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S.R.S. VARADHAN

LECTURE NOTES

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Harmonic Analysis

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Preface

Harmonic analysis is the study of functions by decomposing them into components that are special functions. A prime example is decomposing a periodic function into a linear combination of sines and cosines. The subject is vast, and these notes cover only the material that was dealt with in the course given at the Institute in 2000 and 2019.

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Harmonic Analysis

S.R.S. VARADHAN

Harmonic Analysis is an important tool that plays a vital role in many areas of mathematics as well as applications. It studies functions by decomposing them into components that are special functions. A prime example is decomposing a periodic function into a linear combination of sines and cosines. The subject is vast, and this book covers only the selection of topics that was dealt with in the course given at the Courant Institute in 2000 and 2019. These include standard topics like Fourier series and Fourier transforms of functions, as well as issues of convergence of Abel, Feier, and Poisson sums. At a slightly more advanced level the book studies convolutions with singular integrals, fractional derivatives, Sobolev spaces, embedding theorems, Hardy spaces, and BMO. Applications to elliptic partial differential equations and prediction theory are explored. Some space is devoted to harmonic analysis on compact non-Abelian groups and their representations, including some details about two groups: the permutation group and SO(3).

The text contains exercises at the end of most chapters and is suitable for advanced undergraduate students as well as first- or second-year graduate students specializing in the areas of analysis, PDE, probability or applied mathematics.

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