

C O U R A N T

15

EMIL ARTIN

Notes by Albert A. Blank

LECTURE
NOTES

Algebra with Galois Theory

American Mathematical Society
Courant Institute of Mathematical Sciences



Algebra with Galois Theory

**Courant Lecture Notes
in Mathematics**

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Courant Institute of Mathematical Sciences
New York University
New York, New York

American Mathematical Society
Providence, Rhode Island

2000 *Mathematics Subject Classification*. Primary 12-01, 12F10.

Library of Congress Cataloging-in-Publication Data

Artin, Emil, 1898-1962.

Algebra with Galois theory / E. Artin, notes by Albert A. Blank.

p. cm. — (Courant lecture notes ; 15)

ISBN 978-0-8218-4129-7 (alk. paper)

1. Galois theory. 2. Algebra. I. Blank, Albert A. II. Title.

QA214.A76 2007
512—dc22

2007060799

Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.

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10 9 8 7 6 5 4 3 2 12 11 10

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Editors' Note

Because what was in 1947 “modern” has now become standard, and what was then “higher” has now become foundational, we have retitled this volume *Algebra with Galois Theory* from the original *Modern Higher Algebra. Galois Theory*.

Jalal Shatah, *Executive Editor*

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August 2007

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Notes by Albert A. Blank

The present text was first published in 1947 by the Courant Institute of Mathematical Sciences of New York University. Published under the title *Modern Higher Algebra. Galois Theory*, it was based on lectures by Emil Artin and written by Albert A. Blank. This volume became one of the most popular in the series of lecture notes published by Courant. Many instructors used the book as a textbook, and it was popular among students as a supplementary text as well as a primary textbook. Because of its popularity, Courant has republished the volume under the new title *Algebra with Galois Theory*.

ISBN 978-0-8218-4129-7



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