	Lie algebra of G
$J: P \to \mathfrak{g}^*$	momentum mapping
$\hat{J}(\xi)(p) = J(p)\cdot\xi$	dual momentum mapping
$P_\mu = J^{-1}(\mu)/G_\mu$	reduced phase space
$I_{h,\mu} = (H \times J)^{-1}(h,\mu)$	level surface of $H \times J$
V_μ	amended or effective potential
$\tilde{\omega}$	pullback of ω to $\boldsymbol{R} \times M$
$X: \boldsymbol{R} \times M \to TM$	time-dependent vector field
$\tilde{X}: \boldsymbol{R} \times M \to T(\boldsymbol{R} \times M)$	vector field associated to X
$\underline{t}$	unit time vector field on $\boldsymbol{R} \times M$
$\omega_H = \tilde{\omega} + dH \wedge dt$	Cartan form
$F: \boldsymbol{R} \times M \to \boldsymbol{R} \times M$	canonical transformation
$K_F: \boldsymbol{R} \times M \to \boldsymbol{R}$	generating function of F
$j^t: M \to \boldsymbol{R} \times M$	embedding at time t
S, W	Hamilton principal functions

Supplement for *Foundations of Mechanics*

Second Edition, American Mathematical Society

Version: February 15, 2008

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with the collaboration of **Richard Cushman**[4]

This file contains corrections and additional commentary for the American Mathematical Society edition of *Foundations of Mechanics*. We are grateful to readers of the book over the decades, who provided many thoughtful corrections. There are too many to name individually, but we would especially like to thank Ethan Aikin and David Rod.

This supplement contains not only errata known to us, but also some alternative approaches and updates on some of the topics treated in the book. It will be be updated online at `http://www.cds.caltech.edu/~marsden/books/Foundations_of_Mechanics.html` as additional items are found.

Additional Book References

Some other books on geometric mechanics that are an outgrowth of *Foundations of Mechanics*, along with abbreviations we shall use to simplify references are the following

Abraham, Marsden and Ratiu [1988], cited as [MTA]

Cendra, Marsden, and Ratiu [2001], cited as [LRBS]

Marsden and Ratiu [1999], cited as [MandS]

Marsden [1992], cited as [LoM]

Ortega and Ratiu [2004], cited as [HRed]

Marsden, Misiołek, Ortega, Perlmutter and Ratiu [1998], cited as [HStages].

In addition, there are many other useful books on geometric mechanics that the reader may find useful—these are listed in the references.

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Errata and Supplementary Comments

Preface

Chorosoff (or Khorozov) should be written Horozov; see also page 697. This is how the name appears in several of his later papers.

Museum

The Museum is currently available only electronically; see `http://www.cds.caltech.edu/~marsden/books/Foundations_of_Mechanics.html`. Some corrections are as follows.

Leibnitz should be Leibniz

Kolmogorov (April 25, 1903–October 20, 1987),

Moser (July 4, 1928–December 17, 1999),

Carl Ludwig Siegel: December 31, 1896–April 4, 1981

Chapter 1. Differential Theory

1.1 Topology

Page 17, Exercise 1.1F. This exercise should be stated as follows: Let M be a topological space and $H : M \to \boldsymbol{R}$ continuous. Suppose that $e \in int(H(M))$. Then show that $H^{-1}(e)$ divides M; that is, $M \backslash H^{-1}(e)$ has at least two components.

Chapter 2. Calculus on Manifolds

2.2 Vector Fields as Differential Operators

Page 80, second line from bottom. $X\varphi$ should be X_φ

Page 92. Two lines after Theorem 2.2.24, at the end of the line, the L should be $\boldsymbol{L}$ (it is a Lie derivative)

2.4 Cartan's Calculus of Differential Forms

Page 119, After the proof. Add this remark: The proof shows that the Poincaré lemma in $\mathbb{R}^n$ is valid on any star-shaped region, in particular a ball.

2.7 Some Riemannian Geometry

Page 150, line 9. The third term $Y\langle X, Y\rangle$ should be $Y\langle X, Z\rangle$.

Chapter 3. Hamiltonian and Lagrangian Systems

3.2 Symplectic Geometry

Page 178. In the display in Theorem 3.2.10, $\alpha_q \circ T\tau_Q^*(w_{\alpha_q})$ should be $\alpha_q \cdot T\tau_Q^*(w_{\alpha_q})$.

3.4 Integral Invariants, Energy Surfaces. and Stability

Page 206, line −3. "Exercise 4.3H" should be "Exercise 4.3I".

3.7 Mechanics on Riemannian Manifolds

Page 230. In part (c) of Corollary 3.7.9, it should say that $Z(v)$ is the *negative* of the normal component of $\nabla_v v$. Perhaps some clarifying remarks are also in order regarding the comments on page 231 following the proof of this Corollary. In fact, the simple example of a particle moving around in a circle of radius r in the plane may help clarify things. In this case, the position is $c(t) = (r\cos\omega t, r\sin\omega t)$, the velocity is $v(t) = (-r\omega\sin\omega t, r\omega\cos\omega t)$ and the second derivative, $\dot{v} = -\omega^2 c(t)$. This is the acceleration that also equals $\nabla_v v$ in this case. The negative of this is $Z(v)$. Thus, $Z(v)$ points in the (outward) normal direction and has magnitude $\|v\|^2/r$, consistent with what was stated on page 231. Thus, $Z(v)$ can be thought of as the force pulling outwards, namely the *centrifugal force*, while its negative corresponds to the force of constraint, the force keeping the particle moving in the circle. Thus, the interpretations on page 231 are correct if, in the first line after the proof, $Z(t)$ is changed to $-Z(t)$.

Page 244. The first display above the Exercises should read

$$p|V| = NkT - \frac{1}{3\mu_e(\Sigma_e)} \sum_{j<k} \int_{\Sigma_e} \nabla V_{jk}(q_j - q_k) \cdot (q_j - q_k)\mu_e$$

Chapter 4. Hamiltonian Systems with Symmetry

4.1 Lie Groups and Group Actions

Page 255, last line. "passing through $\psi_\xi(s)$" should be "passing through $\phi_\xi(s)$"

Page 257, line 9 from the bottom. "for every $A \in Gl(2, \boldsymbol{R})$" should read "for every $A \in L(\boldsymbol{R}^2, \boldsymbol{R}^2)$"

Page 258, line 11. The argument starting with the sentence "Also, B cannot..." is incorrect. One has to rule out case (ii) on this page for the matrix B. This is done by first noting that the eigenvalues of matrices of the form (ii) are $e^\alpha(\cos\beta \pm i\sin\beta)$. These are real only when $\beta = 0, \pi, \ldots$ and in this case the two eigenvalues are coincident. Thus, these two numbers cannot equal -2 and -1.

Page 259, line 13. The expression $e^{2\pi i j p_k}$ should be $e^{2\pi i j p_k \alpha}$

Page 262, Theorem 4.1.20. We make a few remarks on an alternative exposition of the ideas on quotient manifolds that are relevant to this theorem. First of all, there is a more general result that we now state. The proof of this more general result may be found in [MTA], towards the end of Section 3.5, in the material on quotient manifolds.

Definition. *An equivalence relation R on a manifold M is called* ***regular*** *if the quotient space M/R carries a manifold structure such that the canonical projection $\pi : M \to M/R$ is a submersion. If R is a regular equivalence relation, then M/R is called the quotient manifold of M by R.*

There is an important characterization of regular equivalence relations due to Godement; the exposition of the proof of this given in [MTA] follows the presentation in Serre [2006].

Theorem. *An equivalence relation R on a manifold M is regular iff*

(i) $\operatorname{graph}(R)$ *is a submanifold of $M \times M$, and*

(ii) $p_1 : \operatorname{graph}(R) \to M$, $p_1(x, y) = x$ *is a submersion.*

For the case of quotients M/G by free and proper group actions one can check these hypotheses as follows. The equivalence relation in this case is of course, xRy when there is a $g \in G$ such that $y = gx$, where here we denote the action of g on x by simple concatenation. Thus,

$$\operatorname{graph}(R) = \{(x, gx) \mid x \in M, g \in G\} \subset M \times M.$$

To show that $\operatorname{graph}(R)$ is a submanifold of $M \times M$, we consider the smooth map

$$F : G \times M \to M \times M; \quad (g, x) \mapsto (x, gx)$$

Note that F is injective because the action is free. The derivative of F is given by

$$T_{(g,x)}F\left(T_e L_g \xi, v_x\right) = \left(v_x, g v_x + g \xi_M(x)\right), \tag{*}$$

where $g \in G$, $x \in M$, $\xi \in \mathfrak{g}$, $v_x \in T_x M$ and again the obvious group actions are denoted by concatenation. We claim that $T_{(g,x)}F$ is injective. Indeed, if the right hand side of (*) is zero, then clearly $v_x = 0$, so $g\xi_M(x) = 0$, and hence $\xi_M(x) = 0$. Thus, $\xi = 0$ because otherwise $\exp(t\xi) \in G$ would be a curve of group elements fixing x, contradicting the freeness of the action. Thus, F is an injective immersion. But F is also a closed map by definition of properness, and hence F is an injective immersion which is a homeomorphism onto its image (with the subset topology). Thus, F is an embedding and so its image, which equals $\operatorname{graph}(R)$ is a submanifold of $M \times M$. This verifies condition (i) of the Theorem.

To prove (ii), note that an arbitrary tangent vector to $\operatorname{graph}(R)$ has the form (*) and from this it is clear that the derivative of p_1 is onto from the tangent space to $\operatorname{graph}(R)$ at (x, gx) to $T_x M$.

Specializing M to be a Lie group G and G to be a closed subgroup H, we see that this result on M/G shows that the quotient G/H is a smooth manifold.

Page 265. The proof of Corollary 4.1.22 is incorrect. A restatement and proof of this Corollary is as follows.

4.1.22 Corollary *If $\Phi : G \times M \to M$ is an action and $x \in M$, then $\widetilde{\Phi}_x : G/G_x \to M$ is an injective immersion whose range is the orbit $G \cdot x$. If Φ is proper, the orbit $G \cdot x$ is a closed submanifold of M, and $\widetilde{\Phi}_x$, regarded as a map of $G/G_x \to G \cdot x$ is a diffeomorphism.*

Proof. First of all, $\widetilde{\Phi}_x : G/G_x \to M$ is smooth because $\widetilde{\Phi}_x \circ \pi = \Phi_x$, where $\pi : G \to G/G_x$ is the projection and Φ_x is smooth (see the Remark following 4.1.20). As we have already noted, $\widetilde{\Phi}_x$ is one-to-one. To show it is an immersion, we show that $T_{[g]}\widetilde{\Phi}_x$ is one-to-one. Since $\widetilde{\Phi}_x \circ \pi = \Phi_x$, we see that $T_{[g]}\widetilde{\Phi}_x([v_g]) = T_g\Phi_x(v_g)$ for all $v_g \in T_gG$. Thus, $T_{[g]}\widetilde{\Phi}_x$ will be one-to-one if we can show that $\ker T_g\Phi_x = T_eL_g\,(T_eG_x)$ for all $g \in G$. Since $\Phi_g \circ \Phi_x = \Phi_x \circ L_g$ we have $T_g\Phi_x \circ T_eL_g = T_x\Phi_g \circ T_e\Phi_x$ which implies that

$$\begin{aligned}\ker T_g\Phi_x &= \{T_eL_g\xi \mid 0 = (T_g\Phi_x \circ T_eL_g)(\xi) = (T_x\Phi_g \circ T_e\Phi_x)(\xi)\}\\ &= \{T_eL_g\xi \mid T_e\Phi_x(\xi) = 0\}\end{aligned}$$

since $T_x\Phi_g$ is an isomorphism. The proof is finished if we show that the Lie algebra of G_x equals $\{\xi \in T_eG \mid T_e\Phi_x(\xi) = 0\}$. However, by Proposition 4.1.13, $\xi \in T_eG$ is an element of T_eG_x if and only if $\exp_G t\xi \in G_x$ for all $t \in \boldsymbol{R}$ which is equivalent to the identity $\Phi(\exp_G t\xi, x) = x$ for all $t \in \boldsymbol{R}$. This in turn is equivalent to $\frac{d}{dt}\Phi(\exp_G t\xi, x) = 0$ for all $t \in \boldsymbol{R}$, because $\Phi(e, x) = 0$. The derivative at $s = 0$ of the identity

$$\Phi(\exp_G(t+s)\xi, x) = \left(\Phi_{\exp_G t\xi} \circ \Phi_x\right)(\exp_G s\xi)$$

gives

$$\begin{aligned}\frac{d}{dt}\Phi(\exp_G t\xi, x) &= \frac{d}{ds}\bigg|_{s=0}\Phi(\exp_G(t+s)\xi, x) = \frac{d}{ds}\bigg|_{s=0}\left(\Phi_{\exp_G t\xi} \circ \Phi_x\right)(\exp_G s\xi)\\ &= \left(T_x\Phi_{\exp_G t\xi} \circ T_e\Phi_x\right)(\xi)\end{aligned}$$

which shows that $T_e\Phi_x(\xi) = 0$ if and only if $\frac{d}{dt}\Phi(\exp_G t\xi, x) = 0$ for all $t \in \boldsymbol{R}$, since $T_x\Phi_{\exp_G t\xi}$ is an isomorphism. Therefore, the Lie algebra of G_x is $\{\xi \in T_eG \mid T_e\Phi_x(\xi) = 0\}$, as required.

If the action is also proper, then one checks from the definition of proper and the quotient topology that $\widetilde{\Phi}_x$ is a closed mapping and hence its image, the orbit $G \cdot x$, is closed. Also, since $\widetilde{\Phi}_x$ is closed, it is a homeomorphism onto its image. The map $\widetilde{\Phi}_x$ is thus an injective immersion that is a homeomorphism onto its image and therefore this image is a closed, embedded submanifold and the map $\widetilde{\Phi}_x$ itself is a diffeomorphism. ■

Page 265, line 1 from bottom. "Here, *ker* Φ_x..." should read "Here, *ker* $T\Phi_x$...".

Page 267, Remark. With suitable interpretations, this remark holds even if the action is not proper and not free. The orbit $G \cdot x$ is endowed, by defintion, with the manifold structure that makes $\widetilde{\Phi}_x$ (defined above Corollary 4.1.22) a diffeomorphism. With this differentiable structure, the orbit $G \cdot x$ is an intial manifold (see the correction for page 557). Since the range of $T_{[x]}\widetilde{\Phi}_x$ is, on one hand, the tangent space to the orbit by the definition of the manifold structure on $G \cdot x$ and, on the other hand, a direct computation shows that it equals $\{\xi_M(x) \mid \xi \in \mathfrak{g}\}$, the statement in the Remark follows. For details see [HRed], Chapter 2.

Page 275. The first displayed formula should read

$$T_g\left(\mathrm{Ad}^*_{\cdot^{-1}}\,\mu\right)(\xi_g) = -\operatorname{ad}\left(T_g R_{g^{-1}}(\xi_g)\right)^* \mathrm{Ad}^*_{g^{-1}}\,\mu$$

and the second displayed formula should read:

$$T_g\left(\mathrm{Ad}^*_{\cdot}\,\mu\right)(\xi_g) = \operatorname{ad}\left(T_g L_{g^{-1}}(\xi_g)\right)^* \mathrm{Ad}^*_g\,\mu$$

For another exposition of formulas like this, see [MandS], §9.3.

4.2 The Momentum Mapping

Page 282, line 2. There should be a minus sign after the equals sign.

Page 285. The momentum map in Corollary 4.2.13 is equivariant.

Page 291, line 17 from the bottom. The kinetic energy expression, namely "$K(q,\dot{q}) = \frac{1}{2}\|q\|^2$" should be "$K(q,\dot{q}) = \frac{1}{2}\|\dot{q}\|^2$"

Page 291, line 7 from the bottom. "$G \circ \mathbf{F}L = \overline{G}$" should be "$\overline{G} \circ \mathbf{F}L = G$"

Page 292, line 5. The last term on this line, namely "$\boldsymbol{d}(G \circ \pi_2)\boldsymbol{d}\pi_1$" should be "$\boldsymbol{d}(G \circ \pi_2) \wedge \boldsymbol{d}\pi_1$"

4.3 Reduction of Phase Spaces with Symmetry

Page 301. The third displayed equation should read

$$T^*_{[x]}(M/G) \cong \{\alpha_x \in T^*_x M \mid \alpha_x(T_x(G \cdot x)) = 0\}$$

See [HStages], §2.2 for details and further explanations.

Page 301, line 3 from the bottom of the main text. One has to assume here that $H^{-1}(e)/\boldsymbol{R}$ is a smooth quotient manifold.

Page 305. The discussion around Figure 4.3-1 is related to the theory of Berry phases and geometric phases, both in classical and quantum mechanics, a topic that has seen much attention in the past couple of decades. While the literature is now quite extensive, a couple of the classical references are Berry [1984] and Wilczek and Shapere [1988]. There is also much mathematical literature, such as Montgomery [1990], Weinstein [1990], and Marsden, Montgomery, and Ratiu [1990]. There is also an extensive Physics literature,

not always connected as well as it could be with the Mathematics literature; see, for instance Chruściński and Jamiołkowski [2004]. Further references and literature can be found in [MandS] and Cushman and Bates [1997].

In the figure itself, the label φ_0 should be p_0 and it should be the point where the two curves intersect.

Page 307, Lemma 4.3.9. There is no need to assume that $E = \ker A \oplus E_1$. Take $e_1 \in E$ such that $A(e_1) = f$. Erase the footnote.

4.5 The Topology of Simple Mechanical Systems

Page 340, line 4 from the bottom of the main text. This line should read "If we assume that f is proper (and so the level sets are compact), then"

Page 343. In line 4, the sentence "If it is surjective then $\mu = J(\alpha)$ is a regular value of J, that is, $\mu \in \mathfrak{g}^* \backslash J(\sigma(J))$." should read "If it is surjective then α is a regular point of J; that is, $\mu \in \mathfrak{g}^* \backslash J(\sigma(J))$."

Chapter 5. Hamilton-Jacobi Theory and Mathematical Physics

5.1 Time-Dependent Systems

Page 372, line −2. The expression $\omega = \boldsymbol{d}\theta$ should be (consistent with the coordinate expression) $\omega = -\boldsymbol{d}\theta$.

Page 373, line −2. Again, it should be $\omega = -\boldsymbol{d}\theta$.

Page 374. Replace the last two lines of **Proposition 5.1.9** with the following: *If $\omega = -\boldsymbol{d}\theta$ and $\widetilde{\theta} = \boldsymbol{d}t + \pi_2^*\theta$, where $t : \mathbb{R} \times P \to \mathbb{R}$ is the projection on the first factor, then $\widetilde{\omega} = -\boldsymbol{d}\widetilde{\theta}$ and $(\mathbb{R} \times P, \widetilde{\theta})$ is an exact contact manifold.*

Page 374. In the third displayed formula."for all ω_p" should be replaced by "for all w_p", in the last line of proof it should read: " since $-\mathbf{d}\widetilde{\theta} = \widetilde{\omega}$ and ...." and on line −2, replace $\widetilde{F}_t(s, m) = (t + s, F_{t,s}(m))$ by $\widetilde{F}_t(s, m) = (t + s, F_{t,0}(m))$.

Page 375, Figure 5.1-1. Replace $\overline{X}$ by $\widetilde{X}$.

Page 376. In the text after the proof of **Proposition 5.1.12**, the positioning of the indices is reversed. It should read

$$\frac{d}{dt}\left[H\left(t, q^i(t), p_j(t)\right)\right] = \frac{\partial H}{\partial t}\left(t, q^i(t), p_j(t)\right)$$

where $q^i(t), p_j(t)$ is an integral .....

Page 376. The last part of (ii) of Theorem 5.1.13 should be $\widetilde{F}_t^*\omega_H = \omega_H$, where $\widetilde{F}_t$ is the flow of $\widetilde{X}_H$. The proof of this is as follows: By the Lie derivative formula

$$\boldsymbol{L}_{\widetilde{X}_H}\omega_H = \boldsymbol{d}\boldsymbol{i}_{\widetilde{X}_H}\omega_H + \boldsymbol{i}_{\widetilde{X}_H}\boldsymbol{d}\omega_H$$

and the first term is zero since, by the first part of (ii), $\boldsymbol{i}_{\widetilde{X}_H}\omega_H = 0$ and the second term is zero since $\boldsymbol{d}\omega_H = 0$.

Page 376. Theorem 5.1.13 (iii) should read as follows: *if* $\omega = -\boldsymbol{d}\theta$ *and* $\theta_H = \pi_2^*\theta - H\boldsymbol{d}t$, *then* $\omega_H = -\boldsymbol{d}\theta_H$; *if* $\theta(X_H) - H$ *is nowhere zero, then* $(\boldsymbol{R}\times P, \theta_H)$ *is an exact contact manifold.*

Page 377. In the fourth line above Theorem 5.1.14, $H + (\theta \circ \pi_2)(X_H)$ should read $\theta(X_H) - H$.

Page 377. In the second line of Theorem 5.1.14, $\omega = \boldsymbol{d}\theta$ should be $\omega = -\boldsymbol{d}\theta$.

Page 377. There are typos in the proof of (iii). It should read as follows: Clearly $-\boldsymbol{d}\theta_H = \omega_H$. Also, using the definition of θ_H,

$$\begin{aligned}\theta_H(\widetilde{X}_H) &= (\pi_2^*\theta)(X_H + \underline{\mathrm{t}}) - H\boldsymbol{d}t((X_H + \underline{\mathrm{t}})\\ &= \theta(X_H) - H\end{aligned}$$

and so θ_H does not vanish on the characteristic bundle of ω_H. Thus, $(\boldsymbol{R}\times P)$ is an exact contact manifold (see 5.1.8) ■

Notice that $\theta(X_H) - H$ is the Lagrangian associated to H according to 3.6.7.

Page 378. In the first line of exercise 5.1.C (ii), insert a minus sign: $\omega = -\boldsymbol{d}\theta$ and in the last line write $(-1/\theta(X_H)) \cdot X_H$.

Page 378. In the first line of Exercise 5.1 G, replace $\theta_H = \widetilde{\theta}_0 + H\boldsymbol{d}t$ by $\theta_H = \widetilde{\theta}_0 - H\boldsymbol{d}t$ and in third line $\boldsymbol{d}g = -\mathbf{i}_Y\omega_H$ by $\boldsymbol{d}g = \mathbf{i}_Y\omega_H$.

5.2 Canonical Transformations and Hamilton-Jacobi Theory

Page 381, lines 7,8,9 from the bottom. The d should be $\boldsymbol{d}$.

Page 384, line 18. [1972) should be [1972]

Page 386, third line of 5.2.8. Replace π by π_2.

Page 386, lines 6 and 8 from the bottom. The d should be $\boldsymbol{d}$. Same correction on the last two lines of the proof of 5.2.13 on page 388.

Page 387. In the third line of the proof of Proposition 5.2.13, "from 5.2.12" should be "from 5.1.13(ii)".

Page 388. In the third line from the end of the proof of Proposition 5.2.13, replace $\mathbf{i}_{F_t^*}\widetilde{\omega}_1$ by $\mathbf{i}_{F^*\underline{t}}\widetilde{\omega}_1$ and replace "and as" by "and by 5.1.13,".

Page 389, the sentence after 5.2.15. This sentence should read: "Notice that if F satisfies (C1) and (C2) then the local existence W is *equivalent* to F being canonical."

Page 389. Two lines above Definition 5.2.17, "All points for $\tilde{X}_K$" should read "At all points, $X_K = 0$ and hence are".

Page 390, lines 3 and 4. These lines need correction and a little more explanation. For clarity, let us write $\widetilde{W}(t, Q^i, P_i)$ for the W which is a time dependent function on P_1, which, in the text, is the W that appears in Definition 5.2.15 and Proposition 5.2.16. Let W be the function of (t, q^i, Q^i) that is obtained by changing variables. That is,

$$W(t, q^i(t, Q^j, P_j), Q^i) = \widetilde{W}(t, Q^j, P_j).$$

By the chain rule,

$$\frac{\partial \widetilde{W}}{\partial t} = \frac{\partial W}{\partial t} + \frac{\partial W}{\partial q^i}\frac{\partial q^i}{\partial t}.$$

One also computes, using the displayed equation preceding Proposition 5.2.16 and the second line on page 390, that

$$\dot{F} = p_i \frac{\partial q^i}{\partial t} = \frac{\partial W}{\partial q^i}\frac{\partial q^i}{\partial t}.$$

Therefore,

$$K = H \circ F + \frac{\partial \widetilde{W}}{\partial t} - \dot{F} = H \circ F + \frac{\partial W}{\partial t}.$$

Remarks. For different arrangement of Hamilton-Jacobi theory that has a simpler treatment of time dependent canonical transformation theory, see [MandS], Section 7.9. Another basic result in this theory, which was an omission in *Foundations* is the Jacobi theorem. Namely, *Allowing L to be time-dependent*, Jacobi in 1866 showed that the ***action integral*** defined by

$$S(q^i, \overline{q}^i, t) = \int_{t_0}^{t} L(q^i(s), \dot{q}^i(s), s)\, ds,$$

where $q^i(s)$ is the solution of the Euler–Lagrange equation subject to the conditions $q^i(t_0) = \overline{q}^i$ and $q^i(t) = q^i$, satisfies the Hamilton–Jacobi equation. There are several implicit assumptions in Jacobi's argument: L is regular and the time $|t - t_0|$ is assumed to be small, so that S is a well-defined function of the endpoints. One can allow $|t - t_0|$ to be large as long as the solution $q(t)$ is near a nonconjugate solution. For a proof and discussion, see [MandS], Section 8.2.

Page 391. In the second paragraph of the proof of Theorem 5.2.19, the third sentence should read: Also, F_λ is symplectic and $\widetilde{F}_* \underline{t} = \widetilde{X}_H$.

Page 392, line 5. The domain of the map α should be $(-\varepsilon, \varepsilon) \times V$.

Page 393. The last part of Definition 5.2.20 defines complete integrability in infinite dimensions. Certainly on line 6, one has to replace T_pP by the tangent space to the common level sets of the integrals. But this topic has many subtleties that are discussed in the literature, such as Kappeler and Pöschel [2003].

Page 394, line -5 of the proof. Replace the lower index k by n.

Page 395, Proposition 5.2.23.

First notice that a proof of the special case for the two torus is given in Example 4.1.11. Since a complete proof in the general case is not so easy to find, we include it here. Our proof of Proposition 5.2.23 will be based on the following classical result that Bröcker and tom Dieck [1995] attribute to Kronecker.

Theorem. *Let $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_n) \in \boldsymbol{R}^n \backslash \{\mathbf{0}\}$. The cyclic group $\{k[\boldsymbol{\nu}] \mid k \in \boldsymbol{Z}\} \subset \boldsymbol{T}^n$ is dense in the n torus $\boldsymbol{T}^n = \boldsymbol{R}^n/\boldsymbol{Z}^n$ if and only if the real numbers $\{1, \nu_1, \ldots, \nu_n\}$ are linearly independent over $\boldsymbol{Z}$.*

Proof. The proof consists of two steps.

Step 1. We will show that $K := \overline{\{k[\boldsymbol{\nu}] \mid k \in \boldsymbol{Z}\}} \neq \boldsymbol{T}^n$ if and only if $[\boldsymbol{\nu}] \in \ker f$ for some non-trivial Lie group homomorphism $f : \boldsymbol{T}^n \to \boldsymbol{T}^1 := S^1$. Indeed, first, note that $K \neq \{[\mathbf{0}]\}$ since $\boldsymbol{\nu} \neq \mathbf{0}$. If $K \neq \boldsymbol{T}^n$, the quotient group $\boldsymbol{T}^n/K$ is a non-trivial compact connected commutative Lie group and hence is isomorphic to a torus $\boldsymbol{T}^p$, $p \geq 1$ (see Exercise 4.1K). Denote this Lie group isomorphism by $\varphi : \boldsymbol{T}^n/K \to \boldsymbol{T}^p$. The canonical projection $\pi : \boldsymbol{T}^n \to \boldsymbol{T}^n/K$ is a surjective submersive Lie group homomorphism. Let $p : \boldsymbol{T}^p \to S^1$ denote the projection on one of the factors in $\boldsymbol{T}^p = S^1 \times \cdots \times S^1$ (p times). Since $\pi(K) = \{[\mathbf{0}]\}$, it follows that $K \subset \ker(p \circ \varphi \circ \pi)$.

Conversely, assume that $[\boldsymbol{\nu}] \in \ker f$ for some nontrivial Lie group homomorphism $f : \boldsymbol{T}^n \to S^1$. Thus $\overline{\{k[\boldsymbol{\nu}] \mid k \in \boldsymbol{Z}\}} \subset \ker f \neq \boldsymbol{T}^n$ since $\ker f$ is a closed Lie subgroup of $\boldsymbol{T}^n$ and f is not the trivial homomorphism mapping $\boldsymbol{T}^n$ to the identity element of S^1.

Step 2. We use Step 1 to prove the theorem. Take a non-trivial Lie group homomorphism $f : \boldsymbol{T}^n \to S^1$. Let $\{\mathbf{e}_1, \ldots, \mathbf{e}_n\}$ be the standard basis of $\boldsymbol{R}^n$ and let $k_i = f'(\mathbf{e}_i) \in \boldsymbol{R}$, where $f' : \boldsymbol{R}^n \to \boldsymbol{R}$ is the induced Lie algebra homomorphism. Since the projections $\pi_n : \boldsymbol{R}^n \to \boldsymbol{R}^n/\boldsymbol{Z}^n = \boldsymbol{T}^n$ are the exponential maps (see Proposition 4.1.7), it follows that $\pi_1 \circ f' = f \circ \pi_n$ and hence $(\pi_1 \circ f')(\boldsymbol{Z}^n) = (f \circ \pi_n)(\boldsymbol{Z}^n) = \{[0]\}$, which implies that $f'(\boldsymbol{Z}^n) \subset \ker \pi_1 = \boldsymbol{Z}$ and hence $k_i \in \mathbf{Z}$. Consequently, $[\boldsymbol{\nu}] \in \ker f$ if and only if

$$[0] = f([\boldsymbol{\nu}]) = (f \circ \pi_n)(\boldsymbol{\nu}) = (\pi_1 \circ f')(\boldsymbol{\nu}) = (\pi_1 \circ f')\left(\sum_{i=1}^n \nu_i \mathbf{e}_i\right) = \pi_1\left(\sum_{i=1}^n \nu_i k_i\right)$$

which is equivalent to $\sum_{i=1}^n \nu_i k_i \in \boldsymbol{Z}$, that is, to the existence of some $k_0 \in \boldsymbol{Z}$ such that $k_0 + \sum_{i=1}^n \nu_i k_i = 0$. We have thus shown that $[\boldsymbol{\nu}] \in \ker f$ if and only if $\{1, \nu_1, \ldots, \nu_n\}$ are linearly dependent over $\boldsymbol{Z}$.

By Step 1, we conclude that $\{1, \nu_1, \ldots, \nu_n\}$ are linearly dependent over $\boldsymbol{Z}$ if and only if $\overline{\{k[\boldsymbol{\nu}] \mid k \in \boldsymbol{Z}\}} \neq \boldsymbol{T}^n$. ■

We will next turn to the proof of Proposition 5.2.23. Note that it assumes something a bit different than Kronecker's result, namely that the numbers $\nu_1, \ldots, \nu_n$ are linearly independent over $\boldsymbol{Z}$, without the extra 1. We will need to show that this condition is equivalent to the denseness of each nontrivial φ_t orbit .

Proof of Proposition 5.2.23. Let $\boldsymbol{\mu} \in \boldsymbol{R}^n$ and recall that $\varphi_t([\boldsymbol{\mu}]) = [\boldsymbol{\mu} + t\boldsymbol{\nu}]$. For fixed $\boldsymbol{\mu} \in \boldsymbol{R}^n$, the map $\psi_{\boldsymbol{\mu}} : [\boldsymbol{\lambda}] \in \boldsymbol{T}^n \mapsto [\boldsymbol{\lambda} + \boldsymbol{\mu}] \in \boldsymbol{T}^n$ is a diffeomorphism and the φ_t-orbit through $[\boldsymbol{\mu}]$ is

$$\begin{aligned}\{\varphi_t([\boldsymbol{\mu}] \mid t \in \boldsymbol{R}\} &= \{[\boldsymbol{\mu} + t\boldsymbol{\nu}] \mid t \in \boldsymbol{R}\} \\ &= \psi_{\boldsymbol{\mu}}\left(\{[t\boldsymbol{\nu}] \mid t \in \boldsymbol{R}\}\right) \\ &= \psi_{\boldsymbol{\mu}}\left(\{\varphi_t([\mathbf{0}]) \mid t \in \boldsymbol{R}\}\right).\end{aligned}$$

Therefore, the φ_t-orbit through $[\boldsymbol{\mu}]$ is dense in $\boldsymbol{T}^n$ if and only if the φ_t-orbit through $[\mathbf{0}]$ is dense in $\boldsymbol{T}^n$. Thus, to prove the proposition, we only need to show that the φ_t-orbit through $[\mathbf{0}]$ is dense in $\boldsymbol{T}^n$ if and only if $\{\nu_1, \ldots, \nu_n\}$ are linearly independent over $\boldsymbol{Z}$.

Since $\boldsymbol{\nu} \neq \mathbf{0}$ by hypothesis, at least one of its components is not zero. So, let us assume that $\nu_n \neq 0$. Then $\{\nu_1, \ldots, \nu_n\}$ are linearly independent over $\boldsymbol{Z}$ if and only if $\{\nu_1/\nu_n, \ldots, \nu_{n-1}/\nu_n, 1\}$ are linearly independent over $\boldsymbol{Z}$ which, by Kronecker's theorem, is equivalent to the fact that the cyclic group C generated by $[\boldsymbol{\nu}']$ is dense in $\boldsymbol{T}^{n-1}$, where $\boldsymbol{\nu}' := (\nu_1/\nu_n, \ldots, \nu_{n-1}/\nu_n) \in \boldsymbol{R}^{n-1}$. However,

$$\begin{aligned}S :&= \{\varphi_t([\mathbf{0}]) \mid t \in \boldsymbol{R}\} \cap \left(\boldsymbol{T}^{n-1} \times \{[0]\}\right) \\ &= \{[t\nu_1, \ldots, t\nu_n] \mid t \in \boldsymbol{R},\ t\nu_n \in \boldsymbol{Z}\} \\ &= \left\{\left[t\nu_n \frac{\nu_1}{\nu_n}, \cdots, t\nu_n \frac{\nu_{n-1}}{\nu_n}, t\nu_n\right] \;\middle|\; t \in \boldsymbol{R},\ t\nu_n \in \boldsymbol{Z}\right\} \\ &= \left\{\left[k\frac{\nu_1}{\nu_n}, \cdots, k\frac{\nu_{n-1}}{\nu_n}, k\right] \;\middle|\; k \in \boldsymbol{Z}\right\} \\ &= \left\{\left[k\frac{\nu_1}{\nu_n}, \cdots, k\frac{\nu_{n-1}}{\nu_n}, 0\right] \;\middle|\; k \in \boldsymbol{Z}\right\}\end{aligned}$$

is exactly this cyclic group C.

In summary, the proposition is proved if we can show that $\{\varphi_t([\mathbf{0}]) \mid t \in \boldsymbol{R}\}$ is dense in $\boldsymbol{T}^n$ if and only if S is dense in $\boldsymbol{T}^{n-1} \times \{[0]\}$. To do this, we first show the "only if" part. So, first assume that $\{\varphi_t([\mathbf{0}]) \mid t \in \boldsymbol{R}\}$ is dense in $\boldsymbol{T}^n$; we need to show that S is dense in $\boldsymbol{T}^{n-1} \times \{[0]\}$. So we work in the unit n-cube with boundaries identified in the usual way. Choose a point $\boldsymbol{x} = (x_1, x_2, \ldots x_{n-1}, 0) \in \boldsymbol{T}^{n-1} \times \{[0]\}$ and pick a Euclidean disk $D_\epsilon(\boldsymbol{x})$ about $\boldsymbol{x}$ of radius $\epsilon > 0$ in $\boldsymbol{T}^n$, so that $D_\epsilon(\boldsymbol{x}) \cap \left(\boldsymbol{T}^{n-1} \times \{[0]\}\right)$ is a neighborhood of $\boldsymbol{x}$ in $\boldsymbol{T}^{n-1} \times \{[0]\}$. Since the orbit $\{\varphi_t([\mathbf{0}]) \mid t \in \boldsymbol{R}\}$ is dense in $\boldsymbol{T}^n$, there is a t_0 such that $\varphi_{t_0}([\mathbf{0}]) = (t_0\nu_1, \ldots t_0\nu_n) \in D_\epsilon(\boldsymbol{x})$. Let $\tau = t_0\nu_n - [\![t_0\nu_n]\!]$, where $[\![t_0\nu_n]\!]$ denotes the integer part of $t_0\nu_n$. Then it is easily checked that the point $\varphi_{t_0 - \tau/\nu_n}([\mathbf{0}])$ lies in $D_\epsilon(\boldsymbol{x}) \cap \left(\boldsymbol{T}^{n-1} \times \{[0]\}\right)$; that is, its last component is zero. This proves the "only if" part as required.

Conversely, showing the "if" part proceeds in a somewhat similar manner; we sketch the main ingredients. So, assume that S is dense in $\boldsymbol{T}^{n-1} \times \{[0]\}$. Now pick a point $\boldsymbol{x} = (x_1, x_2, \ldots x_{n-1}, x_n) \in \boldsymbol{T}^n$; specifically, assume we work with the representative of $\boldsymbol{x}$ that lies in the unit cube. Now again, let $D_\epsilon(\boldsymbol{x})$ be an ϵ-disk about $\boldsymbol{x}$; we need to show that the orbit $\{\varphi_t([\mathbf{0}]) \mid t \in \boldsymbol{R}\}$ intersects

this disk at some point t_0. This is done as follows, consider the new ϵ disk $D_\epsilon(\boldsymbol{y})$ that is obtained by translating the disk $D_\epsilon(\boldsymbol{x})$ as a set, along the orbit through $\boldsymbol{x}$ (which of course need not be the orbit through zero) by an amount $-\tau$, until the center hits the set $\boldsymbol{T}^{n-1} \times \{[0]\}$; that is, $\boldsymbol{y}$ has last component zero. By assumption, there is a point $\boldsymbol{z} \in (S \cap D_\epsilon(\boldsymbol{y})$. Translating this point $\boldsymbol{z}$ along the orbit by the same amount τ produces a point that is close to $\boldsymbol{x}$. One has to take into account the fact that the image of $D_\epsilon(\boldsymbol{y})$ under the flow by an amount τ might not be entirely contained in $D_\epsilon(\boldsymbol{x})$; but this is just a matter of shrinking the original ϵ by a geometrical factor.

Page 395, line 3 of the proof of 5.2.24. Replace 5.2.20 by 5.2.21.

Page 395, the line after the diagram. In the right hand side of the second formula, replace φ_t by χ_t.

Page 396, line −8. Replace $\mathbf{T}^k$ by $\mathbf{T}^n$.

Page 397. In the displayed formula in (ii) of Definition 5.2.25 replace φ^{-1} by ψ^{-1}.

Page 399, the third displayed equations. The various d should be $\boldsymbol{d}$ and the second two integrals should be closed path integrals: $\oint$.

Page 401, Exercise 5.2E. The (i) and (ii) in this Exercise refer to Theorem 5.2.18.

Page 402, Exercise 5.2I. At the end of line 7 of the Exercise, replace $G_\mu \times \mathfrak{g}^*$ by $G_\mu \times \mathfrak{g}^*_\mu$.

5.3 Lagrangian Submanifolds

Page 403. A more direct proof of (iii) of Proposition 5.3.2 is as follows. As in this section, $(\boldsymbol{E}, \boldsymbol{\omega})$ is a finite-dimensional symplectic vector space and $\boldsymbol{F} \subset \boldsymbol{E}$ is a subspace. We want to show that $\dim \boldsymbol{F} + \dim \boldsymbol{F}^\perp = \dim \boldsymbol{E}$. To prove this, note that $\boldsymbol{F}^\perp = \ker(i^* \circ \boldsymbol{\omega}^\flat)$, where $i : \boldsymbol{F} \to \boldsymbol{E}$ is the inclusion and $i^* : \boldsymbol{E}^* \to \boldsymbol{F}^*$ is its dual, which is surjective. But $\boldsymbol{\omega}^\flat$ is an isomorphism and so $i^* \circ \boldsymbol{\omega}^\flat : \boldsymbol{E} \to \boldsymbol{F}^*$ is surjective. Thus, $\boldsymbol{E}/\boldsymbol{F}^\perp$ is isomorphic to $\boldsymbol{F}^*$, so $\dim \boldsymbol{E} - \boldsymbol{F}^\perp = \dim \boldsymbol{F}^* = \dim \boldsymbol{F}$.

Page 423. In Exercise 5.3.K, the "5" is missing in the Exercise label.

5.4 Quantization

Page 445, first paragraph. Assume that P is connected. The last phrase of the paragraph should read as follows: ... are invariant, then $\{H, \widehat{J}(\xi)\}$ is a constant function on P for each $\xi \in \mathfrak{g}$. The reason for this is as follows: Since $H \circ \Phi_{\exp t\xi} - H = c(\exp t\xi)$ is constant on P for all $t \in \mathbb{R}$ and $\xi \in \mathfrak{g}$, taking the derivative at $t = 0$ we conclude that $T_e c(\xi) = \mathbf{d}H\,(\xi_P) = \mathbf{d}H\left(X_{\widehat{J}(\xi)}\right) = \{H, \widehat{J}(\xi)\}$ which is a constant function on P, for every $\xi \in \mathfrak{g}$.

5.5 Introduction to Infinite Dimensional Hamiltonian Systems

Page 467, line 1. $c = 4$ should be $c = -4$.

Page 467, line 9. $f_0(u) = u$ should be $f_0(u) = u/2$ and $\delta f_0/\delta u = 1$ should be $\delta f_0/\delta u = 1/2$.

Chapter 7. Differentiable Dynamics

7.2 Stable Manifolds

Page 527, line 10. The sentence

> "Recall from, Sect. 1.5 that A subset $S \subset M$ is an **immersed submanifold** of it is the image of a mapping $f : V \to M$ that is injective and locally a diffeomorphism onto a submanifold of M."

with the following text:

> "For the statement of the next corollary we need to introduce the concept of an *initial manifold*. Let M and V be smooth manifolds. Then V is said to be an ***initial submanifold*** of M if there is an injective immersion $f : V \to M$ satisfying the following condition: for any smooth manifold P, an arbitrary map $g : P \to V$ is smooth if and only if $f \circ g : P \to M$ is smooth. Often one does not distinguish between V and its image $f(V) \subset M$ and refers to $f(V)$ as an initial submanifold of M.
>
> Useful criteria for when a manifold satisfies this condition may be found in [HRed], Lemma 1.1.11 on page 6 and 1.1.12 in formula (1.1.4) on page 7. The main implications are that any embedded submanifold is an initial manifold which in turn is an injectively immersed submanifold. The reverse implications are, in general, false.

Page 527, last word of line 14. "immersed" should be changed to "initial".

Chapter 8. Hamiltonian Dynamics

8.1 Critical Elements

Page 573, line 2. Replace "Section 3.3" by " Section 3.1".

8.3 Stability of Orbits

Page 585. The famous figure 8.3-3 is of course related to KAM theory about which much has been written since the book was written, such as Gallavotti [1983] and Celletti and Chierchia [2007] and even material in infinite dimensions, such as Kappeler and Pöschel [2003] and Kuksin [2000].

Chapter 9. The Two–Body Problem

Sections 9.2–9.5. The proof of the symplectic nature of the Delaunay variables is incorrect in these sections because of a confusion between the mean and true anomaly, although the overall strategy may be correct. For a correct proof and an interesting approach to this result, the reader can consult the following references:

1. In the paper of Chang and Marsden [2003] an elegant and geometric construction of the Delaunay variables is given as follows: a symplectic $\mathbb{T}^3$ (three torus) action, together with its associated momentum mapping $\mathbf{J}$ is constructed on the elliptic elements of the Kepler problem with the property that the angle variables of $\mathbb{T}^3$ together with the components of $\mathbf{J}$ comprise the Delaunay variables. It follows naturally from this construction that these variables are canonical. This is the main result, attributed to Lagrange, in Theorem 9.4.1 on page 635. The construction of Chang and Marsden [2003] has other nice features, such as yielding an interpretation of the phase shift of satellites, when the bulge of the Earth is taken into account, as a geometric phase.
2. There is another derivation of the Delaunay variables using Hamilton-Jacobi theory, which can be found in § 21, 22 of Born [1927], which is summarized as follows. The rotational symmetry of the Kepler Hamiltonian allows one to use separation of variables in the Hamilton-Jacobi equation, yielding three action variables. This step involves a special integration trick using complex variables due to Sommerfeld. Then, one makes use of the degeneracy of the corresponding angle variables to obtain a new set of angle and action variables so that two of the three angle variables do not change in time. Finally, one seeks the physical meaning of this set of action-angle variables, which requires nontrivial geometric intuition.
3. Another approach to constructing the Delaunay variables can be based on the Liouville-Arnold theorem. This approach is sketched in Arnold, Kozlov, and Neishtadt [2006], which refers to Charlier [1927] for details. In this approach one begins with first integrals in involution. Even though this general machinery guarantees that one gets a set of action-angle variables, it lacks geometric insight and it involves some complicated integrations.

9.6 Poincaré Variables

Page 647. The reference for this section is Poincaré [2005]. In particular, in this reference in Volume I, Chapter III, pages 79-84, it is shown that the transformation between the Poincaré elements and the Cartesian coordinates with their conjugate variables is an analytic diffeomorphism from the set

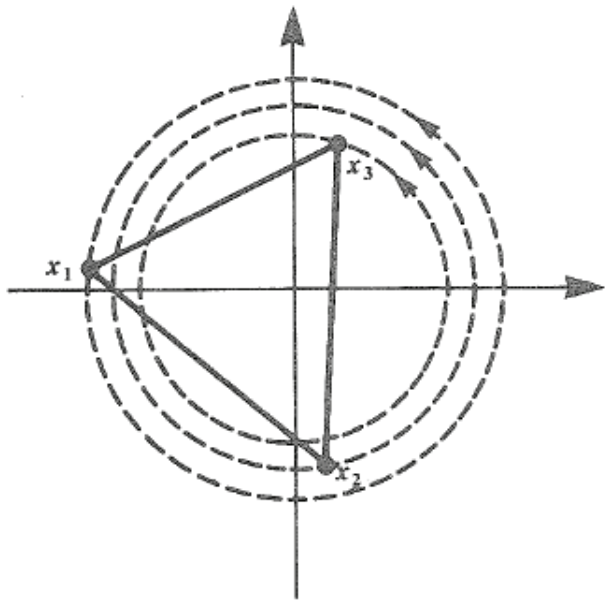
$$\left\{(L, x, y) \in \mathbb{R}^3 \mid L > 0, |x| < \sqrt{2L}, |y| < \sqrt{2L}\right\} \times S^1$$

to the subset of the phase space of the Kepler planar problem consisting of the elliptic Keplarian orbits.

Chapter 10. The Three–Body Problem

10.4 Topology of the Planar n-Body Problem

Page 721. Figure 10.4.1, illustrating Theorem 10.4.13 should look as follows.



Page 740, Conjecture 10.4.25. This conjecture has been solved in Llibre and Simo [1981].

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