COURANT 11

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LECTURE NOTES

Supersymmetry for Mathematicians: An Introduction

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Supersymmetry for Mathematicians: An Introduction

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Preface

These notes are essentially the contents of a minicourse I gave at the Courant Institute in the fall of 2002. I have expanded the lectures by discussing spinors at greater length and by including treatments of integration theory and the local Frobenius theorem, but otherwise have not altered the plan of the course. My aim was and is to give an introduction to some of the mathematical aspects of supersymmetry with occasional physical motivation. I do not discuss supergravity.

Not much is original in these notes. I have drawn freely and heavily from the beautiful exposition of P. Deligne and J. Morgan, which is part of the AMS volumes on quantum field theory and strings for mathematicians, and from the books and articles of D. S. Freed and D. A. Leites, all of which and more are referred to in the introduction.

I have profited greatly from the lectures that Professor S. Ferrara gave at UCLA as well as from many extended conversations with him, both at UCLA and at CERN, where I spent a month in 2001. He introduced me to this part of mathematical physics and was a guide and participant on a seminar on supersymmetry that I ran in UCLA in 2000 with Rita Fioresi. I am deeply grateful to him for his unfailing patience and courtesy. I also gave a course in UCLA and a miniworkshop on supersymmetry in 2000 in Genoa, Italy, in the Istituto Nazionale di Fisica Nucleare. I am very grateful to Professors E. Beltrametti and G. Cassinelli, who arranged that visit; to Paolo Aniello, who made notes of my UCLA course; to Ernesto De Vito and Alberto Levrero, whose enthusiasm and energy made the Genoa workshop so memorable; and finally to Lauren Caston, who participated in the Courant course with great energy and enthusiasm. I also wish to thank Alessandro Toigo and Claudio Carmeli of INFN, Genoa, who worked through the entire manuscript and furnished me with a list of errors and misprints in the original version of the notes, and whose infectious enthusiasm lifted my spirits in the last stages of this work. I am very grateful to Julie Honig for her help during all stages of this work. Last, but not least, I wish to record my special thanks to Paul Monsour and Reeva Goldsmith whose tremendous effort in preparing and editing the manuscript has made this book enormously better than what it was when I sent it to them.

The course in the Courant Institute was given at the suggestion of Professor S. R. S. Varadhan. My visit came at a time of mourning and tragedy for him in the aftermath of the 9/11 catastrophe, and I do not know how he found the time and energy to take care of all of us. It was a very special time for us and for him, and in my mind this course and these notes will always appear as a small effort on my part to alleviate the pain and grief by thinking about some beautiful things that are far bigger than ourselves.

V. S. Varadarajan
Pacific Palisades
March 2004

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Supersymmetry for Mathematicians: An Introduction

V. S. VARADARAJAN

Supersymmetry has been the object of study by theoretical physicists since the early 1970's. In recent years it has attracted the interest of mathematicians because of its novelty, and because of significance, both in mathematics and physics, of the main issues it raises.

This book presents the foundations of supersymmetry to the mathematically minded reader in a cogent and self-contained manner. It begins with a brief introduction to the physical foundations of the theory, especially the classification of relativistic particles and their wave equations, such as the equations of Dirac and Weyl. It then continues the development of the theory of supermanifolds stressing the analogy with the Grothendieck theory of schemes. All the super linear algebra needed for the book is developed here and the basic theorems are established: differential and integral calculus in supermanifolds, Frobenius theorem, foundations of the theory of super Lie groups, and so on. A special feature of the book is the treatment in depth of the theory of spinors in all dimensions and signatures, which is the basis of all developments of supergeometry both in physics and mathematics, especially in quantum field theory and supergravity.

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