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S. R. S. VARADHAN

Probability Theory

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Preface

These notes are based on a first-year graduate course on probability and limit theorems given at Courant Institute of Mathematical Sciences. Originally written during the academic year 1996-97, they have been subsequently revised during the academic year 1998-99 as well as in the Fall of 1999. Several people have helped me with suggestions and corrections and I wish to express my gratitude to them. In particular, I want to mention Prof. Charles Newman, Mr. Enrique Loubet, and Ms. Vera Peshchansky. Chuck used it while teaching the same course in 1998–99, Enrique helped me as TA when I taught from these notes again in the fall of 1999, and Vera, who took the course in the fall of 2000, provided me with a detailed list of corrections. These notes cover about three-fourths of the course, essentially discrete time processes. Hopefully there will appear a companion volume some time in the near future that will cover continuous time processes. A small amount of measure theory is included. While it is not meant to be complete, it is my hope that it will be useful.

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Probability Theory

S. R. S. VARADHAN

This volume presents topics in probability theory covered during a first-year graduate course given at the Courant Institute of Mathematical Sciences. The necessary background material in measure theory is developed, including the standard topics, such as extension theorem, construction of measures, integration, product spaces, Radon-Nikodym theorem, and conditional expectation.

In the first part of the book, characteristic functions are introduced, followed by the study of weak convergence of probability distributions. Then both the weak and strong limit theorems for sums of independent random variables are proved, including the weak and strong laws of large numbers, central limit theorems, laws of the iterated logarithm, and the Kolmogorov three series theorem. The first part concludes with infinitely divisible distributions and limit theorems for sums of uniformly infinitesimal independent random variables.

The second part of the book mainly deals with dependent random variables, particularly martingales and Markov chains. Topics include standard results regarding discrete parameter martingales and Doob's inequalities. The standard topics in Markov chains are treated, i.e., transience, and null and positive recurrence. A varied collection of examples is given to demonstrate the connection between martingales and Markov chains.

Additional topics covered in the book include stationary Gaussian processes, ergodic theorems, dynamic programming, optimal stopping, and filtering. A large number of examples and exercises is included. The book is a suitable text for a first-year graduate course in probability.

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