

American Mathematical Society

Colloquium Publications

Volume 23

Orthogonal Polynomials

Gabor Szegő



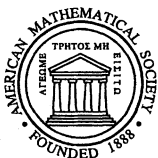
American Mathematical Society

Colloquium Publications

Volume 23

Orthogonal Polynomials

Gabor Szegő



American Mathematical Society
Providence, Rhode Island

2000 *Mathematics Subject Classification*. Primary 33-XX.

Library of Congress Card Number 39-33497
ISBN 0-8218-1023-5
ISSN 0065-9258

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to reprint-permission@ams.org.

© 1939 by the American Mathematical Society. All rights reserved.

Reprinted with corrections, 2003

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.

15 14 13 12

14 13 12 11 10 09

TO MY WIFE

This page intentionally left blank

PREFACE

Recent years have seen a great deal of progress in the field of orthogonal polynomials, a subject closely related to many important branches of analysis. Orthogonal polynomials are connected with trigonometric, hypergeometric, Bessel, and elliptic functions, are related to the theory of continued fractions and to important problems of interpolation and mechanical quadrature, and are of occasional occurrence in the theories of differential and integral equations. In addition, they furnish comparatively general and instructive illustrations of certain situations in the theory of orthogonal systems. Recently, some of these polynomials have been shown to be of significance in quantum mechanics and in mathematical statistics.

The origins of the subject are to be found in the investigation of a certain type of continued fractions, bearing the name of Stieltjes. Special cases of these fractions were studied by Gauss, Jacobi, Christoffel, and Mehler, among others, while more general aspects of their theory were given by Tchebichef, Heine, Stieltjes, and A. Markoff.

Despite the close relationship between continued fractions and the problem of moments, and notwithstanding recent important advances in this latter subject, continued fractions have been gradually abandoned as a starting point for the theory of orthogonal polynomials. In their place, the orthogonal property itself has been taken as basic, and it is this point of view which has been adopted in the following exposition of the subject. Choosing this same basic property, we discuss certain special orthogonal polynomials, which have been treated in great detail independently of the general theory, and indeed, even before this theory existed at all. In this connection we add the names of Laplace, Legendre, Fourier, Abel, Laguerre, and Hermite to those previously mentioned.

As regards treatises on the subject, we note that the only systematic treatment thus far given is found in J. Shohat's monograph, *Théorie Générale des Polynomes Orthogonaux de Tchebichef*, Mémorial des Sciences Mathématiques, Paris, 1934. Limitations of space have compelled that work to be brief, and consequently, it does not enter into a detailed treatment of many problems which have been especially advanced in recent years. It has therefore seemed desirable to attempt a new and detailed development of the main ideas of this field, devoting, in particular, some space to recent investigations of the distribution of the zeros, of asymptotic representations, of expansion problems, and of certain questions of interpolation and mechanical quadrature.

In what follows, we are concerned partly with the general theory of orthogonal polynomials, and partly with the study of special classes of these polynomials. As might be expected, we have more exhaustive results for these special classes, and we cite as an instance the classical polynomials satisfying linear differential

equations of the second order. Also, when the primary importance of these special classes in applications is taken into account, it should not be at all surprising that the present book is mainly devoted to their study. The general theory, however, as developed in Chapters XII and XIII, doubtless represents the most important progress made in recent years.

In the present work, no claim is made for completeness of treatment. On the contrary, the aim has purposely been to make the material suggestive rather than exhaustive. An attempt has been made to indicate the main and characteristic methods and to point out the relation of these to some general ideas in modern analysis. As a rule, preference has been given to those topics to which we were able to make some new, though modest, contributions, or which we could present in a new setting. Thus the book contains a number of results not previously published, some of which originated several years ago. For instance, we have included a discussion of the Cesàro summability of the Jacobi series at the end-points of the orthogonality interval (the method used here is of interest even in the classical case of Legendre series). Further, a new and simpler approach has been given to S. Bernstein's asymptotic formula for orthogonal polynomials. We also refer to certain details of minor importance, such as: simplifications and additions in the asymptotic investigation of Jacobi and Laguerre polynomials and in the discussion of the expansions in terms of these polynomials; the discussion of the cases in which the Jacobi differential equation has only polynomial solutions; the evaluation of the number of zeros of general Jacobi polynomials in the intervals $[-\infty, -1]$, $[-1, +1]$, $[+1, +\infty]$; a new proof of the Heine-Stieltjes theorem on linear differential equations of the second order with polynomial coefficients and polynomial solutions, and so on.

In general, we have preferred to discuss problems which may be stated and treated simply, and which could be presented in a more or less complete form. This was the main reason for devoting no space to the extremely interesting arithmetic and algebraic properties of orthogonal polynomials, such as, for instance, the recent important investigations of I. Schur concerning the irreducibility and related properties of Laguerre and Hermite polynomials. Furthermore, we have attached great importance to the idea of replacing incomplete and overlapping theorems, scattered in the literature, by complete results involving only intrinsic or necessary restrictions. We have also tried to exploit, as far as seemed to be at all possible, definite methods, such as, for instance, Sturm's methods in differential equations (see §§6.3, 6.31, 6.32, 6.83).

A complete treatment of Legendre polynomials was not feasible, and probably not desirable, in the framework of the general theory. Besides, there are already complete treatises on spherical and other harmonics.¹ We have selected and considered only those properties of Legendre polynomials which are the starting points of generalizations to ultraspherical, Jacobi, or to more general polynomials. Another subject which could not be included was Stieltjes'

¹ For instance, E. W. Hobson 1 (see bibliography).

problem of moments, which has been omitted in spite of its great interest; for this subject would have necessitated the development of a complicated apparatus of results and methods. Orthogonal polynomials of more than one variable also have not been treated.²

The book is based on a course given at Washington University during the academic year 1935–1936. Acquaintance with the general ideas and methods of the theory of functions of real and complex variables is naturally required. Occasionally, Stieltjes-Lebesgue and Lebesgue integrals are considered. In the greater part of the book, however, these integrals have been avoided, and, except in a very few places, no detailed properties of them were used.

The problems at the end of this book are, with few exceptions, not new, and they are not interconnected as are, for instance, those in Pólya-Szegő's *Aufgaben und Lehrsätze*. They are more or less supplementary in character and serve as illustrations and exercises; they sometimes differ widely from one another both as to subject and method.

The list of references is not complete; it contains only original memoirs, a few text books of primary importance, and monographs to which references are made in the text.

For the suggestion of preparing a book on orthogonal polynomials for the Colloquium Publications, I am indebted to Professor J. D. Tamarkin, who has also participated in the present work by offering a great number of valuable suggestions. It is with the greatest gratitude that I mention his friendly interest.

I have also received valuable advice from my friends and teachers L. Fejér (Budapest), and G. Pólya (Zürich). My colleagues P. Erdős (Manchester), G. Grünwald (Budapest), W. H. Roever (St. Louis), A. Ross (St. Louis), J. Shohat (Philadelphia), and P. Turán (Budapest) gave generously and unstintingly of their time. F. A. Butter, Jr. (at present in Los Angeles) collaborated with me in the preparation of the manuscript. This last aid was made possible through a grant from the Rockefeller Research Fund of Washington University (1936–1937). My student L. H. Kanter also rendered valuable assistance in the preparation of the manuscript.

My gratitude for the encouragement and help of these friends, colleagues, and institutions can hardly be measured by any formal acknowledgment. Lastly, I wish to express to the American Mathematical Society my great appreciation for the inclusion of the present book in its Colloquium Series.

G. SZEGÖ

WASHINGTON UNIVERSITY, 1938.

² Cf. the bibliography in Jackson 8, p. 423.

This page intentionally left blank

PREFACE TO THE REVISED EDITION

The first printing of this book published in 1939 was about exhausted in 1948. Reprinting was arranged then but for various reasons no change in the text was made. During the past twenty years since the preparation of the original edition was completed, considerable progress was made in this field. A glance at the pertinent section of the Mathematical Reviews suggests that the interest in this topic is still very much alive. Systematic treatment of orthogonal polynomials has been incorporated in various modern texts published in the meantime. We refer only to the *Higher Transcendental Functions* published by the Bateman Manuscript Project Staff (cf. in particular, vol. 2, Chapter X, edited by Professor A. Erdélyi), and to the book of F. Tricomi, *Vorlesungen über Orthogonalreihen* (Chapters IV–VI).

Recently the council of the American Mathematical Society has authorized the author to prepare a revised edition of the book, adding a moderate amount of material in order to bring it up to date. Naturally, limitations of space and time did not allow including all new results (or, for that matter, the old ones which were missing from the original edition). Only a few particularly interesting new items have been added as well as some details which deserve attention because of elegance of the method or originality of ideas. We mention here in particular the important Pollaczek polynomials; they are treated in an Appendix. Further new material was incorporated in the form of Problems and Exercises. New bibliographic items have been included, again in a rather selective way. Finally, misprints have been corrected and numerous minor improvements and additions made.

The author recollects again, as was stated in the Preface of 1938, that the preparation of this book was suggested to him by the late Professor J. D. Tamarkin. Since his untimely death in 1945 his name is not too frequently mentioned. It is justified and probably necessary to remind the younger mathematical generation, in the rush of modern developments, how much American mathematics owes to his great energy and far-sighted intelligence.

STANFORD UNIVERSITY, 1958

G. SZEGÖ

This page intentionally left blank

PREFACE TO THE THIRD EDITION

The interest of the mathematical community for orthogonal polynomials, classical and non-classical, is still not entirely exhausted. During the past years I lectured about this subject several times at Stanford. The attendants of the course were upper division and graduate students, specializing in mathematics, mathematical statistics, calculus of probability, etc.

Only minor changes have been made in the text. I owe numerous improvements and corrections to various friends and colleagues. I mention particularly Professor Paul Turan (Budapest, Hungary) and Professor Lee Lorch (Edmonton, Canada). New references, published in the time interval 1958-1966, have been included.

STANFORD UNIVERSITY, 1966

G. SZEGÖ

PREFACE TO THE FOURTH EDITION

Again the American Mathematical Society has taken the initiative to reprint the present book, allowing some minor changes and new material. Among the persons interested in the field of orthogonal polynomials who have contributed to these changes and additions, I mention with particular indebtedness my friend and colleague Professor Richard Askey (Madison, Wisconsin) and the very active and original group of mathematicians around him. A very important set of lectures by Askey entitled, "Orthogonal Polynomials and Special Functions," reached me too late to be incorporated in the present edition.

Further material has been furnished by Professor Paul Turán (Budapest, Hungary) and Professor Lee Lorch (Toronto, Canada). New problems and exercises have also been included. Peter Szego (Redwood City, California) gave me valuable assistance in preparing the present manuscript.

My gratitude goes to all these friends and colleagues.

STANFORD UNIVERSITY, 1975

G. SZEGÖ

This page intentionally left blank

TABLE OF CONTENTS

	PAGE
PREFACE	v
PREFACE TO THE REVISED EDITION	ix
PREFACE TO THE THIRD EDITION.	xi
PREFACE TO THE FOURTH EDITION	xi
CHAPTER I. PRELIMINARIES	1
CHAPTER II. DEFINITION OF ORTHOGONAL POLYNOMIALS; PRINCIPAL EXAMPLES	23
CHAPTER III. GENERAL PROPERTIES OF ORTHOGONAL POLYNOMIALS	38
CHAPTER IV. JACOBI POLYNOMIALS	58
CHAPTER V. LAGUERRE AND HERMITE POLYNOMIALS	100
CHAPTER VI. ZEROS OF ORTHOGONAL POLYNOMIALS	111
CHAPTER VII. INEQUALITIES	159
CHAPTER VIII. ASYMPTOTIC PROPERTIES OF THE CLASSICAL POLYNOMIALS	191
CHAPTER IX. EXPANSION PROBLEMS ASSOCIATED WITH THE CLASSICAL POLYNOMIALS	244
CHAPTER X. REPRESENTATION OF POSITIVE FUNCTIONS	274
CHAPTER XI. POLYNOMIALS ORTHOGONAL ON THE UNIT CIRCLE	287
CHAPTER XII. ASYMPTOTIC PROPERTIES OF GENERAL ORTHOGONAL POLYNOMIALS	296
CHAPTER XIII. EXPANSION PROBLEMS ASSOCIATED WITH GENERAL ORTHOGONAL POLYNOMIALS	313
CHAPTER XIV. INTERPOLATION	329
CHAPTER XV. MECHANICAL QUADRATURE	349
CHAPTER XVI. POLYNOMIALS ORTHOGONAL ON AN ARBITRARY CURVE	364
PROBLEMS AND EXERCISES	377
FURTHER PROBLEMS AND EXERCISES	387
APPENDIX	393
LIST OF REFERENCES	401
FURTHER REFERENCES	416
INDEX	425

This page intentionally left blank

LIST OF REFERENCES⁷³

ABEL, N. H.

- *1. *Oeuvres Complètes*. Vol. 2, 1881.

ACHESER, N. I.

1. *Über eine Eigenschaft der "elliptischen" Polynome*. Communications de la Société Mathématique de Kharkoff, (4), vol. 9 (1934), pp. 3-8.
2. *Verallgemeinerung einer Korkine-Zolotareffschen Minimum-Aufgabe*. Ibid., (4), vol. 13 (1936), pp. 3-14.

ADAMOFF, A.

1. *On the asymptotic expansion of the polynomials $U_n(x) = e^{ax^2/2} d^n[e^{-ax^2/2}]/dx^n$ for large values of n* (in Russian). Annals of the Polytechnic Institute of St. Petersburg, vol. 5 (1906), pp. 127-143.
2. *Expansions of an Arbitrary Function of a Single Real Variable in Series of Functions of a Preassigned Kind* (in Russian). Thesis. St. Petersburg, 1907, 191 pp.

BANACH, S.

- *1. *Théorie des Opérations Linéaires*. Warszawa-Lwów, 1932.

BERNSTEIN, S.

- *1. *Leçons sur les Propriétés Extrémales et la Meilleure Approximation des Fonctions Analytiques d'une Variable Réelle*. Paris, 1926.
2. *Sur les polynomes orthogonaux relatifs à un segment fini*. Journal de Mathématiques, (9), vol. 9 (1930), pp. 127-177; vol. 10 (1931), pp. 219-286.
3. *Sur une classe de polynomes orthogonaux*. Communications de la Société Mathématique de Kharkoff, (4), vol. 4 (1930), pp. 79-93. *Complément*. Ibid., vol. 5 (1932), pp. 59-60.
4. *Sur la limitation des valeurs d'un polynôme $P_n(x)$ de degré n sur tout un segment par ses valeurs en $(n + 1)$ points du segment*. Bulletin de l'Académie des Sciences de l'URSS, 1931, pp. 1025-1050.

BLUMENTHAL, O.

1. *Ueber die Entwicklung einer willkürlichen Funktion nach den Nennern des Kettenbruches für $\int_{-\infty}^0 [\phi(\xi)/(z - \xi)]d\xi$* . Inaugural-Dissertation. Göttingen, 1898.

BOCHNER, S.

1. *Über Sturm-Liouvillesche Polynomsysteme*. Mathematische Zeitschrift, vol. 29 (1929), pp. 730-736.

BOTTEMA, O.

1. *Die Nullstellen der Hermiteschen Polynome*. Koninklijke Akademie van Wetenschappen te Amsterdam, Proceedings, vol. 33 (1930), pp. 495-503.
2. *Die Nullstellen gewisser durch Rekursionsformeln definierten Polynome*. Ibid., vol. 34 (1931), pp. 681-691.

BRAUER, A.

1. *Über die Nullstellen der Hermiteschen Polynome*. Mathematische Annalen, vol. 107 (1932), pp. 87-89.

BRUNS, H.

1. *Zur Theorie der Kugelfunctionen*. Journal für die reine und angewandte Mathematik, vol. 90 (1881), pp. 322-328.

BUELL, C. E.

1. *The zeros of Jacobi and related polynomials*. Duke Mathematical Journal, vol. 2 (1936), pp. 304-316.

⁷³ The asterisks indicate items not dealing with orthogonal polynomials.

CARATHÉODORY, C.

- *1. *Conformal Representation*. Cambridge, 1932.

CARLEMAN, T.

1. *Über die Approximation analytischer Funktionen durch lineare Aggregate von vorgegebenen Potenzen*. Arkiv för Matematik, Astronomi och Fysik, vol. 17 (1923), no. 9, 30 pp.

CHRISTOFFEL, E. B.

1. *Über die Gaussische Quadratur und eine Verallgemeinerung derselben*. Journal für die reine und angewandte Mathematik, vol. 55 (1858), pp. 61-82.

CRAMÉR, H.

1. *On some classes of series used in mathematical statistics*. Comptes Rendus du Sixième Congrès des Mathématiciens Scandinaves, Stockholm, 1926, pp. 399-425.

DARBOUX, G.

1. *Mémoire sur l'approximation des fonctions de très grands nombres*. Journal de Mathématiques, (3), vol. 4 (1878), pp. 5-56, 377-416.

DIRICHLET, G. L.

1. *Sur les séries dont le terme général dépend de deux angles, et qui servent à exprimer des fonctions arbitraires entre des limites données*. Journal für die reine und angewandte Mathematik, vol. 17 (1837), pp. 35-56.

DOETSCH, G.

1. *Integraleigenschaften der Hermiteschen Polynome*. Mathematische Zeitschrift, vol. 32 (1930), pp. 587-599.
 2. *Die in der Statistik seltener Ereignisse auftretenden Charlierschen Polynome und eine damit zusammenhängende Differentialdifferenzgleichung*. Mathematische Annalen, vol. 109 (1933), pp. 257-266.

DU BOIS-REYMOND, P.

- *1. *Untersuchungen über die Convergenz und Divergenz der Fourierschen Darstellungsformeln*. Abhandlungen der Akademie München, vol. 12 (1876), pp. 1-103.

ERDÉLYI, A.

1. *Über eine Integraldarstellung der $M_{k,m}$ -Funktionen und ihre asymptotische Darstellung für grosse Werte von $\Re k$* . Mathematische Annalen, vol. 113 (1936), pp. 357-362.
 2. *Über eine erzeugende Funktion von Produkten Hermitescher Polynome*. Mathematische Zeitschrift, vol. 44 (1938), pp. 201-211.

ERDÖS, P., and FELDHEIM, E.

1. *Sur le mode de convergence pour l'interpolation de Lagrange*. Comptes Rendus de l'Académie des Sciences, Paris, vol. 203 (1936), pp. 913-915.

ERDÖS, P., and TURÁN, P.

1. *On interpolation. I. Quadrature- and mean-convergence in the Lagrange interpolation*. Annals of Mathematics, (2), vol. 38 (1937), pp. 142-155.
 2. *On interpolation. II. On the distribution of the fundamental points of Lagrange and Hermite interpolation*. Ibid., vol. 39 (1938), pp. 703-724.

EULER, L.

- *1. *Institutiones Calculi Integralis*. Vol. 2, sect. I, chap. X, problem 130. *Opera Omnia*. Ser. 1, vol. 12, p. 224.

FABER, G.

1. *Über polynomische Entwicklungen*. Mathematische Annalen, vol. 57 (1903), pp. 389-408.
 *2. *Über die interpolatorische Darstellung stetiger Funktionen*. Jahresbericht der Deutschen Mathematiker-Vereinigung, vol. 23 (1914), pp. 192-210.
 3. *Tschebyscheffsche Polynome*. Journal für die reine und angewandte Mathematik, vol. 150 (1919), pp. 79-106.

4. *Über nach Polynomen fortschreitende Reihen.* Sitzungsberichte der Bayrischen Akademie der Wissenschaften, 1922, pp. 157–178.
- FATOU, P.
 *1. *Séries trigonométriques et séries de Taylor.* Acta Mathematica, vol. 30 (1906), pp. 335–400.
- FAVARD, J.
 1. *Sur les polynomes de Tchebicheff.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 200 (1935), pp. 2052–2053.
- FEJÉR, L.
 *1. *Sur les fonctions bornées et intégrables.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 131 (1900), pp. 984–987.
 *2. *Untersuchungen über Fouriersche Reihen.* Mathematische Annalen, vol. 58 (1904), pp. 51–69.
 3. *Asymptotikus értékek meghatározásáról.* Matematikai és Természettudományi Értesítő, vol. 27 (1909), pp. 1–33.
 4. *Über die Laplacesche Reihe.* Mathematische Annalen, vol. 67 (1909), pp. 76–109.
 *5. *Über trigonometrische Polynome.* Journal für die reine und angewandte Mathematik, vol. 146 (1915), pp. 53–82.
 6. *Über Interpolation.* Nachrichten der Gesellschaft der Wissenschaften zu Göttingen, 1916, pp. 66–91.
 7. *Über die Lage der Nullstellen von Polynomen, die aus Minimumforderungen gewisser Art entspringen.* Mathematische Annalen, vol. 85 (1922), pp. 41–48.
 8. *Über die Summabilität der Laplaceschen Reihe durch arithmetische Mittel.* Mathematische Zeitschrift, vol. 24 (1925), pp. 267–284.
 9. *Abschätzungen für die Legendreschen und verwandte Polynome.* Mathematische Zeitschrift, vol. 24 (1925), pp. 285–298.
 10. *Über Weierstrass'sche Approximation, besonders durch Hermitesche Interpolation.* Mathematische Annalen, vol. 102 (1930), pp. 707–725.
 11. *Die Abschätzung eines Polynoms in einem Intervalle, wenn Schranken für seine Werte und ersten Ableitungswerte in einzelnen Punkten des Intervalles gegeben sind, und ihre Anwendung auf die Konvergenzfrage Hermitescher Interpolationsreihen.* Mathematische Zeitschrift, vol. 32 (1930), pp. 426–457.
 12. *Ultrasphärikus polynomok összegéről.* Matematikai és Fizikai Lapok, vol. 38 (1931), pp. 161–164.
 13. *Lagrangesche Interpolation und die zugehörigen konjugierten Punkte.* Mathematische Annalen, vol. 106 (1932), pp. 1–55.
 14. *Bestimmung derjenigen Abszissen eines Intervalles, für welche die Quadratsumme der Grundfunktionen der Lagrangeschen Interpolation im Intervalle ein möglichst kleines Maximum besitzt.* Annali della Scuola Normale Superiore di Pisa, (2), vol. 1 (1932), pp. 3–16.
 15. *Mechanische Quadraturen mit positiven Cotesschen Zahlen.* Mathematische Zeitschrift, vol. 37 (1933), pp. 287–309.
 16. *On the characterization of some remarkable systems of points of interpolation by means of conjugate points.* American Mathematical Monthly, vol. 41 (1934), pp. 1–14.
 17. *Potenzreihen mit mehrfach monotoner Koeffizientenfolge und ihre Legendre Polynome.* Proceedings of the Cambridge Philosophical Society, vol. 31 (1935), pp. 307–316.
 18. *A hatványsorról és a vele kapcsolatos Legendre-féle többtagúakról.* Matematikai és Természettudományi Értesítő, vol. 53 (1935), pp. 1–17.
 19. *Bestimmung von Grenzen für die Nullstellen des Legendreschen Polynoms aus der Stieltjeschen Integraldarstellung desselben.* Monatshefte für Mathematik und Physik, vol. 43 (1936), pp. 193–209.
 20. *Trigonometrische Reihen und Potenzreihen mit mehrfach monotoner Koeffizientenfolge.* Transactions of the American Mathematical Society, vol. 39 (1936), pp. 18–59.

FEJÉR, L., and SZEGÖ, G.

- *1. *Über die monotone Konvergenz von Potenzreihen mit mehrfach monotoner Koeffizientenfolge.* Prace Matematyczno-Fizyczne, vol. 44 (1935), pp. 15-25.

FEKETE, M.

- *1. *Über die Verteilung der Wurzeln bei gewissen algebraischen Gleichungen mit ganzzahligen Koeffizienten.* Mathematische Zeitschrift, vol. 17 (1923), pp. 228-249.

FELDHEIM, E.

1. *Sur l'orthogonalité des fonctions fondamentales de l'interpolation de Lagrange.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 203 (1936), pp. 650-652.
2. *Sur le mode de convergence dans l'interpolation de Lagrange.* Comptes Rendus de l'Académie des Sciences de l'URSS, vol. 14 (1937), pp. 327-331.
3. *Sur l'orthogonalité des fonctions fondamentales, et sur la forte convergence en moyenne des polynômes d'interpolation de Lagrange dans le cas des abscisses de Tchebycheff.* Bulletin de la Société Mathématique de France, vol. 65 (1937), pp. 1-40.
4. *Théorie de la convergence des procédés d'interpolation et de quadrature mécanique.* Mémoires des Sciences Mathématiques, vol. 95, Paris, 1939.

FRIEDRICH, K.

1. *On certain inequalities and characteristic value problems for analytic functions and for functions of two variables.* Transactions of the American Mathematical Society, vol. 41 (1937), pp. 321-364.

FUJIWARA, M.

1. *On the zeros of Jacobi polynomials.* Japanese Journal of Mathematics, vol. 2 (1925), pp. 1-2.

GALBRUN, H.

1. *Sur un développement d'une fonction à variable réelle en séries de polynômes.* Bulletin de la Société Mathématique de France, vol. 41 (1913), pp. 24-47.

GAUSS, C. F.

- *1. *Summatio quarundam serierum singularium.* Werke. Vol. 2, pp. 9-45.
2. *Methodus nova integralium valores per approximationem inveniendi.* Werke. Vol. 3, pp. 163-196.

GEGENBAUER, L.

1. *Über einige bestimmte Integrale.* Sitzungsberichte der mathematisch-naturwissenschaftlichen Klasse der Akademie der Wissenschaften in Wien, Abteilung IIa, vol. 70 (1874), pp. 433-443.
2. *Über die Functionen $C_n^{\nu}(x)$.* Ibid., vol. 75 (1877), pp. 891-905.
3. *Über die Functionen $C_n^{\nu}(x)$.* Ibid., vol. 97 (1888), pp. 259-270.
4. *Zur Theorie der hypergeometrischen Reihe.* Ibid., vol. 100 (1891), pp. 225-244.
5. *Das Additionstheorem der Functionen $C_n^{\nu}(x)$.* Ibid., vol. 102 (1893), pp. 942-950.
6. *Zur Theorie der Functionen $C_n^{\nu}(x)$.* Denkschriften der Akademie der Wissenschaften in Wien, Mathematisch-naturwissenschaftliche Klasse, vol. 48 (1884), pp. 293-316.
7. *Einige Sätze über die Functionen $C_n^{\nu}(x)$.* Ibid., vol. 57 (1890), pp. 425-480.

G(Ü)ERONIMUS, J.

1. *Sur le polynôme multiplement monotone qui s'écarte le moins de zéro, dont un coefficient est donné.* Bulletin de l'Académie des Sciences de l'URSS, 1929, pp. 377-389.
2. *Sur l'écart minimal quadratique de zéro d'un polynôme.* Rendiconti del Circolo Matematico di Palermo, vol. 54 (1930), pp. 298-313.
3. *On some problems of Tchebycheff.* American Journal of Mathematics, vol. 53 (1931), pp. 597-604.
4. *On a problem of M. J. Shohat.* Ibid., vol. 54 (1932), pp. 85-91.
5. *On some extremal properties of polynomials.* Annals of Mathematics, (2), vol. 37 (1936), pp. 483-517.

- GOTTLIEB, M. J.
 1. *Concerning some polynomials orthogonal on a finite or enumerable set of points.* American Journal of Mathematics, vol. 60 (1938), pp. 453-458.
- GRONWALL, T. H.
 1. *Über die Laplacesche Reihe.* Mathematische Annalen, vol. 74 (1913), pp. 213-270.
 2. *Über die Summierbarkeit der Reihen von Laplace und Legendre.* Ibid., vol. 75 (1914), pp. 321-375.
- GRÜNWARD, G.
 1. *Über Divergenzerscheinungen der Lagrangeschen Interpolationspolynome stetiger Funktionen.* Annals of Mathematics, (2), vol. 37 (1936), pp. 908-918.
- GRÜNWARD, G., and TURÁN, P.
 1. *Über Interpolation.* Annali della Scuola Normale Superiore de Pisa, (2), vol. 7 (1938), pp. 137-146.
- HAAR, A.
 *1. *Zur Theorie der orthogonalen Funktionensysteme. I.* Mathematische Annalen, vol. 69 (1910), pp. 331-371.
 2. *Reihenentwicklungen nach Legendreschen Polynomen.* Ibid., vol. 78 (1917), pp. 121-136.
- HAHN, W.
 1. *Die Nullstellen der Laguerreschen und Hermiteschen Polynome.* Inauguraldissertation, Berlin. Schriften des Mathematischen Seminars und des Instituts für angewandte Mathematik der Universität Berlin, vol. 1 (1933), pp. 213-244.
 2. *Bericht über die Nullstellen der Laguerreschen und der Hermiteschen Polynome.* Jahresbericht der Deutschen Mathematiker-Vereinigung, vol. 44 (1934), pp. 215-236. *Nachtrag.* Ibid., vol. 45 (1935), p. 211.
 3. *Über die Jacobischen Polynome und zwei verwandte Polynomklassen.* Mathematische Zeitschrift, vol. 39 (1935), pp. 634-638.
 4. *Über höhere Ableitungen von Orthogonalpolynomen.* Ibid. vol. 43 (1937), p. 101.
- HAMBURGER, H.
 1. *Beiträge zur Konvergenztheorie der Stieltjesschen Kettenbrüche.* Mathematische Zeitschrift, vol. 4 (1919), pp. 186-222.
 2. *Über eine Erweiterung des Stieltjesschen Momentenproblems.* Mathematische Annalen, vol. 81 (1920), pp. 235-319; vol. 82 (1920), pp. 120-164, 168-187.
- HARDY, G. H.
 1. *Summation of a series of polynomials of Laguerre.* Journal of the London Mathematical Society, vol. 7 (1932), pp. 138-139, 192.
- HARDY, G. H., LITTLEWOOD, J. E., and PÓLYA, G.
 *1. *Inequalities.* Cambridge, 1934.
- HAUSDORFF, F.
 *1. *Momentprobleme für ein endliches Intervall.* Mathematische Zeitschrift, vol. 16 (1923), pp. 220-248.
- HEINE, E.
 1. *Mittheilung über Kettenbrüche.* Journal für die reine und angewandte Mathematik, vol. 67 (1867), pp. 315-326.
 2. *Die Fourier-Besselsche Function.* Ibid., vol. 69 (1869), pp. 128-141.
 3. *Handbuch der Kugelfunctionen.* Vols. I, II. 2d edition. Berlin, 1878, 1881.
- HELLY, E.
 *1. *Über lineare Funktionaloperationen.* Sitzungsberichte der mathematisch-naturwissenschaftlichen Klasse der Akademie in Wien, Abteilung IIa, vol. 121 (1912), pp. 265-297.
- HERMITE, C.
 1. *Sur les polynômes de Legendre.* Rendiconti del Circolo Matematico di Palermo, vol. 4 (1890), pp. 146-152. *Oeuvres.* Vol. 4, pp. 314-320.

2. *Sur les polynômes de Legendre*. Journal für die reine und angewandte Mathematik, vol. 107 (1891), pp. 80-83. *Oeuvres*. Vol. 4, pp. 321-326.
 3. *Sur les racines de la fonction sphérique de seconde espèce*. Annales de la Faculté des Sciences de Toulouse, vol. 4 (1890), 10 pp. *Oeuvres*. Vol. 4, pp. 327-336.
- HERMITE, C., and STIELTJES, T. J.
1. *Correspondance d'Hermite et de Stieltjes*. Vols. I, II. Paris, 1905.
- HILB, E.
1. *Über die Laplacesche Reihe*. Mathematische Zeitschrift, vol. 5 (1919), pp. 17-25; vol. 8 (1920), pp. 79-90.
- HILBERT, D.
1. *Über die Discriminante der im Endlichen abbrechenden hypergeometrischen Reihe*. Journal für die reine und angewandte Mathematik, vol. 103 (1888), pp. 337-345.
- HILBERT, D., and COURANT, R.
1. *Methoden der mathematischen Physik*. Vol. 1. 2d edition. Berlin, 1931.
- HILDEBRANDT, T. H.
- *1. *On integrals related to and extensions of the Lebesgue integrals*. Bulletin of the American Mathematical Society, vol. 24 (1918), pp. 113-144, 177-202.
- HILLE, E.
1. *A class of reciprocal functions*. Annals of Mathematics, (2), vol. 27 (1926), pp. 427-464.
 2. *On Laguerre's series*. I, II, III. Proceedings of the National Academy of Sciences, vol. 12 (1926), pp. 261-265, 265-269, 348-352.
 3. *Bemerkung zu einer Arbeit des Herrn Müntz*. Mathematische Zeitschrift, vol. 32 (1930), pp. 422-425.
 4. *Über die Nullstellen der Hermiteschen Polynome*. Jahresbericht der Deutschen Mathematiker-Vereinigung, vol. 44 (1933), pp. 162-165.
- HOBSON, E. W.
1. *The Theory of Spherical and Ellipsoidal Harmonics*. Cambridge, 1931.
- HOLLÓ, Á.
1. *A mechanikus quadraturáról*. Thesis. Budapest, 1939, 23 pp.
- JACKSON, D.
1. *On functions of closest approximation*. Transactions of the American Mathematical Society, vol. 22 (1921), pp. 117-128.
 2. *Note on a class of polynomials of approximation*. Ibid., vol. 22 (1921), pp. 320-326.
 3. *A generalized problem in weighted approximation*. Ibid., vol. 26 (1924), pp. 133-154.
 4. *The Theory of Approximation*. American Mathematical Society Colloquium Publications, vol. 11, 1930.
 5. *Series of orthogonal polynomials*. Annals of Mathematics, (2), vol. 34 (1933), pp. 527-545.
 6. *Certain problems of closest approximation*. Bulletin of the American Mathematical Society, vol. 39 (1933), pp. 889-906.
 7. *The summation of series of orthogonal polynomials*. Ibid., vol. 40 (1934), pp. 743-752.
 8. *Formal properties of orthogonal polynomials in two variables*. Duke Mathematical Journal, vol. 2 (1936), pp. 423-434.
- JACOB, M. M.
1. *Sullo sviluppo di una funzione di ripartizione in serie di polinomi di Hermite*. Giornale dell'Istituto Italiano degli Attuari, vol. 2 (1931), pp. 100-106, 356-368.
 2. *Sur le phénomène de Gibbs dans les développements de séries de polynômes d'Hermite*. Comptes Rendus de l'Académie des Sciences, Paris, vol. 204 (1937), pp. 1540-1543.
- JACOBI, C. G. J.
1. *Ueber Gauss' neue Methode, die Werthe der Integrale näherungsweise zu finden*. Journal für die reine und angewandte Mathematik, vol. 1 (1826), pp. 301-308. *Gesammelte Werke*. Vol. 6, pp. 3-11.

2. *Über die Entwicklung des Ausdrucks* $(aa - 2aa' [\cos \omega \cos \phi + \sin \omega \sin \phi \cdot \cos (\theta - \theta')] + a'a')^{-1/2}$ Ibid., vol. 26, 1843, pp. 81-87. *Gesammelte Werke*. Vol. 6, pp. 148-155.
 3. *Untersuchungen über die Differentialgleichung der hypergeometrischen Reihe*. Ibid., vol. 56 (1859), pp. 149-165. *Gesammelte Werke*. Vol. 6, pp. 184-202.
- JORDAN, CAMILLE.
1. *Cours d'Analyse de l'École Polytechnique*. Vol. 3, 2d edition. Paris, 1896.
- JORDAN, CHARLES.
1. *Sur une série de polynomes dont chaque somme partielle représente la meilleure approximation d'un degré donné suivant la méthode des moindres carrés*. Proceedings of the London Mathematical Society, (2), vol. 20 (1920), pp. 297-325.
- JULIA, G.
1. *Sur les polynomes de Tchebichef*. Comptes Rendus de l'Académie des Sciences, Paris, vol. 182 (1926), pp. 1201-1202.
- KACZMARZ, ST., and STEINHAUS, H.
- *1. *Theorie der Orthogonalreihen*. Warszawa-Lwów, 1935.
- KELDYSCH, M., and LAVRENTIEFF, M.
1. *Sur la représentation conforme des domaines limités par des courbes rectifiables*. Annales Scientifiques de l'École Normale Supérieure, (3), vol. 54 (1937), pp. 1-38.
- KLEIN, F.
1. *Ueber die Nullstellen der hypergeometrischen Reihe*. Mathematische Annalen, vol. 37 (1890), pp. 573-590. *Gesammelte Abhandlungen*. Vol. 2, pp. 550-567.
- KOGBETLIANTZ, E.
1. *Sur les séries de fonctions ultrasphériques*. Comptes Rendus de l'Académie des Sciences, Paris, vol. 163 (1916), pp. 601-603.
 2. *Sur la sommation des séries ultrasphériques*. Ibid., vol. 164 (1917), pp. 510-513, 626-628, 778-780; vol. 169 (1919), pp. 54-57.
 3. *Sur les développements de Jacobi*. Ibid., vol. 168 (1919), pp. 992-994.
 4. *Sur les séries ultrasphériques*. Ibid., vol. 169 (1919), pp. 322-324.
 5. *Nouvelles observations sur les séries ultrasphériques*. Ibid., vol. 169 (1919), pp. 423-426.
 6. *Sur l'unicité des développements ultrasphériques*. Ibid., vol. 169 (1919), pp. 769-770, 950-953.
 7. *Sur les développements de Jacobi*. Ibid., vol. 172 (1921), pp. 1333-1334; vol. 192 (1931), pp. 915-918.
 8. *Sur la sommabilité (C, δ) de développements suivant les polynomes d'Hermite*. Ibid., vol. 192 (1931), pp. 662-663.
 9. *Nouvelles observations sur le système orthogonal de polynomes d'Hermite*. Ibid., vol. 192 (1931), pp. 1696-1698.
 10. *Sur les séries d'Hermite et de Laguerre*. Ibid., vol. 193 (1931), pp. 386-389.
 11. *Sur la convergence des séries d'Hermite*. Ibid., vol. 194 (1932), pp. 161-163.
 12. *Sur les développements de Laguerre*. Ibid., vol. 194 (1932), pp. 1422-1424.
 13. *Sur la série de Laguerre*. Ibid., vol. 196 (1933), pp. 523-525.
 14. *Expression approchée du polynome de Laguerre $L_n^{(\alpha)}(x)$* . Ibid., vol. 196 (1933), pp. 1079-1080.
 15. *Sur la détermination du saut $D(x_0)$ de $f(x)$* . Ibid., vol. 196 (1933), pp. 464-466.
 16. *Über die (C, δ) -Summierbarkeit der Laplaceschen Reihe für $1/2 < \delta < 1$* . Mathematische Zeitschrift, vol. 14 (1922), pp. 99-109.
 17. *Analogie entre les séries trigonométriques et les séries sphériques au point de vue de leur sommabilité par les moyennes arithmétiques*. Thèse. Paris, 1923, 65 pp.
 18. *Sur la sommabilité de la série ultrasphérique à l'intérieur de l'intervalle $(-1, +1)$ par la méthode des moyennes arithmétiques*. Bulletin de la Société Mathématique de France, vol. 51 (1923), pp. 244-295.

19. *Recherches sur la sommabilité des séries ultrasphériques par la méthode des moyennes arithmétiques.* Journal de Mathématiques, (9), vol. 3 (1924), pp. 107–187.
 20. *Recherches sur l'unicité des séries ultrasphériques.* Ibid., (9), vol. 5 (1926), pp. 125–196.
 21. *Sommation des séries et intégrales divergentes par les moyennes arithmétiques et typiques.* Mémoires des Sciences Mathématiques. Vol. 51. Paris, 1931.
 22. *Recherches sur la sommabilité des séries d'Hermite.* Annales Scientifiques de l'École Normale Supérieure, (3), vol. 49 (1932), pp. 137–221.
 23. *Sur les moyennes arithmétiques des séries-noyaux des développements en séries d'Hermite et de Laguerre et sur celles de ces séries-noyaux dérivées terme à terme.* Journal of Mathematics and Physics, Massachusetts Institute of Technology, vol. 14 (1935), pp. 37–99.
 24. *Contribution à l'étude du saut d'une fonction donnée par son développement en séries d'Hermite ou de Laguerre.* Transactions of the American Mathematical Society, vol. 38 (1935), pp. 10–47.
- KOROUS, J.
1. *O rozvoji funkcí jedné reálné proměnné v řadu Hermiteových polynomů.* Rozpravy České Akademie, (2), vol. 37 (1928), no. 11, 34 pp.
 2. *O řadách Laguerrových polynomů* (with French abstract). Ibid., no. 40, 23 pp.
 3. *O rozvoji funkcí jedné reálné proměnné v řadu jistých ortogonálních polynomů* (with English abstract). Ibid., vol. 48 (1938), 12 pp.
 4. *Über Reihenentwicklungen nach verallgemeinerten Laguerreschen Polynomen mit drei Parametern.* Věstník Královské České Společnosti Nauk, Třída Matemat.-Přírodověd, 1937, 26 pp.
 5. *Über Entwicklungen der Funktionen einer reellen Veränderlichen in Reihen einer gewissen Klasse orthogonaler Polynome im unendlichen Intervalle.* Ibid., 1937, 19 pp.
- KOSCHMIEDER, L.
1. *Über besondere Jacobische Polynome.* Mathematische Zeitschrift, vol. 8 (1920), pp. 123–137.
- KOWALEWSKI, G.
- *1. *Einführung in die Determinantentheorie.* Leipzig, 1909.
- KOWALLIK, U.
1. *Entwicklung einer willkürlichen Funktion nach Hermite'schen Orthogonalfunktionen.* Mathematische Zeitschrift, vol. 31 (1930), pp. 498–518.
- KRALL, H. L.
1. *On derivatives of orthogonal polynomials.* Bulletin of the American Mathematical Society, vol. 42 (1936), pp. 423–428.
 2. *On higher derivatives of orthogonal polynomials.* Ibid., vol. 42 (1936), pp. 867–870.
- KRAWTCHOUK, M.
1. *Sur une généralisation des polynomes d'Hermite.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 189 (1929), pp. 620–622.
 2. *Sur la distribution des racines des polynomes orthogonaux.* Ibid., vol. 196 (1933), pp. 739–741.
- KRONECKER, L.
- *1. *Zur Theorie der Elimination einer Variablen aus zwei algebraischen Gleichungen.* Monatsberichte der Preussischen Akademie der Wissenschaften zu Berlin, 1881, pp. 535–600.
- LAGRANGE, J. L.
1. *Oeuvres.* Vol. 1, pp. 534–539. 1867.
- LAGUERRE, E. N.
1. *Sur l'intégrale $\int_x^\infty x^{-1} e^{-x} dx$.* Bulletin de la Société Mathématique de France, vol. 7 (1879), pp. 72–81. *Oeuvres.* Vol. 1, pp. 428–437.

2. *Sur l'approximation des fonctions circulaires au moyen des fonctions algébriques.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 90 (1880), pp. 304-307. *Oeuvres.* Vol. 1, pp. 104-107.
 3. *Sur les équations algébriques dont le premier membre satisfait à une équation linéaire du second ordre.* Ibid., vol. 90 (1880), pp. 809-812. *Oeuvres.* Vol. 1, pp. 126-132.
- LANGER, R. E.
- *1. *The asymptotic solutions of ordinary linear differential equations of the second order, with special reference to the Stokes phenomenon.* Bulletin of the American Mathematical Society, vol. 40 (1934), pp. 545-582.
 - *2. *The asymptotic solutions of certain linear ordinary differential equations of the second order.* Transactions of the American Mathematical Society, vol. 36 (1934), pp. 90-106.
 - *3. *On the asymptotic solutions of ordinary differential equations, with reference to the Stokes' phenomenon about a singular point.* Ibid., vol. 37 (1935), pp. 397-416.
- LAWTON, W.
1. *On the zeros of certain polynomials related to Jacobi and Laguerre polynomials.* Bulletin of the American Mathematical Society, vol. 38, (1932), pp. 442-448.
- LEBESGUE, H.
- *1. *Sur la divergence et la convergence non-uniforme des séries de Fourier.* Comptes Rendus, de l'Académie des Sciences, Paris, vol. 141 (1905), pp. 875-877.
 - *2. *Leçons sur les Séries Trigonométriques.* Paris, 1906.
- LEGENDRE, A. M.
1. *Exercices de Calcul Intégral sur Divers Ordres de Transcendantes.* Vol. 2. Paris, 1817.
- LE ROY, É.
1. *Sur les séries divergentes et les fonctions définies par un développement de Taylor.* Annales de la Faculté des Sciences de Toulouse, (2), vol. 2 (1900), pp. 317-430.
- LUKÁCS, F.
1. *Verschärfung des ersten Mittelwertsatzes der Integralrechnung für rationale Polynome.* Mathematische Zeitschrift, vol. 2 (1918), pp. 295-305.
 2. *Über die Laplacesche Reihe.* Ibid., vol. 14 (1922), pp. 250-262.
- MARCINKIEWICZ, J.
- *1. *Quelques remarques sur l'interpolation.* Acta Litterarum ac Scientiarum Regiae Universitatis Hungaricae Francisco-Josephinae, vol. 8 (1937), pp. 127-130.
 2. *Sur la divergence des polynomes d'interpolation.* Ibid., vol. 8 (1937), pp. 131-135.
- MARKOFF, A.
1. *On some applications of algebraic continued fractions* (in Russian). Thesis. St. Petersburg, 1884, 131 pp.
 2. *Démonstration de certaines inégalités de M. Tchêbycheff.* Mathematische Annalen, vol. 24 (1884), pp. 172-180.
 3. *Extrait d'une lettre.* Annales Scientifiques de l'École Normale Supérieure, (3), vol. 2 (1885), p. 183.
 4. *Sur les racines de certaines équations (second note).* Mathematische Annalen, vol. 27 (1886), pp. 177-182.
 5. *Differenzenrechnung.* Leipzig, 1896.
- MEHLER, F. G.
1. *Bemerkungen zur Theorie der mechanischen Quadraturen.* Journal für die reine und angewandte Mathematik, vol. 63 (1864), pp. 152-157.
 2. *Ueber die Entwicklung einer Function von beliebig vielen Variablen nach Laplaceschen Functionen höherer Ordnung.* Ibid., vol. 66 (1866), pp. 161-176.

3. Ueber die Vertheilung der statischen Elektrizität in einem von zwei Kugelkalotten begrenzten Körper. *Ibid.*, vol. 68 (1868), pp. 134–150.
 4. Ueber die Darstellung einer willkürlichen Function zweier Variablen durch Cylinderfunctionen. *Mathematische Annalen*, vol. 5 (1872), pp. 135–140.
 5. Notiz über die Dirichlet'schen Integralausdrücke für die Kugelfunction $P^n(\cos \theta)$ und über eine analoge Integralform für die Cylinderfunction $J(x)$. *Ibid.*, vol. 5 (1872), pp. 141–144.
- MEIXNER, J.
1. Orthogonale Polynomsysteme mit einer besonderen Gestalt der erzeugenden Function. *Journal of the London Mathematical Society*, vol. 9 (1934), pp. 6–13.
 2. Erzeugende Funktionen der Charlierschen Polynome. *Mathematische Zeitschrift*, vol. 44 (1938), pp. 531–535.
- MOECKLIN, E.
1. Asymptotische Entwicklungen der Laguerreschen Polynome. *Commentarii Mathematici Helvetici*, vol. 7 (1934), pp. 24–46.
- MÜNTZ, C.
1. Über die Potenzsummation einer Entwicklung nach Hermiteschen Polynomen. *Mathematische Zeitschrift*, vol. 31 (1929), pp. 350–355.
- MYLLER-LEBEDEFF, V.
1. Die Theorie der Integralgleichungen in Anwendung auf einige Reihenentwicklungen. *Mathematische Annalen*, vol. 64 (1907), pp. 388–416.
- NEUMANN, E. R.
1. Die Entwicklung willkürlicher Funktionen nach den Hermiteschen und Laguerreschen Orthogonalfunktionen auf Grund der Theorie der Integralgleichungen. *Inaugural-Dissertation*. Breslau, 1912.
 2. Beiträge zur Kenntnis der Laguerreschen Polynome. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 30 (1921), pp. 15–35.
- OBRECHKOFF, N.
1. Sur la sommation de la série ultrasphérique par la méthode des moyennes arithmétiques. *Rendiconti del Circolo Matematico di Palermo*, vol. 59 (1936), pp. 266–287.
 2. Formules asymptotiques pour les polynomes de Jacobi et sur les séries suivant les mêmes polynomes. *Annuaire de l'Université de Sofia, Faculté Physico-Mathématique*, vol. 1 (1936), pp. 39–133.
- PERRON, O.
1. Über das infinitäre Verhalten der Koeffizienten einer gewissen Potenzreihe. *Archiv der Mathematik und Physik*, (3), vol. 22 (1914), pp. 329–340.
 2. Über das Verhalten einer ausgearteten hypergeometrischen Reihe bei unbegrenztem Wachstum eines Parameters. *Journal für die reine und angewandte Mathematik*, vol. 151 (1921), pp. 63–78.
 3. *Die Lehre von den Kettenbrüchen*. 2d edition. Leipzig, 1929.
 - *4. *Algebra*. Vols. I, II, 2d edition. Berlin, 1932, 1933.
- PLANCHEREL, M., and ROTACH, W.
1. Sur les valeurs asymptotiques des polynomes d'Hermite $H_n(x) = (-1)^n e^{x^2/2} \cdot d^n(e^{-x^2/2})/dx^n$. *Commentarii Mathematici Helvetici*, vol. 1 (1929), pp. 227–254.
- PÓLYA, G.
1. Sur un théorème de Stieltjes. *Comptes Rendus de l'Académie des Sciences, Paris*, vol. 155 (1912), pp. 767–769.
 2. Sur un algorithme toujours convergent pour obtenir les polynomes de meilleure approximation de Tchebychef pour une fonction continue quelconque. *Ibid.*, vol. 157 (1913), pp. 840–843.

- *3. *Über die Nullstellen gewisser ganzer Funktionen.* *Mathematische Zeitschrift*, vol. 2*(1918), pp. 352-383.
4. *Über die Konvergenz von Quadraturverfahren.* *Ibid.*, vol. 37 (1933), pp. 264-286.
- PÓLYA, G., and SZEGÖ, G.
1. *Aufgaben und Lehrsätze aus der Analysis.* Vols. I, II. Berlin, 1925.
 2. *Über den transfiniten Durchmesser (Kapazitätskonstante) von ebenen und räumlichen Punktmengen.* *Journal für die reine und angewandte Mathematik*, vol. 165 (1931), pp. 4-49.
- POPOVICIU, T.
1. *Sur la distribution des zéros de certains polynomes minimisants.* *Bulletin de l'Académie Roumaine*, vol. 16 (1934), pp. 214-217.
 2. *Sur certains problèmes de maximum de Stieltjes.* *Bulletin Mathématique de la Société Roumaine des Sciences*, vol. 38 (1936), pp. 73-96.
- RAU, H.
1. *Über die Lebesgueschen Konstanten der Reihenentwicklungen nach Jacobischen Polynomen.* *Journal für die reine und angewandte Mathematik*, vol. 161 (1929), pp. 237-254.
 2. *Über eine asymptotische Darstellung der Jacobischen Polynome durch Besselsche Funktionen.* *Mathematische Zeitschrift*, vol. 40 (1936), pp. 683-692.
- RIESZ, F.
- *1. *Sur certains systèmes singuliers d'équations intégrales.* *Annales Scientifiques de l'École Normale Supérieure*, (3), vol. 28 (1911), pp. 33-62.
 - *2. *Über die Randwerte einer analytischen Funktion.* *Mathematische Zeitschrift*, vol. 18 (1923), pp. 87-95.
- RIESZ, M.
1. *Eine trigonometrische Interpolationsformel und einige Ungleichungen für Polynome.* *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 23 (1915), pp. 354-368.
 2. *Sur le problème des moments.* *Arkiv för Matematik, Astronomi och Fysik*, vol. 16 (1921), no. 12, 23 pp.; vol. 16 (1922), no. 19, 21 pp.; vol. 17 (1923), no. 16, 52 pp.
- ROTACH, W.
1. *Reihenentwicklungen einer willkürlichen Funktion nach Hermite'schen und Laguerre'schen Polynomen.* *Inauguraldissertation, Eidgenössische Technische Hochschule Zürich*, 1925.
- SCHMIDT, E.
1. *Über die Charlier-Jordansche Entwicklung einer willkürlichen Funktion nach der Poissonschen Funktion und ihren Ableitungen.* *Zeitschrift für angewandte Mathematik und Mechanik*, vol. 13 (1933), pp. 139-142.
- SCHUR, I.
1. *Über die Verteilung der Wurzeln bei gewissen algebraischen Gleichungen mit ganzzahligen Koeffizienten.* *Mathematische Zeitschrift*, vol. 1 (1918), pp. 377-402.
 2. *Affektlose Gleichungen in der Theorie der Laguerreschen und Hermite'schen Polynome.* *Journal für die reine und angewandte Mathematik*, vol. 165 (1931), pp. 52-58.
- SCHWID, N.
1. *The asymptotic forms of the Hermite and Weber functions.* *Transactions of the American Mathematical Society*, vol. 37 (1935), pp. 339-362.
- SEN, D. N., and RANGACHARIAR, V.
1. *Generalized Jacobi polynomials.* *Bulletin of the American Mathematical Society*, vol. 42 (1936), pp. 901-908.
- SHERMAN, J.
1. *On the numerators of the convergents of the Stieltjes continued fractions.* *Transactions of the American Mathematical Society*, vol. 35 (1933), pp. 64-87.

SHIBATA, K.

1. *On the distribution of the roots of a polynomial satisfying a certain differential equation of the second order.* Japanese Journal of Mathematics, vol. 1 (1924), pp. 147-153.

SHOHAT, J.

1. *On the polynomial of the best approximation to a given continuous function.* Bulletin of the American Mathematical Society, vol. 31 (1925), pp. 509-514.
2. *On a general formula in the theory of Tchebycheff polynomials and its applications.* Transactions of the American Mathematical Society, vol. 29 (1927), pp. 569-583.
3. *On a certain formula of mechanical quadratures with non-equidistant ordinates.* Ibid., vol. 31 (1929), pp. 448-463.
4. *On the polynomial and trigonometric approximation of measurable bounded functions on a finite interval.* Mathematische Annalen, vol. 102 (1929), pp. 157-175.
5. *On interpolation.* Annals of Mathematics, (2), vol. 34 (1933), pp. 130-146.
6. *Théorie Générale des Polynomes Orthogonaux de Tchebichef.* Mémoires des Sciences Mathématiques. Vol. 66. Paris, 1934.
7. *On mechanical quadratures, in particular, with positive coefficients.* Transactions of the American Mathematical Society, vol. 42 (1937), pp. 461-496.
8. *On the convergence properties of Lagrange interpolation based on the zeros of orthogonal Tchebycheff polynomials.* Annals of Mathematics, (2), vol. 38 (1937), pp. 758-769.

SMIRNOFF, V. J.

1. *Sur la théorie des polynomes orthogonaux à une variable complexe.* Journal de la Société Physico-Mathématique de Léningrad, vol. 2 (1928), pp. 155-179.
2. *Sur les formules de Cauchy et de Green et quelques problèmes qui s'y rattachent.* Bulletin de l'Académie des Sciences de l'URSS, 1932, pp. 337-372.

SMITH, E. R.

1. *Zeros of the Hermitian polynomials.* American Mathematical Monthly, vol. 43 (1936), pp. 354-358.

SONIN(Э), N. J.

1. *Recherches sur les fonctions cylindriques et le développement des fonctions continues en séries.* Mathematische Annalen, vol. 16 (1880), pp. 1-80.
2. *On the precision of the determination of limiting values of definite integrals (in Russian).* Zapiski Akademii Nauk, vol. 69 (1892), pp. 1-30.

SPENCER, V. E.

1. *Asymptotic expressions for the zeros of generalized Laguerre polynomials and Weber functions.* Duke Mathematical Journal, vol. 3 (1937), pp. 667-675.

STEKLOFF, W.

1. *Sur les expressions asymptotiques de certaines fonctions, définies par les équations différentielles linéaires du second order, et leurs applications au problème du développement d'une fonction arbitraire en séries procédant suivant les-dites fonctions.* Communications de la Société Mathématique de Kharkow, (2), vol. 10 (1907), pp. 97-200. *Remarque complémentaire*, p. 201.
2. *On approximate evaluation of definite integrals by means of formulas of mechanical quadratures. I. Convergence of formulas of mechanical quadratures (in Russian).* Bulletin de l'Académie Impériale des Sciences, Petrograd, (6), vol. 10 (1916), pp. 169-186.

STIELTJES, T. J.

1. *Quelques recherches sur la théorie des quadratures dites mécaniques.* Annales Scientifiques de l'École Normale Supérieure, (3), vol. 1 (1884), pp. 409-426. *Oeuvres Complètes.* Vol. 1, pp. 377-394.
2. *Note à l'occasion de la réclamation de M. Markoff.* Ibid., (3), vol. 2 (1885), pp. 183-184. *Oeuvres Complètes.* Vol. 1, pp. 430-431.

3. *Sur certains polynômes qui vérifient une équation différentielle linéaire du second ordre et sur la théorie des fonctions de Lamé.* Acta Mathematica, vol. 6 (1885), pp. 321-326. *Oeuvres Complètes.* Vol. 1, pp. 434-439.
 4. *Sur quelques théorèmes d'algèbre.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 100 (1885), pp. 439-440. *Oeuvres Complètes.* Vol. 1, pp. 440-441.
 5. *Sur les polynômes de Jacobi.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 100 (1885), pp. 620-622. *Oeuvres Complètes.* Vol. 1, pp. 442-444.
 6. *Sur les racines de l'équation $X_n = 0$.* Acta Mathematica, vol. 9 (1886), pp. 385-400. *Oeuvres Complètes.* Vol. 2, pp. 73-88.
 7. *Sur la valeur asymptotique des polynômes de Legendre.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 110 (1890), pp. 1026-1027. *Oeuvres Complètes.* Vol. 2, pp. 234-235.
 8. *Sur les polynômes de Legendre.* Annales de la Faculté des Sciences de Toulouse, vol. 4 (1890), 17 pp. *Oeuvres Complètes.* Vol. 2, pp. 236-252.
 9. *Sur les racines de la fonction sphérique de seconde espèce.* Annales de la Faculté des Sciences de Toulouse, vol. 4 (1890), 10 pp. *Oeuvres Complètes.* Vol. 2, pp. 253-262.
 10. *Recherches sur les fractions continues.* Comptes Rendus de l'Académie des Sciences, Paris, vol. 118 (1894), pp. 1401-1403. *Oeuvres Complètes.* Vol. 2, pp. 398-401.
 11. *Recherches sur les fractions continues.* Annales de la Faculté des Sciences de Toulouse, vol. 8 (1894), 122 pp.; vol. 9, 1895, 47 pp. *Oeuvres Complètes.* Vol. 2, pp. 402-566.
 12. *Sur certaines inégalités dues à M. P. Tchebychef.* Article rédigé d'après un manuscrit inédit. *Oeuvres Complètes.* Vol. 2, pp. 586-593.
- STONE, M. H.
1. *Developments in Hermite polynomials.* Annals of Mathematics, (2), vol. 29 (1928), pp. 1-13.
 - *2. *Linear Transformations in Hilbert Space and their Applications to Analysis.* American Mathematical Society Colloquium Publications, vol. 15, 1932.
- SZÁSZ, O.
- *1. *Korlátos hatványsorokról.* Matematikai és Természettudományi Értesítő, vol. 43 (1926), pp. 504-520.
- SZEGÖ, G.
1. *A Hankel-féle formákról.* Matematikai és Természettudományi Értesítő, vol. 36 (1918), pp. 497-538.
 2. *Ein Beitrag zur Theorie der Polynome von Laguerre und Jacobi.* Mathematische Zeitschrift, vol. 1 (1918), pp. 341-356.
 3. *Über Orthogonalsysteme von Polynomen.* Ibid., vol. 4 (1919), pp. 139-151.
 4. *Beiträge zur Theorie der Toeplitzschen Formen.* II. Ibid., vol. 9 (1921), pp. 167-190.
 5. *Über orthogonale Polynome, die zu einer gegebenen Kurve der komplexen Ebene gehören.* Ibid., vol. 9 (1921), pp. 218-270.
 6. *Über die Entwicklung einer analytischen Funktion nach den Polynomen eines Orthogonalsystems.* Mathematische Annalen, vol. 82, 1921, pp. 188-212.
 7. *Über die Randwerte analytischer Funktionen.* Ibid., vol. 84 (1921), pp. 232-244.
 8. *Über den asymptotischen Ausdruck von Polynomen, die durch eine Orthogonalitätseigenschaft definiert sind.* Ibid., vol. 86 (1922), pp. 114-139.
 9. *Über die Entwicklung einer willkürlichen Funktion nach den Polynomen eines Orthogonalsystems.* Mathematische Zeitschrift, vol. 12 (1921), pp. 61-94.
 10. *Beiträge zur Theorie der Laguerreschen Polynome.* I. *Entwicklungssätze.* Ibid., vol. 25 (1926), pp. 87-115.

11. *Bemerkungen zu einer Arbeit von Herrn Fejér über die Legendreschen Polynome.* Ibid., vol. 25 (1926), pp. 172-187.
12. *Ein Beitrag zur Theorie der Thetafunktionen.* Sitzungsberichte der Preussischen Akademie der Wissenschaften, physikalisch-mathematische Klasse, 1926, pp. 242-252.
13. *Koeffizientenabschätzungen bei ebenen und räumlichen harmonischen Entwicklungen.* Mathematische Annalen, vol. 96 (1927), pp. 601-632.
14. *Über gewisse Interpolationspolynome, die zu den Jacobischen und Laguerreschen Abszissen gehören.* Mathematische Zeitschrift, vol. 35 (1932), pp. 579-602.
15. *Über einige asymptotische Entwicklungen der Legendreschen Funktionen.* Proceedings of the London Mathematical Society, (2), vol. 36 (1932), pp. 427-450.
16. *Über eine von Herrn S. Bernstein herrührende Abschätzung der Legendreschen Polynome.* Mathematische Annalen, vol. 108 (1933), pp. 360-369.
17. *Asymptotische Entwicklungen der Jacobischen Polynome.* Schriften der Königsberger Gelehrten Gesellschaft, naturwissenschaftliche Klasse, vol. 10 (1933), pp. 35-112.
18. *Bemerkungen zu einem Satz von E. Schmidt über algebraische Gleichungen.* Sitzungsberichte der Preussischen Akademie der Wissenschaften, physikalisch-mathematische Klasse, 1934, pp. 3-15.
19. *Über gewisse orthogonale Polynome, die zu einer oszillierenden Belegungsfunktion gehören.* Mathematische Annalen, vol. 110 (1934), pp. 501-513.
20. *Inequalities for the zeros of Legendre polynomials and related functions.* Transactions of the American Mathematical Society, vol. 39 (1936), pp. 1-17.
21. *An integral equation for the square of a Laguerre polynomial.* Journal of the London Mathematical Society, vol. 12 (1937), pp. 162-163.

TAMARKIN, J. D.

1. *On the Theory of Polynomials of Approximation.* Lecture delivered at Brown University, 1935-1936.

TCHAKALOFF, L.

1. *Sur la structure des ensembles linéaires définis par une certaine propriété minimale.* Acta Mathematica, vol. 63 (1934), pp. 77-97.

TCHEBICHEF, P. I.

1. *Sur les fractions continues.* Journal de Mathématiques, (2), vol. 3 (1858), pp. 289-323. *Oeuvres.* Vol. 1, pp. 201-230.
2. *Sur l'interpolation par la méthode des moindres carrés.* Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg, (7), vol. 1 (1859), pp. 1-24. *Oeuvres.* Vol. 1, pp. 471-498.
3. *Sur le développement des fonctions à une seule variable.* Bulletin de l'Académie Impériale des Sciences de St. Pétersbourg, vol. 1 (1859), pp. 193-200. *Oeuvres.* Vol. 1, pp. 499-508.
4. *Sur l'interpolation.* Zapiski Akademii Nauk, vol. 4, Supplement no. 5, 1864. *Oeuvres.* Vol. 1, pp. 539-560.
5. *Sur les fonctions analogues à celles de Legendre.* Zapiski Akademii Nauk, vol. 16 (1870), pp. 131-140. *Oeuvres.* Vol. 2, pp. 59-68.
6. *Sur les valeurs limites des intégrales.* Journal de Mathématiques, (2), vol. 19 (1874), pp. 157-160. *Oeuvres.* Vol. 2, pp. 181-185.
7. *Sur le rapport de deux intégrales étendues aux mêmes valeurs de la variable.* Zapiski Akademii Nauk, vol. 44, Supplement no. 2, 1883. *Oeuvres.* Vol. 2, pp. 375-402.
8. *Sur la représentation des valeurs limites des intégrales par des résidus intégraux.* Ibid., vol. 51, Supplement no. 4, 1885. *Oeuvres.* Vol. 2, pp. 419-440. Acta Mathematica, vol. 9 (1886) pp. 35-56.

TITCHMARSH, E. C.

- *1. *The Theory of Functions*. Oxford, 1932.

TRICOMI, F.

1. *Trasformazione di Laplace e polinomi di Laguerre*. I. *Inversione della trasformazione*. Rendiconti della R. Accademia dei Lincei, (6), vol. 21 (1935), pp. 232-239.

USPENSKY, J. V.

1. *On the development of arbitrary functions in series of Hermite's and Laguerre's polynomials*. Annals of Mathematics, (2), vol. 28 (1927), pp. 593-619.

VAN VEEN, S. C.

1. *Asymptotische Entwicklung und Nullstellenabschätzung der Hermiteschen Funktionen*. Proceedings, Koninklijke Akademie van Wetenschappen te Amsterdam, vol. 34 (1931), pp. 257-267.
 2. *Asymptotische Entwicklung und Nullstellenabschätzung der Hermiteschen Funktionen*. Mathematische Annalen, vol. 105 (1931), pp. 408-436.
 3. *Zusatz zum vorangehenden Berichte* (of W. Hahn). Jahresbericht der Deutschen Mathematiker-Vereinigung, vol. 44 (1934), pp. 236-238.

VITALI, G., and SANSONE, G.

1. *Moderna Teoria delle Funzioni di Variabile Reale*. II. *Sviluppi in Serie di Funzioni Ortogonali*. Bologna, 1935.

WALSH, J. L.

1. *Interpolation and Approximation by Rational Functions in the Complex Domain*. American Mathematical Society Colloquium Publications, vol. 20, 1935.

WANGERIN, A.

1. *Theorie der Kugelfunktionen und der verwandten Funktionen, insbesondere der Laméschen und Besselschen (Theorie spezieller, durch lineare Differentialgleichungen definierter Funktionen)*. Encyklopädie der Mathematischen Wissenschaften, vol. II.1.2, pp. 695-759.

WATSON, G. N.

1. *The harmonic functions associated with the parabolic cylinder*. Proceedings of the London Mathematical Society, (2), vol. 8 (1910), pp. 393-421; (2), vol. 17 (1918), pp. 116-148.
 2. *Approximate formulae for Legendre functions*. Messenger of Mathematics, vol. 47 (1918), pp. 151-160.
 3. *A Treatise on the Theory of Bessel Functions*. Cambridge, 1922.
 4. *Notes on generating functions of polynomials: (1) Laguerre polynomials*. Journal of the London Mathematical Society, vol. 8 (1933), pp. 189-192.
 5. *Notes on generating functions of polynomials: (2) Hermite polynomials*. Ibid., vol. 8 (1933), pp. 194-199.
 6. *Über eine Reihe aus verallgemeinerten Laguerre'schen Polynomen*. Sitzungsberichte der mathematisch-naturwissenschaftlichen Klasse der Akademie Wien, IIa, vol. 147 (1938), pp. 151-159.

WEYL, H.

1. *Singuläre Integralgleichungen mit besonderer Berücksichtigung des Fourierschen Integraltheorems*. Inauguraldissertation, Göttingen, 1908. Mathematische Annalen, vol. 66 (1909), pp. 273-324.

WHITTAKER, E. T., AND WATSON, G. N.

1. *A Course of Modern Analysis*. 4th edition. Cambridge, 1935.

WIGERT, S.

1. *Contributions à la théorie des polynomes d'Abel-Laguerre*. Arkiv för Matematik, Astronomi och Fysik, vol. 15 (1921), no. 25, 22 pp.
 2. *Sur les polynomes orthogonaux et l'approximation des fonctions continues*. Ibid., vol. 17 (1923), no. 18, 15 pp.

WIMAN, A.

1. *Über eine asymptotische Eigenschaft der Ableitungen der ganzen Funktionen von den Geschlechtern 1 und 2 mit einer endlichen Anzahl von Nullstellen.* *Mathematische Annalen*, vol. 104 (1931), pp. 169-181.

WINSTON, C.

1. *On mechanical quadratures formulae involving the classical orthogonal polynomials.* *Annals of Mathematics*, (2), vol. 35 (1934), pp. 658-677.

WRIGHT, E. M.

1. *The coefficients of a certain power series.* *Journal of the London Mathematical Society*, vol. 7 (1932), pp. 256-262.

YOUNG, W. H.

1. *On the connexion between Legendre series and Fourier Series.* *Proceedings of the London Mathematical Society*, (2), vol. 18 (1919), pp. 141-162.

ZERNIKE, F.

1. *Eine asymptotische Entwicklung für die grösste Nullstelle der Hermiteschen Polynome.* *Koninklijke Akademie van Wetenschappen te Amsterdam, Proceedings*, vol. 34 (1931), pp. 673-680.

ZYGMUND, A.

1. *Sur la théorie riemannienne de certains systèmes orthogonaux*, II. *Prace Matematyczno-Fizyczne*, vol. 39 (1932), pp. 73-117.
2. *Trigonometrical Series.* Warszawa-Lwów, 1935.

FURTHER REFERENCES

ACHIESER, N. I. (See List of References above.)

3. *Theory of Approximation.* New York, 1956.
4. *The Classical Moment Problem.* Oliver and Boyd, Edinburgh and London, 1965.

ASKEY, R.

1. *Orthogonal expansions with positive coefficients.* *Proceedings of the American Mathematical Society*, vol. 16 (1965), pp. 1191-1194.
2. *Jacobi polynomial expansions with positive coefficients and imbeddings of projective spaces.* *Bulletin of the American Mathematical Society*, vol. 74 (1968), pp. 301-304.
3. *A transplantation theorem for Jacobi series.* *Illinois Journal of Mathematics*, vol. 13 (1969), pp. 583-590.
4. *Linearization of the product of orthogonal polynomials*, in *Problems in analysis*. Edited by R. Gunning, Princeton University Press, Princeton, N. J., 1970, pp. 223-228.
5. *Positivity of the Cotes numbers for some Jacobi abscissas.* *Numerische Mathematik*, vol. 19 (1972), pp. 46-48.
6. *Mean convergence of orthogonal series and Lagrange interpolation.* *Acta Mathematica Academiae Scientiarum Hungaricae*, vol. 23 (1972), pp. 71-85.
7. *Grünbaum's inequality for Bessel functions.* *Journal of Mathematical Analysis and Applications*, vol. 41 (1973), pp. 122-124.
8. *Summability of Jacobi series.* *Transactions of the American Mathematical Society*, vol. 179 (1973), pp. 71-84.
9. *Certain rational functions whose power series have positive coefficients.* II. *SIAM Journal on Mathematical Analysis*, vol. 5 (1974), pp. 53-57.
10. *Jacobi polynomials. I. New proofs of Koornwinder's Laplace type integral representation and Bateman's bilinear sum.* *SIAM Journal on Mathematical Analysis*, vol. 5 (1974), pp. 119-124.

ASKEY, R., AND FITCH, J.

1. *Positivity of the Cotes numbers for some ultraspherical abscissas.* *SIAM Journal on Numerical Analysis*, vol. 5 (1968), pp. 199-201.
2. *Integral representations for Jacobi polynomials and some applications.* *Journal of Mathematical Analysis and Applications*, vol. 26 (1969), pp. 411-437.

ASKEY, R., AND GASPER, G.

1. *Linearization of the product of Jacobi polynomials*. III. Canadian Journal of Mathematics, vol. 23 (1971), pp. 332-338.
2. *Jacobi polynomial expansions of Jacobi polynomials with non-negative coefficients*. Proceedings of the Cambridge Philosophical Society, vol. 70 (1971), pp. 243-255.
3. *Certain rational functions whose power series have positive coefficients*. American Mathematical Monthly, vol. 79 (1972), pp. 327-341.
4. *Positive Jacobi polynomial sums*. II. American Journal of Mathematics.

ASKEY, R., AND POLLARD, H.

1. *Some absolutely monotonic and completely monotonic functions*. SIAM Journal on Mathematical Analysis, vol. 5 (1974), pp. 58-63.

ASKEY, R., AND STEINIG, J.

1. *Some positive trigonometric sums*. Transactions of the American Mathematical Society, vol. 187 (1974), pp. 295-307.
2. *A monotonic trigonometric sum*. American Journal of Mathematics.

ASKEY, R., AND WAINGER, S.

1. *Mean convergence of expansions in Laguerre and Hermite series*. American Journal of Mathematics, vol. 87 (1965), pp. 695-708.
2. *A transplantation theorem between ultraspherical series*. Illinois Journal of Mathematics, vol. 10 (1966), pp. 322-344.
3. *A transplantation theorem for ultraspherical coefficients*. Pacific Journal of Mathematics, vol. 16 (1966), pp. 393-405.
4. *A convolution structure for Jacobi series*. American Journal of Mathematics, vol. 91 (1969), pp. 463-485.

BAILEY, W. N.

1. *The generating function of Jacobi polynomials*. Journal of the London Mathematical Society, vol. 13 (1938), pp. 8-12.
2. *On the product of two Laguerre polynomials*. Quarterly Journal of Mathematics, vol. 10 (1939), pp. 60-66.

BALÁZS, J., AND TURÁN, P.

1. *Notes on Interpolation*. II. (*Explicit formulae*). Acta Mathematica Academiae Scientiarum Hungaricae, vol. 8 (1957), pp. 201-215.
2. *Notes on Interpolation*. III. (*Convergence*). Ibid., vol. 9 (1958), pp. 195-214.

BATEMAN, H.

1. *A generalization of the Legendre polynomial*. Proceedings of the London Mathematical Society, ser. 2, vol. 3 (1905), pp. 111-123.
2. *The solution of linear differential equations by means of definite integrals*. Transactions of the Cambridge Philosophical Society, vol. 21 (1909), pp. 171-196.
3. *Partial Differential Equations*. Cambridge University Press, Cambridge, 1932.

BATEMAN MANUSCRIPT PROJECT (Director: A. ERDÉLYI).

1. *Higher Transcendental Functions*. Vols. 1, 2, 3. New York-Toronto-London, 1953, 1955.

BOCHNER, S. (See List of References above.)

2. *Positive zonal functions on spheres*. Proceedings of the National Academy of Sciences, vol. 40 (1954), pp. 1141-1147.

BONAMI, A., AND CLERC, J.-L.

1. *Sommes de Cesàro et multiplicateurs des développements en harmoniques sphériques*. Transactions of the American Mathematical Society, vol. 183 (1973), pp. 223-263.

BUTLEWSKI, Z.

1. *Sur les intégrales d'une équation différentielle du second ordre*. Mathematica (Cluj), vol. 12 (1936), pp. 36-48.

CARLESON, L.

1. *On convergence and growth of partial sums of Fourier series*. Acta Mathematica, vol. 116 (1966), pp. 135-157.

COIFMAN, R., AND WEISS, G.

1. *Analyse harmonique non-commutative sur certaines espaces homogènes*. Lecture Notes in Mathematics, vol. 242, Springer-Verlag, Berlin and New York, 1971.

CSORDAS, G., AND WILLIAMSON, J.

1. *On polynomials satisfying a Turán type inequality*. Proceedings of the American Mathematical Society, vol. 43 (1974), pp. 367-372.

DAVIS, J., AND HIRSCHMAN, I. I., JR.

1. *Toeplitz forms and ultraspherical polynomials*. Pacific Journal of Mathematics, vol. 18 (1966), pp. 73-95.

DAVIS, J., AND RABINOWITZ, P.

1. (a) *Some geometrical theorems for abscissas and weights of Gauss type*. Journal of Mathematical Analysis and Applications, vol. 2 (1961), pp. 428-437. (b) *Erratum*. Ibid., vol. 3 (1961), p. 619.

DE BRUIJN, N. G.

1. *Uncertainty principles in Fourier analysis, Inequalities*, Edited by O. Shisha, Academic Press, New York, 1967, pp. 57-71.

DOETSCH, G. (See List of References above.)

3. *Handbuch der Laplace-Transformationen*. Vol. 2: *Anwendungen der Laplace-Transformationen*, 1. Abteilung. Basel-Stuttgart, 1955.

EAGLESON, G.

1. *A characterization theorem for positive definite sequences on the Krawtchouk polynomials*. Australian Journal of Statistics, vol. 11 (1969), pp. 29-38.

EGERVÁRY, E., AND TURÁN, P.

1. *Notes on interpolation*. V. (*On the stability of interpolation*). Acta Mathematica Academiae Scientiarum Hungaricae, vol. 9 (1958), pp. 259-267.
2. *Notes on interpolation*. VI. (*On the stability of the interpolation on an infinite interval*). Acta Mathematica Academiae Scientiarum Hungaricae, vol. 10 (1959), pp. 55-62.

ERDÉLYI, A. (See List of References above.)

3. *Asymptotic forms for Laguerre polynomials*. Journal of the Indian Mathematical Society, Golden Jubilee Volume, vol. 24 (1960), pp. 235-250.

ERDÉLYI, A., AND SWANSON, C. A.

1. *Asymptotic forms of Whittaker's confluent hypergeometric functions*. Memoirs of the American Mathematical Society, No. 25, 1957.

ERDÖS, P.

1. *On the distribution of normal point groups*. Proceedings of the National Academy of Sciences, vol. 26 (1940), pp. 294-297.
2. *On divergence properties of the Lagrange interpolation parabolas*. Annals of Mathematics, vol. 42 (1941), pp. 309-315.
3. *Problems and results on the theory of interpolation*. II. Acta Mathematica Academiae Scientiarum Hungaricae, vol. 12 (1961), pp. 235-244.

ERDÖS, P., AND GRÜNWARD, G.

1. *Über einen Faber'schen Satz*. Annals of Mathematics, vol. 39 (1938), pp. 257-261.

ERDÖS P., AND LENGYEL, B. A.

1. *On fundamental functions of Lagrangean interpolation*. Bulletin of the American Mathematical Society, vol. 44 (1938), pp. 828-834.

ERDÖS, P., AND TURÁN, P. (See List of References above.)

3. *On Interpolation*. III. *Interpolatory theory of polynomials*. Annals of Mathematics, vol. 41 (1940), pp. 510-553.
4. *An extremal problem in the theory of interpolation*. Acta Mathematica Academiae Scientiarum Hungaricae, vol. 12 (1961), pp. 221-234.

FELDHEIM E. (See List of References above.)

5. *On the positivity of certain sums of ultraspherical polynomials*. (Translated and edited by G. Szegő.) Journal d'Analyse Mathématique, vol. 11 (1963), pp. 275-284.

FREUD, G.

1. *Az Hermite-Fejér-féle interpolációs eljárás konvergenciájáról*. Magyar Tudományos Akadémia III. osztályának közleményei, vol. 5 (1955), pp. 29-47.
2. *Ortogonalis polinómokról*. Dissertation. Budapest, 1956.
3. *Über die Asymptotik orthogonaler Polynome*. Académie Serbe des Sciences, vol. 11 (1957), pp. 19-32.
4. *Orthogonale Polynome*. Basel, 1969. English translation, New York, 1971.

GASPER, G.

1. *Linearization of the product of Jacobi polynomials*. I. Canadian Journal of Mathematics, vol. 22 (1970), pp. 171-175.
2. *Linearization of the product of Jacobi polynomials*. II. Canadian Journal of Mathematics, vol. 22 (1970), pp. 582-593.
3. *Positivity and the convolution structure for Jacobi series*. Annals of Mathematics, (2), vol. 93 (1971), pp. 112-118.
4. *Banach algebras for Jacobi series and positivity of a kernel*. Annals of Mathematics, (2), vol. 95 (1972), pp. 261-280.
5. *An inequality of Turán type for Jacobi polynomials*. Proceedings of the American Mathematical Society, vol. 32 (1972), pp. 435-439.
6. *Nonnegativity of a discrete Poisson kernel for the Hahn polynomials*. Journal of Mathematical Analysis and Applications, vol. 42 (1973), pp. 438-451.
7. *Projection formulas for orthogonal polynomials of a discrete variable*. Journal of Mathematical Analysis and Applications, vol. 45 (1974), pp. 176-198.

GATTESCHI, L.

1. *Approssimazione asintotica degli zeri dei polinomi ultrasferici*. Rendiconti di Matematica e delle sue Applicazioni, (5), vol. 8 (1949), pp. 399-411.
2. *Limitazione degli errori nelle formule asintotiche per le funzioni speciali*. Rendiconti del Seminario Matematico dell'Università e del Politecnico di Torino, vol. 16 (1956-1957), pp. 83-94.

GINIBRE, R.

1. *General formulation of Griffiths' inequalities*. Communications on Mathematical Physics, vol. 16 (1970), pp. 310-328.

GRENDER, U., AND SZEGÖ, G.

1. *Toeplitz Forms and their Applications*. Berkeley-Los Angeles, 1958.

GRÜNBAUM, F. A.

1. *A property of Legendre polynomials*. Proceedings of the National Academy of Sciences, vol. 67 (1970), pp. 959-960.
2. *A new kind of inequality for Bessel functions*. Journal of Mathematical Analysis and Applications, vol. 41 (1973), pp. 115-121.

GRÜNWARD, G. (See List of References above.)

2. *On a convergence theorem for the Lagrange interpolation polynomials*. Bulletin of the American Mathematical Society, vol. 47 (1941), pp. 271-275.
3. *On the theory of Interpolation*. Acta Mathematica, vol. 75 (1943), pp. 219-245.

HAHN, W. (See List of References above.)

5. *Über Orthogonalpolynome, die q -Differenzgleichungen genügen*. Mathematische Nachrichten, vol. 2 (1949), pp. 4-34.

HARTMAN, P., AND WINTNER, A.

1. *On non-conservative linear oscillators of low frequency*. American Journal of Mathematics, vol. 70 (1948), pp. 529-539.

HIRSCHMAN, I. I., JR.

1. *Variation diminishing transformations and orthogonal polynomials*. Journal d'Analyse Mathématique, vol. 9 (1961), pp. 177-193.
2. *The strong Szegő limit theorem for Toeplitz determinants*. American Journal of Mathematics, vol. 88 (1966), pp. 73-95.

HORTON, R.

1. *Expansions using orthogonal polynomials*. Ph.D. Thesis, University of Wisconsin, Madison, 1973, 60 pp.

Hsü, H.

1. *Certain integrals and infinite series involving ultraspherical polynomials and Bessel functions*. Duke Mathematical Journal, vol. 4 (1938), pp. 374-383.

HUA, L. K.

1. *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains*, Translations of Mathematical Monographs, vol. 6 (1963), American Mathematical Society, Providence, R. I.

HUNT, R

1. *On the convergence of Fourier series*, Orthogonal Expansions and their Continuous Analogues, Edited by D. Haimo, Southern Illinois University Press, Carbondale, Illinois, 1968, pp. 235-255.

KARLIN, S., AND MCGREGOR, J. L.

1. *The differential equations of birth-and-death processes, and the Stieltjes moment problem*. Transactions of the American Mathematical Society, vol. 85 (1957), pp. 489-546.
2. *The Hahn polynomials, formulas and an application*. Scripta Mathematica, vol. 26 (1961), pp. 33-46.
3. *Classical diffusion processes and total positivity*. Journal of Mathematical Analysis and Applications, vol. 1 (1960), pp. 163-183.

KARLIN, S., AND STUDDEN, W. J.

1. *Tchebycheff Systems with Applications in Analysis and Statistics*. Interscience Publishers, New York, 1966.

KARLIN, S., AND SZEGÖ, G.

1. *On certain determinants whose elements are orthogonal polynomials*. Journal d'Analyse Mathématique, vol. 8 (1960), pp. 1-157.

KOORNWINDER, T.

1. *The addition formula for Jacobi polynomials*. I. *Summary of results*. Indagationes Mathematicae, vol. 34 (1972), pp. 188-191.
2. *The addition formula for Jacobi polynomials and spherical harmonics*. SIAM Journal on Applied Mathematics, vol. 25 (1973), pp. 236-246.
3. *Jacobi polynomials*. II. *An analytic proof of the product formula*. SIAM Journal on Mathematical Analysis, vol. 5 (1974), pp. 125-137.
4. *Jacobi polynomials*. III. *An analytic proof of the addition formula*. SIAM Journal on Mathematical Analysis, vol. 6 (1975), pp. 533-543.

LOCHER, F.

1. *Norm bounds of quadrature processes*. SIAM Journal on Numerical Analysis, vol. 10 (1973), pp. 553-558.

LORCH, L.

1. *The Lebesgue constants for Jacobi series*. I. Proceedings of the American Mathematical Society, vol. 10 (1959), pp. 756-761.
2. *The Lebesgue constants for Jacobi series*. II. American Journal of Mathematics, vol. 81 (1959), pp. 875-888.
3. *Comparison of two formulations of Sonin's theorem and of their respective applications to Bessel functions*. Studia Scientiarum Mathematicarum Hungarica, vol. 1 (1966), pp. 141-145.

LORCH, L., MULDOON, M. E., AND SZEGO, P.

1. *Higher monotonicity properties of certain Sturm-Liouville functions*. III. Canadian Journal of Mathematics, vol. 22 (1970), pp. 1238-1265.
2. *Higher monotonicity properties of certain Sturm-Liouville functions*. IV. Canadian Journal of Mathematics, vol. 24 (1972), pp. 349-368.

LORCH, L., AND SZEGO, P.

1. *Higher monotonicity properties of certain Sturm-Liouville functions*. Acta Mathematica, vol. 109 (1963), pp. 55-73.
2. *Higher monotonicity properties of certain Sturm-Liouville functions*. II. Bulletin de l'Académie Polonaise des Sciences. Série des Sciences Mathématiques, Astronomiques et Physiques, vol. 11 (1963), pp. 455-457.

MAGNUS, W., AND OBERHETTINGER, F.

1. *Formeln und Lehrsätze für die speziellen Funktionen der mathematischen Physik*. Berlin, 1948.

MAKAI, E.

1. *Über die Nullstellen von Funktionen, die Lösungen Sturm-Liouville'scher Differentialgleichungen sind*. Commentarii Mathematici Helvetici, vol. 16 (1944), pp. 153-199.
2. *On a monotonic property of certain Sturm-Liouville functions*. Acta Mathematica Academiae Scientiarum Hungaricae, vol. 3 (1952), pp. 165-172.

3. *On systems of polynomials orthogonal in two intervals*. *Publicationes Mathematicae*, vol. 2 (1952), pp. 222-228.
- MAKAI, E., AND TURÁN, P.
1. *Hermite expansions and distribution of zeros of polynomials*. *Magyar Tud. Akadémia Mat. Kutató Int. Közl.* vol. 8 (1963), pp. 157-163.
- MUCKENHOUPT, B.
1. *Asymptotic forms for Laguerre polynomials*. *Proceedings of the American Mathematical Society*, vol. 24 (1970), pp. 288-292.
 2. *Mean convergence of Hermite and Laguerre series*. I. *Transactions of the American Mathematical Society*, vol. 147 (1970), pp. 419-432.
 3. *Mean convergence of Hermite and Laguerre series*. II. *Transactions of the American Mathematical Society*, vol. 147 (1970), pp. 433-460.
 4. *Equiconvergence and almost everywhere convergence of Hermite and Laguerre series*. *SIAM Journal on Mathematical Analysis*, vol. 1 (1970), pp. 295-321.
 5. *Mean convergence of Jacobi series*. *Proceedings of the American Mathematical Society*, vol. 23 (1969), pp. 306-310.
- MUCKENHOUPT, B., AND STEIN, E.
1. *Classical expansions and their relation to conjugate harmonic functions*. *Transactions of the American Mathematical Society*, vol. 118 (1965), pp. 17-92.
- NEWMAN, J., AND RUDIN, W.
1. *Mean convergence of orthogonal series*. *Proceedings of the American Mathematical Society*, vol. 3 (1952), pp. 219-222.
- NOVIKOFF, A.
1. *On a special system of orthogonal polynomials*. Dissertation, Stanford University, 1954.
- OLVER, F. W. J.
1. *A paradox in asymptotics*. *SIAM Journal on Mathematical Analysis*, vol. 1 (1970), pp. 533-534.
- PEETRE, J.
1. *The Weyl transform and Laguerre polynomials*. *Le Matematiche*, vol. 27 (1972), pp. 301-323.
- POLLACZEK, F.
1. *Sur une généralisation des polynomes de Legendre*. *Comptes Rendus de l'Académie des Sciences*, Paris, vol. 228 (1949), pp. 1363-1365.
 2. *Systèmes de polynomes biorthogonaux qui généralisent les polynomes ultrasphériques*. *Ibid.*, vol. 228 (1949), pp. 1998-2000.
 3. *Sur une famille de polynomes orthogonaux qui contient les polynomes d'Hermite et de Laguerre comme cas limites*. *Ibid.*, vol. 230 (1950), pp. 1563-1565.
 4. *Sur une généralisation des polynomes de Jacobi*. *Mémorial des Sciences Mathématiques*, vol. 131 (1956).
- POLLARD, H.
1. *The mean convergence of orthogonal series*. I. *Transactions of the American Mathematical Society*, vol. 62 (1947), pp. 387-403.
 2. *The mean convergence of orthogonal series*. II. *Transactions of the American Mathematical Society*, vol. 63 (1948), pp. 355-367.
 3. *The mean convergence of orthogonal series*. III. *Duke Mathematical Journal*, vol. 16 (1949), pp. 189-191.
- ROOSENRAAD, C. T.
1. *Inequalities with orthogonal polynomials*. Ph.D. Thesis, University of Wisconsin, Madison, 1969, 52 pp.
- ŠAPIRO, R. L.
1. *The special functions connected with the representations of the group $SU(n)$ of class one with respect to $SU(n-1)$ ($n \geq 3$)*. *Izvestija Vysših Učebnyh Zavedenii, Matematika*, vol. 71 (1968), pp. 97-107, (Russian).
- SARMANOV, I. O.
1. *A generalized symmetric gamma correlation*. (Russian; translation in) *Soviet Mathematics, Doklady*, vol. 9 (1968), pp. 547-550.

SCHMEISSER, G.

1. *Optimale Schranken zu einem Satz über Nullstellen Hermitescher Trinome*. Journal für die reine und angewandte Mathematik, vol. 246 (1971), pp. 147-160.

SCHOENBERG, I., AND SZEGÖ, G.

1. *An extremum problem for polynomials*. Compositio Mathematica, vol. 14 (1960), pp. 260-268.

SEIDEL, W., AND SZÁSZ, O.

1. *On positive harmonic functions and ultraspherical polynomials*. Journal of the London Mathematical Society, vol. 26 (1951), pp. 36-41.

SHOHAT, J. A., HILLE, E., AND WALSH, J. L.

1. *A Bibliography on Orthogonal Polynomials*. Washington, 1940.

SURÁNYI, J., AND TURÁN, P.

1. *Notes on interpolation*. I. (On some interpolational properties of the ultraspherical polynomials). Acta Mathematica Academiae Scientiarum Hungaricae, vol. 6 (1955), pp. 67-79.

SZÁSZ, O. (See List of References above.)

2. *On the relative extrema of ultraspherical polynomials*. Bollettino della Unione Matematica Italiana, series 7, vol. 5 (1950), pp. 125-127.
3. *On the relative extrema of the Hermite orthogonal functions*. Journal of the Indian Mathematical Society, vol. 25 (1951), pp. 129-134.

SZEGÖ, G. (See List of References above.)

22. *On an inequality of P. Turán concerning Legendre polynomials*. Bulletin of the American Mathematical Society, vol. 54 (1948), pp. 401-405.
23. *On the relative extrema of Legendre polynomials*. Bollettino della Unione Matematica Italiana, (3), vol. 5 (1950), pp. 120-121.
24. *On certain special sets of orthogonal polynomials*. Proceedings of the American Mathematical Society, vol. 1 (1950), pp. 731-737.
25. *Ultrasphaerikus polinomok összegéről*. Matematikai és Fizikai Lapok, vol. 45 (1938), pp. 36-38.
26. *Über gewisse Potenzreihen mit lauter positiven Koeffizienten*. Mathematische Zeitschrift, vol. 37 (1933), pp. 674-688.
27. *On some Hermitian forms associated with two given curves of the complex plane*. Transactions of the American Mathematical Society, vol. 40 (1936), pp. 450-461.

SZEGÖ, G., AND TURÁN, P.

1. *On the monotone convergence of certain Riemann sums*. Publicationes Mathematicae (Debrecen), vol. 8 (1961), pp. 326-335.

THORNE, R. C.

1. *The asymptotic expansion of Legendre functions of large degree and order*. Technical Report, Office of Naval Research, California Institute of Technology, 1956.

TRICOMI, F. G. (See List of References above.)

2. *Sul comportamento asintotico dell' n -esimo polinomio di Laguerre nell'intorno dell'ascissa $4n$* . Commentarii Mathematici Helvetici, vol. 22 (1949), pp. 150-167.
3. *Sul comportamento asintotico dei polinomi di Laguerre*. Annali di Matematica, (4), vol. 28 (1949), pp. 263-289.
4. *Sugli zeri dei polinomi sferici ed ultrasferici*. Ibid., (4), vol. 31 (1950), pp. 93-97.
5. *Vorlesungen über Orthogonalreihen*. Berlin-Göttingen-Heidelberg, 1955.

TURÁN, P.

1. *On the zeros of the polynomials of Legendre*. Časopis pro Pěstování Matematiky a Fysiky, vol. 75 (1950), pp. 113-122.
2. *On Descartes-Herriot's rule*. Bulletin of the American Mathematical Society, vol. 55 (1949), pp. 797-800.
3. *Remark on a theorem of Erhard Schmidt*. Mathematica, vol. 2 (1960), pp. 373-378.

VIETORIS, L.

1. *Über das Vorzeichen gewisser trigonometrischer Summen*. Sitzungsberichte der mathematisch-naturwissenschaftlichen Klasse der Akademie der Wissenschaften in Wien, vol. 167 (1958), pp. 125-135.

VITALI, G., AND SANSONE, G.

See List of References above. Third edition. Bologna, 1952.

WEBSTER, M. S.

1. *A convergence theorem for certain Lagrange interpolation polynomials.* Bulletin of the American Mathematical Society, vol. 49 (1943), pp. 114-119.

WIDOM, H., AND WILF, H.

1. *Small eigenvalues of large Hankel matrices.* Proceedings of the American Mathematical Society, vol. 17 (1966), pp. 338-344.

ZYGMUND, A.

See List of References above. Second edition. New York, 1952.

This page intentionally left blank

INDEX

The numbers refer to pages

- Abel, v, 100, 401.
 —'s continuity theorem, 92.
 —'s transformation, 2, 91, 135.
 —summability, 245, 273.
 See Inequality.
- Achieser, 5, 37, 42, 401, 416.
 Adamoff, 203, 248, 249, 250, 401.
 Addition theorems, 58, 97 ff.
 Additive operation, 11, 12.
 Airy's function, 18, 19, 131, 201, 234, 239,
 243, 377, 382.
 See Zeros.
- Antipole condition, 246, 249, 265.
 Approximation, 5 ff.
- Askey, xi, 98, 99, 110, 273, 348, 363, 391,
 392, 416.
 Askey-Fitch, 96, 363, 416.
 Askey-Gasper, 97, 99, 110, 190, 392, 417.
 Askey-Pollard, 273, 417.
 Askey-Steinig, 97, 158, 363, 417.
 Askey-Wainger, 99, 190, 273, 417.
 Associated function, Legendre, 84.
 Asymptotic formula of Bessel functions,
 15, 16.
 — — — classical polynomials, 191 ff.
 — — — general orthogonal polynomials, v,
 vi, 296 ff.
 — — — Hermite polynomials, 132, 191, 194,
 198 ff., 218 ff., 235, 242.
 — — — Jacobi functions of second kind,
 201, 225.
 — — — polynomials, 58, 167, 168, 191 ff.,
 201, 202, 212, 225.
 — — — of Hilb's type, 169, 197, 202,
 214.
 — — — of Mehler-Heine type, 167,
 192 ff., 202.
 See Darboux's formula.
 — — — kernel polynomials, 369 ff.
 — — — Laguerre polynomials, 132, 191, 193,
 194, 198 ff., 236, 237, 240 ff.
 — — — of Hilb's type, 177, 199, 203,
 216 ff., 219 ff., 384.
 — — — — of Mehler-Heine type, 193, 194,
 217.
 — — — — of Plancherel-Rotach type, 200,
 201, 227 ff.
 See Fejér's formula, Perron's formula.
 — — — Legendre functions of second kind,
 198, 212, 222 ff.
 — — — polynomials, 191, 193, 194 ff., 243.
 Asymptotic formula of Legendre polynomials
 of Hilb's type, 195, 202, 212 ff.
 See Laplace formula, Laplace-Heine
 formula.
 — — — polynomials orthogonal on a curve,
 371 ff.
 — — — — the unit circle, 297 ff.
 — — — ultraspherical polynomials, 196, 197,
 206, 208, 209, 212.
- Bailey, 273, 392, 417.
 Balázs-Turán, 347, 417.
 Banach, 13, 401.
 Bateman, 96, 98, 99, 273, 417.
 Bateman Project, ix, 95, 110, 243, 393, 394,
 417.
 Bernstein, S., vi, 9, 31, 42, 159, 165, 168 ff.,
 296, 299, 300, 303, 314, 315, 330, 401.
 —'s theorem on trigonometric polynomials,
 5, 280, 304.
 See Polynomials.
- Bessel functions, v, 14 ff., 95, 96, 102 ff.,
 126 ff., 140, 166, 167, 191, 193, 202, 203,
 212 ff., 225, 243, 272, 353, 362, 363, 380,
 384, 387, 392, 400.
 See Asymptotic formula, Zeros.
 —'s inequality, 25, 38, 289, 316, 322, 333, 367.
- Blumenthal, 268, 310, 401.
 Bochner, 108, 273, 401, 417.
 Bonami-Clerc, 273, 417.
 Bottema, 132, 401.
 Bounded variation, functions of, 12.
 Brauer, A., 131, 401.
 Bruns, 122, 125, 136, 138, 401.

- Buell, 124 ff., 401.
 Butlewski, 166, 417.
- Capacity, see Transfinite diameter.
 Carathéodory, 364, 402.
 See Osgood-Carathéodory.
- Carleman, 366, 402.
 Carleson, 273, 417.
 Cauchy-Hadamard formula, 248, 253, 312.
 Cauchy's principle value, 278.
 – theorem, 69, 106, 221, 275, 289, 324, 370, 372.
 Centroid, 189.
 Cesàro summability, vi, 244, 246 ff., 258.
 – – (means) of Fourier series, 12, 14, 246, 250.
 – – – Hermite series, 250, 251.
 – – – Jacobi series, vi, 246, 248, 249, 256 ff.
 Cesàro summability of Laguerre series, 247, 248, 250, 251, 271 ff.
 – – – Legendre series, 14, 249.
 – – – power series, 325.
 – – – ultraspherical series, 248.
 Characteristic values, 49, 187.
 Charlier, see Polynomials.
 Christoffel, iii, 42, 43, 47, 402.
 – (-Darboux) formula, 42 ff., 318, 326.
 – formula of, 29 ff., 397.
 – numbers, 46 ff., 114, 115, 187, 351 ff., 378.
 Clerc, see Bonomi-Clerc.
 Closure, see Orthogonal polynomials.
 Coifman-G. Weiss, 273, 417.
 Completely monotonic sequence, 136 ff., 155.
 Confluent hypergeometric function, 104.
 Conformal mapping, 21, 160, 364, 365, 372.
 Conjugate function, 279.
 – points (interpolation), 332, 347.
 Continued fractions, v, 54 ff.
 Continuous operation, 12.
 Convergence in mean, 330.
 – – –, generalized, 330.
 – of interpolation, 330.
 – , quadrature, 330, 333, 350 ff.
 Convergent (continued fraction), 55 ff.
 Cotes numbers, 12, 349 ff., 363.
 Courant-Hilbert, see Hilbert-Courant.
 Cramér, 251, 402.
 Csordas-Williamson, 388, 418.
- Darboux, 42, 43, 195, 196, 211, 402.
 Darboux's formula for Jacobi polynomials, 168, 196, 225, 236, 237, 248, 253, 337, 353.
 – method, 202, 206 ff., 265, 269, 395.
- Davis-Hirschman, 99, 418.
 Davis-Rabinowitz, 158, 418.
 De Bruijn, 110, 418.
 de la Vallée-Poisson summability, 273.
 Deviation, quadratic, 38, 41, 288.
 – , Tchebichef, 41, 368.
 Differences, 34, 35, 136.
 Differential equation, v, 16 ff., 37, 152, 159, 164, 166, 210.
 – – of Bessel functions, 15.
 – – – Hermite polynomials, 106, 176, 380.
 – – – Jacobi (hypergeometric) polynomials, iv, 60 ff., 117, 141.
 – – – Laguerre polynomials, 100, 117, 176.
 – – – ultraspherical polynomials, 80, 81.
 Dirichlet, 85, 402.
 Dirichlet-Mehler, see Integral representation.
 Dirichlet's integral, 12, 14.
 Discriminant of classical polynomials, 143 ff.
 Distribution, 8, 9, 26, 29, 38, 40, 57, 181, 274, 330, 351, 378.
 – of Stieltjes type, 9, 33 ff., 38.
 Doetsch, 34, 35, 380, 402, 418.
 Du Bois, Reymond, 14, 402.
- Eagleson, 37, 418.
 Egerváry-Turán, 348, 418.
 Ellipse, 8, 21, 251 ff.
 – of convergence, 245, 248, 252, 311, 312.
 Elliptic functions, v, 60.
 Equiconvergence, 31, 244, 246, 247, 249, 313 ff.
 Erdélyi, ix, 104, 243, 273, 380, 402, 418.
 Erdélyi-Swanson, 243, 418.
 Erdős, vii, 347, 348, 418.
 Erdős-Feldheim, 334, 402.
 Erdős-Grunwald, 347, 418.
 Erdős-Lengyel, 347, 418.
 Erdős-Turán, 113, 114, 115, 332, 333, 335, 347, 348, 402, 418.
 Euler, 72, 402.
 –'s constant, 15.
 –'s integral of the first kind, 14.
 – – – second kind, see Gamma function.
 Expansion, v, vi, 58.
 – , finite cosine, of Legendre polynomials, 90 ff.
 – , infinite sine, of Legendre polynomials, 90 ff.
 – in power series, 313, 314.
 – in series of classical polynomials, 244 ff., 251 ff.

- - - - general orthogonal polynomials, 313 ff.
- - - - Hermite polynomials, 244, 245, 247, 251, 253, 269 ff.
- - - - Jacobi polynomials, 244 ff.
- - - - Laguerre polynomials, 244 ff., 249 ff., 253, 266 ff., 313.
- - - - Legendre polynomials, 248, 249.
- - - - Tchebichef polynomials, 248, 314.
- - - - ultraspherical polynomials, 248, 249.
- See Fourier series, Orthogonal polynomials.
- Faber, 299, 330, 368, 369, 372, 402.
- See Polynomials.
- Fatou, 275, 403.
- 's theorem, 274.
- Favard, 43, 403.
- Fejér, vii, 14, 45, 58, 89, 91, 96, 134, 136 ff., 157, 165, 172, 174, 175, 179, 198, 202, 249, 264, 330 ff., 336, 348, 350, 386, 403.
- See Stekloff-Fejér.
- 's asymptotic formula for Laguerre polynomials, 198, 202, 203, 237, 240, 269.
- Fejér's generalization of Legendre polynomials, 135 ff., 174 ff., 206.
- 's second generalization of Legendre polynomials, 137, 138, 155.
- 's integral, 12.
- 's representation of positive trigonometric polynomials, 3, 4, 274.
- - - - - , generalization of, 275 ff.
- Fejér-Szegő, 175, 404.
- Fekete, 369, 404.
- Feldheim, 95, 96, 97, 334, 335, 386, 404, 418.
- See Erdős-Feldheim.
- Fitch, see Askey-Fitch.
- Fourier, v.
- coefficient (constant), 24, 287.
- 's inversion formula, 387.
- series, 12, 14, 24, 25, 28, 38, 39, 244, 246, 253, 274, 289, 311, 314, 323, 347, 367.
- See Cesàro means.
- Freud, 309, 418.
- Friedrichs, 404.
- Fujiwara, 145, 404.
- Functions of second kind, Jacobi's, 73 ff., 251.
- - - - , Legendre's, 74, 78, 88, 89, 92, 379, 383.
- See Asymptotic formula, Integral representation, Zeros.
- fundamental polynomials of Hermite interpolation, 330 ff.
- - - Lagrange interpolation, 12, 47, 329 ff.
- Galbrun, 251, 404.
- Gamma-function, 14, 75.
- Gasper, 37, 98, 99, 190, 273, 419.
- See Askey-Gasper.
- Gatteschi, 242, 419.
- Gauss, iii, 33, 47, 49, 62, 404.
- See Mean-value theorem, Mechanical quadrature.
- Gauss-Weierstrass summability, 273.
- Gegenbauer, 80, 94, 98, 99, 404.
- "Gegenbeispiel", 250, 272.
- Generating function, 35, 36.
- - of Hermite polynomials, 106, 380.
- - - Jacobi polynomials, 69 ff., 82, 83, 95.
- - - Laguerre polynomials, 101, 202, 379.
- - - Legendre polynomials, 90, 206, 207, 383.
- - - Pollaczek polynomials, 393.
- - - ultraspherical polynomials, 82, 83.
- Geometric mean, 275, 296, 385.
- Geronimus, 42, 189, 404.
- Gibbs' phenomenon, 251.
- Ginibre, 392, 419.
- Gottlieb, 37, 405.
- Grenander-Szegő, 274, 287, 419.
- Gronwall, 165, 249, 405.
- Grünbaum, 392, 419.
- Grünwald, vii, 330, 347, 405, 419.
- See Erdős-Grünwald.
- Grünwald-Turán, 346, 405.
- Haar, 14, 248, 405.
- Hahn, W., 33, 107, 111, 112, 129, 132, 151, 405, 419.
- Hamburger, 57, 110, 405.
- Hankel, see Quadratic form.
- Hardy, 102, 380, 405.
- Hardy-Littlewood-Pólya, 2, 405.
- Hartman-Wintner, 20, 419.
- Hausdorff, 136, 155, 405.
- Heine, iii, 27, 37, 47, 54, 80, 90 ff., 151 ff., 192, 194, 251, 405.
- Heine-Stieltjes theorem, iv, 151 ff.
- Helly, 13, 405.
- 's theorem, 13, 14, 258, 272, 330, 339, 350.
- Hermite, v, 156, 405, 406.
- interpolation, 330 ff., 340 ff., 347, 348.
- Hermite-Stieltjes, 89, 156, 157, 172, 406.

- Hermitian form, 287.
 Hilb, 195, 249, 406.
 Hilbert, 142, 145, 406.
 Hilbert-Courant, 58, 59, 100, 106, 109, 406.
 Hildebrandt, 8, 10, 406.
 Hille, 102, 105, 126, 131, 242, 251, 406.
 See Shohat-Hille-Walsh.
 Hirschman, 99, 273, 419.
 See Davis-Hirschman.
 Hobson, iv, 58, 84, 92, 382, 406.
 Holló, 335, 406.
 Horton, 99, 273, 419.
 l'Hospital, rule of, 307.
 Howell, 392.
 Hsü, 390, 419.
 Hua, 99, 419.
 Hunt, 273, 420.
 Hurwitz, theorem of, 21, 150, 193, 239, 371.
 Hypergeometric function, iii, 62 ff., 83, 84, 96.
 See Differential equation.
- Inequalities, 159 ff.
 Inequality, Abel's, 2, 174, 205.
 —, Cauchy's, 2, 39, 120, 160, 183, 291, 302, 306, 321, 370.
 — for the arithmetic and geometric mean, 2, 300.
 —, Schwarz's, 2, 9, 110, 136, 162, 268, 276, 317, 376.
 —, Turán's, 190, 388.
 See Bessel's inequality.
- Integral, Lebesgue, vii, 9, 26, 159, 246 ff., 274, 275, 287, 291, 364, 384.
 —, Riemann, 9, 10, 280, 281, 282, 298, 310, 333, 335, 351, 361.
 —, Riemann-Stieltjes, 8, 9, 11, 50, 333, 350, 351.
 —, Stieltjes-Lebesgue, v, 1, 8, 10, 38, 187.
 — equations, v, 218, 251.
 — representation of Legendre functions of second kind, 88 ff.
 — — — — — polynomials, 85 ff.
 — — — — —, Dirichlet-Mehler, 85 ff., 96.
 — — — — —, Laplace (first), 86, 176.
 — — — — —, Laplace (second), 86 ff.
 — — — — —, Stieltjes, 87 ff.
 — — — ultraspherical polynomials, Dirichlet-Mehler, 89.
 — — — — —, Stieltjes, 89 ff.
- Interpolation, v, ix, 12, 14, 58, 329 ff., 347.
 —, Lagrange, 12, 47, 180, 329 ff., 348, 349, 389.
 — on Hermite abscissas, 340.
 — — Jacobi abscissas, 335 ff.
 — — Laguerre abscissas, 340, 344 ff.
 — — Legendre abscissas, 386.
 — — Tchebichef abscissas, 330, 334, 335, 386.
 — — ultraspherical abscissas, 386.
 See Conjugate points, Fundamental polynomials, Hermite interpolation, Lagrange polynomials.
- Jackson, vii, 6, 7, 42, 331, 406.
 Jacob, 251, 406.
 Jacobi, iii, 49, 58, 69, 86, 406, 407.
 See Functions of second kind, Mechanical quadrature, Polynomials, Series.
- Jensen's theorem, 301.
 Joó, 391.
 Jordan, C., 58, 407.
 — arc, 8, 364.
 — curve, 8, 21, 364, 365.
 —, Ch., 34, 407.
- Jordan-Pochhammer integral, 75.
 Julia, 369, 407.
- Kaczmarz-Steinhaus, 1, 10, 407.
 Kanter, vii.
 Karlin-McGregor, 37, 273, 389, 420.
 Karlin-Studden, 5, 420.
 Karlin-Szegö, 190, 388, 420.
 Keldysch-Lawrentieff, 368, 407.
- Kernel polynomials, 39, 40, 44, 71, 101, 180, 183, 249, 290, 322, 323, 333, 351, 368, 377.
 See Asymptotic formula, Zeros.
- Klein, 145, 407.
 Kogbetliantz, 102, 168, 172, 203, 240, 248, 249, 251, 256, 273, 380, 407.
- Koornwinder, 98, 99, 420.
 Korous, 131, 132, 162, 167, 212, 251, 303, 408.
 Koschmieder, 60, 408.
- Kowalewski, 24, 408.
 Kowallik, 251, 408.
- Krall, 107, 408.
 Krawtchouk, 113, 408.
 See Polynomials.
- Kronecker, 408.
- Lagrange, 100, 408.
 — polynomials, 329 ff., 347.
 — series, 70.
 See Interpolation.
- Laguerre, v, 100, 117, 131, 408, 409.
 See Polynomials.

- Lamé function, 151.
 Langer, 204, 210, 409.
 Laplace, iv, 86.
 Laplace's formula for Legendre polynomials, 194, 198, 201, 202, 204 ff., 211 ff., 224.
 — — — — —, Darboux's generalization, 195, 201, 206 ff., 211.
 — — — — —, Stieltjes' generalization, 195, 202, 209 ff.
 Laplace-Heine formula, 194, 204 ff., 208, 243.
 — —, generalization, 194.
 Laplace series, 96, 249.
 — transform, 379.
 See Integral representation.
 Laurent expansion, 252.
 Lawton, 151, 409.
 Lebesgue, 14, 15, 409.
 See Integral.
 — constant, 13, 258, 330, 336, 338, 339, 350.
 Legendre, v, 70, 409.
 See Associated function, Polynomials.
 Lengyel, 113.
 See Erdős-Lengyel.
 LeRoy, 103, 409.
 Level curve, 8.
 Leibniz, rule of, 67, 68, 101.
 Limited operation, 12.
 Linear operation, see Operation.
 Liouville-Stekloff, method of, 202 ff., 210 ff., 299.
 Lipschitz, 383.
 — condition, 6, 162, 163, 186.
 Lipschitz-Dini condition, 279, 297, 324.
 Littlewood, see Hardy-Littlewood-Pólya.
 Locher, 363, 420.
 Lorch, xi, 190, 249, 420.
 Lorch-Muldoon-P. Szego, 158, 420.
 Lorch-P. Szego, 158, 420.
 Lukács, 4, 178 ff., 249, 409.

 Magnus-Oberhettinger, 420.
 Makai, 20, 148, 157, 190, 420, 421.
 Makai-Turán, 158, 421.
 Marcinkiewicz, 273, 330, 347, 409.
 Markoff, A., v, 33, 37, 50, 57, 115, 116, 121, 122, 139, 259, 378, 409.
 McGregor, see Karlin-McGregor.
 Mean approximation, 10, 11.
 Mean-Value theorem, 383.
 — — — of Gauss, 275, 310, 311.
 — — —, second, 2, 202, 357, 363.
 Mechanical quadrature, v, ix, 12, 14, 58, 187, 329, 348 ff.
 — — for classical abscissas, 352 ff.
 — — — Jacobi abscissas, 355 ff., 378, 379.
 — —, Gauss-Jacobi, 47 ff., 111, 348 ff.
 Mehler, iii, 47, 49, 85, 192, 380, 394, 409, 410.
 Mehler-Heine, see Asymptotic formula.
 Meixner, 34, 35, 410.
 Method of Liouville-Stekloff, see Liouville-Stekloff.
 — — steepest descent, 202, 203, 221 ff.
 Modulus of continuity, 6, 7, 335, 336, 340, 346.
 Moecklin, 194, 203, 410.
 Moment problem of Stieltjes, iii, iv, v, 40.
 Muckenhoupt, 190, 243, 273, 421.
 Muckenhoupt-Stein, 273, 421.
 Muldoon, see Lorch-Muldoon-P. Szego.
 Müntz, 251, 410.
 Myller-Lebedeff, 251, 410.

 Neumann, E. R., 130, 251, 380, 410.
 Neumann, F., 248.
 Neumann, J. von, 108.
 Newman-Rudin, 273, 421.
 Newton's formula, 259.
 Normalization, 28, 58, 160.
 Norm function, see Weight function.
 Norm of operation, 12.
 Novikoff, 394, 396, 421.

 Obrechhoff, 198, 248, 249, 410.
 Olver, 243, 421.
 Operation, linear functional, 11 ff.
 Orthogonal polynomials and continued fractions, 54 ff.
 — —, asymptotic formula of, v, ix, 4, 296 ff.
 — —, Christoffel-Darboux formula for, 42 ff.
 — —, classical, 29, 49.
 — —, closure, 40, 108 ff.
 — —, definition, ix, 23 ff.
 — —, expansion in series of, v, ix, 28, 38, 39, 41, 311, 312, 313 ff.
 — —, extremum properties of, 28, 38 ff.
 — —, general properties of, 38 ff.
 — —, highest coefficient of, 28.
 — —, recurrence formula for, 42 ff., 55, 391.
 — —, representation of, 27.
 — —, zeros of, v, ix, 44 ff., 121 ff., 188, 189.
 Orthogonality, Orthogonalization, v, vi, 8, 23 ff., 44.
 Orthonormal set, 23, 25, 68.
 Osgood-Carathéodory's theorem, 364.

 Parabola of convergence, 253.

- Parseval's formula, 40, 289, 368.
 Peano, 1.
 Peetre, 110, 421.
 Pencil, 49.
 Perron, 45, 54, 144, 202, 410.
 —'s formula for Laguerre polynomials, 198, 199, 202, 203, 220, 221, 225 ff.
P-function, contiguous Riemann, 71.
 Plancherel, 250.
 Plancherel-Rotach, 132, 201, 203, 410.
 Pochhammer-Barnes, notation of, 103.
 Poincaré, 310.
 Poisson's integral, 276, 292.
 See Polynomials.
 Pollaczek, ix, 37, 393, 421.
 Pollard, 273, 421.
 See Askey-Pollard.
 Pólya, vii, 42, 53, 117, 134, 154, 166, 350, 388, 410, 411.
 See Hardy-Littlewood-Pólya.
 Pólya-Szegő, vii, 5, 11, 21, 24, 34, 40, 70, 87, 105, 117, 134, 135, 175, 178, 179, 184, 212, 310, 311, 411.
 Polynomials associated with a curve (Tchebichef polynomials), 368, 385.
 —, classical, v, 29, 159.
 See Asymptotic formula, Expansion, Mechanical quadrature.
 —, Faber, 372, 374.
 —, Fejér's generalization of Legendre, see Fejér.
 —, Gegenbauer, see Polynomials, ultraspherical.
 —, Hermite, vi, ix, 29, 36, 37, 105 ff., 110, 111, 176 ff., 190, 331, 388 ff., 392.
 See Asymptotic formula, Differential equation, Expansion, Generating function, Interpolation, Recurrence formula, Rodrigues' formula, Zeros.
 —, Jacobi, vi, ix, 3, 29, 58 ff., 94 ff., 103, 105, 107, 161, 167 ff., 172 ff., 179 ff., 243, 249, 295, 348, 383, 385, 399.
 See Asymptotic formula, Differential equation, Expansion, Generating function, Interpolation, Mechanical quadrature, Recurrence formula, Rodrigues' formula, Zeros.
 —, Laguerre, vi, ix, 29, 35, 100 ff., 110, 111, 164, 176 ff., 184, 185, 190, 243, 379, 380, 382, 387 ff., 391, 393, 395.
 See Asymptotic formula, Differential equation, Expansion, Generating function, Interpolation, Recurrence formula, Rodrigues' formula.
 —, Legendre, vi, 29, 30, 33, 34, 48, 58, 63, 70, 85 ff., 95, 96, 136 ff., 162, 164 ff., 167, 172, 189, 346, 348, 379, 382 ff., 392, 393.
 See Asymptotic formula, Expansion, Generating function, Integral representation, Interpolation, Zeros.
 —, Krawtchouk, 35 ff.
 —, of S. Bernstein and Szegő, 31 ff.
 —, Poisson-Charlier, 34, 35, 37, 377, 389, 390.
 —, Pollaczek, 37, 393 ff.
 — orthogonal on a curve, vii, 364 ff.
 See Asymptotic formula, Zeros.
 — — — segment, see Orthogonal polynomials.
 Polynomials orthogonal on the unit circle, ix, 287 ff., 384.
 —, Stieltjes-Wigert, 33.
 —, Tchebichef (of the first and second kind), 3, 26, 29, 30, 60, 63, 112, 136 ff., 162, 347, 348, 387, 391.
 See Expansion, Interpolation, Zeros.
 —, ultraspherical, vi, 29, 58 ff., 80 ff., 93 ff., 107, 135 ff., 167, 170 ff., 379, 387, 390, 394.
 See Asymptotic formula, Differential equation, Expansion, Generating function, Integral representation, Interpolation, Recurrence formula, Rodrigues' formula, Zeros.
 Popoviciu, 46, 140, 142, 411.
 Principle of argument, 21, 157.
 Probability, calculus of, 35.
 Quadratic form, 24, 123, 187, 366.
 — — of Hankel (recurrent) type, 27, 309.
 Quantum mechanics, iii.
 Rabinowitz, see Davis-Rabinowitz.
 Rau, 197, 214, 249, 411.
 Recurrence formula, general, 42 ff.
 — — of Hermite polynomials, 106.
 — — — Jacobi functions of second kind, 78 ff., 379.
 — — — polynomials, 71 ff.
 — — — Laguerre polynomials, 101.
 — — — polynomials orthogonal on the unit circle, 293.
 — — — ultraspherical polynomials, 81, 82.
 Riemann, see Integral, *P*-function.
 —'s lemma, 254, 267, 319.
 —'s theory of trigonometric series, 248.
 Riesz, F., 12, 275, 411.
 Riesz, M., 5, 57, 304, 411.
 Robin's constant, see Transfinite diameter.

- Rodrigues' formula for Hermite polynomials, 106.
- — — Jacobi polynomials, 67 ff., 73, 74, 94, 99, 117.
- — — Laguerre polynomials, 101, 117, 388.
- — — ultraspherical polynomials, 81.
- Roever, vii.
- Rogosinski, 347.
- Rolle's theorem, 51, 53, 117, 378.
- Roosenrad, 110, 421.
- Ross, vii.
- Rotach, 203, 249, 250, 411.
See Plancherel-Rotach.
- Rouché's theorem, 21, 149.
- Rudin, see Newman-Rudin.
- Runge, 7.
- Runge-Walsh, theorem of, 7.
- Saddle-point, 219, 231.
- Šapiro, 98, 421.
- Sarmonov, 392, 421.
- Scalar product, 8, 9, 10, 25, 365, 367, 378.
- Schmeiser, 158, 422.
- Schmidt, E., 34, 35, 411.
- Schoenberg, 273.
- Schoenberg-Szegő, 190, 422.
- Schur, I., vi, 140, 142, 411.
- Schwid, 204, 416.
- Seidel-Szász, 94, 422.
- Sen-Rangachariar, 145, 411.
- Series, Jacobi, vi, 273.
- , Laguerre, 240.
- , Legendre, vi, 42.
See Expansion, Orthogonal polynomials.
- Sherman, 57, 411.
- Shibata, 145, 412.
- Shohat, v, vii, 42, 48, 159, 163, 189, 309, 333, 336, 340, 345, 347, 350, 385, 386, 412.
- Shohat-Hille-Walsh, 422.
- Singular integral, 13.
- Smirnof, 275, 276, 289, 368, 369, 412.
- Smith, 412.
- Sonin, 96, 100, 102, 104, 159, 166, 169, 176, 189, 379, 412.
- Spencer, 132, 412.
- Statistics, mathematical, v.
- Stein, see Muckenhoupt-Stein.
- Steinig, see Askey-Steinig.
- Stekloff, 9, 210, 350, 412.
See Liouville-Stekloff.
- Stekloff-Fejér, theorem of, 350 ff., 360.
- Step function, 34, 35, 37, 164.
- polynomial, 329, 339 ff.
- —, generalized, 331.
- Stieltjes, v, 33, 37, 46, 50, 51, 53, 54, 58, 87, 89, 90, 92, 93, 121 ff., 136, 139, 140, 142, 145, 151 ff., 156, 165, 172, 174, 175, 193, 195, 211, 382, 412, 413.
See Heine-Stieltjes, Hermite-Stieltjes, Integral, Laplace, Moments, Polynomials.
- Stirling's formula, 227.
- series, 212.
- Stone, 10, 23, 251, 413.
- Strip of convergence, 253.
- Studden, see Karlin-Studden.
- Sturm-Liouville type, 210.
- Sturm's theorem on differential equations, 19 ff.
- — (method) — zeros, vi, 45, 111, 121, 124 ff., 139.
- Summability, see Abel, Cesàro.
- Surányi-Turán, 347, 422.
- Szász, 5, 190, 413, 422.
See Seidel-Szász.
- Szegő, G., 19, 27, 31, 33, 37, 45, 59, 87, 93, 94, 96, 110, 125, 127, 128, 135, 137, 140, 158, 163, 164, 167 ff., 189, 190, 197, 198, 203, 206, 214, 243, 248, 249, 251, 274 ff., 287, 289, 294 ff., 299, 300, 309, 310, 312, 313, 340, 346, 355, 360, 361, 366, 369, 371, 377, 384, 388, 392, 393, 394, 413, 414, 422.
See Fejér-Szegő, Grenander-Szegő, Karlin-Szegő, Pólya-Szegő, Polynomials, Schoenberg-Szegő.
- Szego, P., xi.
See Lorch-P. Szego, Lorch-Muldoon-P. Szego.
- Szegő-Turán, 157, 158, 422.
- Tamarkin, vii, ix, 42, 414.
- Tchakaloff, 189, 383, 414.
- Tchebichef, v, 33, 47, 50, 54, 70, 100, 186, 188, 189, 414.
See Deviation, Polynomials.
- Thorne, 243, 422.
- Titchmarsh, 37, 57, 301, 310, 311, 415.
- Toeplitz matrix, 287, 399.
- , operator, 99.
- Total variation, 12, 34, 35.
- Transfinite diameter, 364, 369.
- Tricomi, ix, 242, 243, 415, 422.
- Trigonometric polynomials, 3, 5 ff., 11.
— representation, 90 ff.

- Turán, vii, ix, 158, 190, 388, 389, 422.
 See Balázs-Turán, Egerváry-Turán,
 Erdős-Turán, Grünwald-Turán,
 Makai-Turán, Surányi-Turán,
 Szegő-Turán.
- Uniqueness theorems, 248
 Uspensky, 53, 107, 203, 219, 220, 251, 415.
- Van Veen, 132, 204, 415.
- Vector, 9.
 — space, 10.
- Vietoris, 363, 422.
- Vitali-Sansone, 415, 423.
- Vitali's theorem, 57.
- Volterra equation, 211.
- Wainger, see Askey-Wainger.
- Walsh, 7, 366, 415.
 See Sholat-Hille-Walsh.
- Wangerin, 84, 415.
- Watson, 18, 20, 96, 102, 104, 107, 159, 162,
 192, 193, 202, 203, 222, 251, 253, 363, 380,
 384, 415.
 See Whittaker-Watson.
- Webster, 347, 423.
- Weierstrass, theorem of, 5 ff., 10, 110.
- Weight function, 9, 37, 68, 159, 160, 162 ff.,
 185, 188, 287, 294, 296, 297, 299, 309, 312,
 314, 316, 347, 380, 385.
- Weiss, G., see Coifman-G. Weiss.
- Weyl, 110, 251, 310, 415.
- Whittaker-Watson, 14 ff., 58, 63, 65, 66, 71,
 75, 84, 86, 88, 93, 103, 248, 415.
- Widom-Wilf, 243, 423.
- Wigert, 33, 102, 251, 413.
 See Polynomials.
- Wilf, see Widom-Wilf.
- Williamson, see Csordas-Williamson.
- Wiman, 131, 416.
- Winston, 131, 351, 416.
- Wintner, see Hartman-Wintner.
- Wright, 203, 384, 416.
- Young, W. H., 10, 248, 414.
- Zernike, 132, 416.
- Zero-function, 9, 40, 45.
- Zeros, distribution of, v, 310.
 —, electrostatical interpretation of, 140, 382.
 — of Airy's function, 18, 19, 377, 382.
 — — analytic functions, 21.
 — — Bessel functions, 126 ff., 140, 192, 193,
 381.
 — — Hermite polynomials, 117 ff., 123,
 127 ff., 141 ff., 240, 353.
 — — Jacobi polynomials, vi, 116 ff., 140 ff.,
 144 ff., 192, 193, 237 ff., 250 ff., 379, 381.
 — — kernel polynomials, 369, 377.
 — — Laguerre polynomials, 116 ff., 122 ff.,
 127 ff., 141 ff., 150 ff., 237 ff., 353, 381, 382.
 — — Legendre functions of second kind,
 155 ff.
 — — — polynomials, 111, 122, 125, 353.
 — — numerators of continued fractions, 57.
 — — polynomials orthogonal on a curve, 369.
 — — — — the unit circle, 292, 384.
 — — solutions of differential equations, see
 Sturm's theorem.
 — — Tchebichef polynomials, 330, 351.
 — — ultraspherical polynomials, 119, 121 ff.,
 138, 352, 381.
 See Orthogonal polynomials.
- Zygmund, 248, 253, 254, 274, 276, 279, 281,
 359, 416, 423.

The general theory of orthogonal polynomials was developed in the late 19th century from a study of continued fractions by P. L. Chebyshev, even though special cases were introduced earlier by Legendre, Hermite, Jacobi, Laguerre, and Chebyshev himself. It was further developed by A. A. Markov, T. J. Stieltjes, and many other mathematicians. The book by Szegő, originally published in 1939, is the first monograph devoted to the theory of orthogonal polynomials and its applications in many areas, including analysis, differential equations, probability and mathematical physics. Even after all the years that have passed since the book first appeared, and with many other books on the subject published since then, this classic monograph by Szegő remains an indispensable resource both as a textbook and as a reference book. It can be recommended to anyone who wants to be acquainted with this central topic of mathematical analysis.

ISBN 978-0-8218-1023-1



9 780821 810231

COLL/23

AMS on the Web
www.ams.org