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Orthogonal Polynomials

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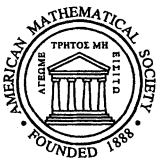

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TO MY WIFE

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PREFACE

Recent years have seen a great deal of progress in the field of orthogonal polynomials, a subject closely related to many important branches of analysis. Orthogonal polynomials are connected with trigonometric, hypergeometric, Bessel, and elliptic functions, are related to the theory of continued fractions and to important problems of interpolation and mechanical quadrature, and are of occasional occurrence in the theories of differential and integral equations. In addition, they furnish comparatively general and instructive illustrations of certain situations in the theory of orthogonal systems. Recently, some of these polynomials have been shown to be of significance in quantum mechanics and in mathematical statistics.

The origins of the subject are to be found in the investigation of a certain type of continued fractions, bearing the name of Stieltjes. Special cases of these fractions were studied by Gauss, Jacobi, Christoffel, and Mehler, among others, while more general aspects of their theory were given by Tchebichef, Heine, Stieltjes, and A. Markoff.

Despite the close relationship between continued fractions and the problem of moments, and notwithstanding recent important advances in this latter subject, continued fractions have been gradually abandoned as a starting point for the theory of orthogonal polynomials. In their place, the orthogonal property itself has been taken as basic, and it is this point of view which has been adopted in the following exposition of the subject. Choosing this same basic property, we discuss certain special orthogonal polynomials, which have been treated in great detail independently of the general theory, and indeed, even before this theory existed at all. In this connection we add the names of Laplace, Legendre, Fourier, Abel, Laguerre, and Hermite to those previously mentioned.

As regards treatises on the subject, we note that the only systematic treatment thus far given is found in J. Shohat's monograph, *Théorie Générale des Polynomes Orthogonaux de Tchebichef*, Mémorial des Sciences Mathématiques, Paris, 1934. Limitations of space have compelled that work to be brief, and consequently, it does not enter into a detailed treatment of many problems which have been especially advanced in recent years. It has therefore seemed desirable to attempt a new and detailed development of the main ideas of this field, devoting, in particular, some space to recent investigations of the distribution of the zeros, of asymptotic representations, of expansion problems, and of certain questions of interpolation and mechanical quadrature.

In what follows, we are concerned partly with the general theory of orthogonal polynomials, and partly with the study of special classes of these polynomials. As might be expected, we have more exhaustive results for these special classes, and we cite as an instance the classical polynomials satisfying linear differential

equations of the second order. Also, when the primary importance of these special classes in applications is taken into account, it should not be at all surprising that the present book is mainly devoted to their study. The general theory, however, as developed in Chapters XII and XIII, doubtless represents the most important progress made in recent years.

In the present work, no claim is made for completeness of treatment. On the contrary, the aim has purposely been to make the material suggestive rather than exhaustive. An attempt has been made to indicate the main and characteristic methods and to point out the relation of these to some general ideas in modern analysis. As a rule, preference has been given to those topics to which we were able to make some new, though modest, contributions, or which we could present in a new setting. Thus the book contains a number of results not previously published, some of which originated several years ago. For instance, we have included a discussion of the Cesàro summability of the Jacobi series at the end-points of the orthogonality interval (the method used here is of interest even in the classical case of Legendre series). Further, a new and simpler approach has been given to S. Bernstein's asymptotic formula for orthogonal polynomials. We also refer to certain details of minor importance, such as: simplifications and additions in the asymptotic investigation of Jacobi and Laguerre polynomials and in the discussion of the expansions in terms of these polynomials; the discussion of the cases in which the Jacobi differential equation has only polynomial solutions; the evaluation of the number of zeros of general Jacobi polynomials in the intervals $[-\infty, -1]$, $[-1, +1]$, $[+1, +\infty]$; a new proof of the Heine-Stieltjes theorem on linear differential equations of the second order with polynomial coefficients and polynomial solutions, and so on.

In general, we have preferred to discuss problems which may be stated and treated simply, and which could be presented in a more or less complete form. This was the main reason for devoting no space to the extremely interesting arithmetic and algebraic properties of orthogonal polynomials, such as, for instance, the recent important investigations of I. Schur concerning the irreducibility and related properties of Laguerre and Hermite polynomials. Furthermore, we have attached great importance to the idea of replacing incomplete and overlapping theorems, scattered in the literature, by complete results involving only intrinsic or necessary restrictions. We have also tried to exploit, as far as seemed to be at all possible, definite methods, such as, for instance, Sturm's methods in differential equations (see §§6.3, 6.31, 6.32, 6.83).

A complete treatment of Legendre polynomials was not feasible, and probably not desirable, in the framework of the general theory. Besides, there are already complete treatises on spherical and other harmonics.[1] We have selected and considered only those properties of Legendre polynomials which are the starting points of generalizations to ultraspherical, Jacobi, or to more general polynomials. Another subject which could not be included was Stieltjes'

[1] For instance, E. W. Hobson 1 (see bibliography).

problem of moments, which has been omitted in spite of its great interest; for this subject would have necessitated the development of a complicated apparatus of results and methods. Orthogonal polynomials of more than one variable also have not been treated.[2]

The book is based on a course given at Washington University during the academic year 1935–1936. Acquaintance with the general ideas and methods of the theory of functions of real and complex variables is naturally required. Occasionally, Stieltjes-Lebesgue and Lebesgue integrals are considered. In the greater part of the book, however, these integrals have been avoided, and, except in a very few places, no detailed properties of them were used.

The problems at the end of this book are, with few exceptions, not new, and they are not interconnected as are, for instance, those in Pólya-Szegö's *Aufgaben und Lehrsätze*. They are more or less supplementary in character and serve as illustrations and exercises; they sometimes differ widely from one another both as to subject and method.

The list of references is not complete; it contains only original memoirs, a few text books of primary importance, and monographs to which references are made in the text.

For the suggestion of preparing a book on orthogonal polynomials for the Colloquium Publications, I am indebted to Professor J. D. Tamarkin, who has also participated in the present work by offering a great number of valuable suggestions. It is with the greatest gratitude that I mention his friendly interest.

I have also received valuable advice from my friends and teachers L. Fejér (Budapest), and G. Pólya (Zürich). My colleagues P. Erdös (Manchester), G. Grünwald (Budapest), W. H. Roever (St. Louis), A. Ross (St. Louis), J. Shohat (Philadelphia), and P. Turán (Budapest) gave generously and unstintingly of their time. F. A. Butter, Jr. (at present in Los Angeles) collaborated with me in the preparation of the manuscript. This last aid was made possible through a grant from the Rockefeller Research Fund of Washington University (1936–1937). My student L. H. Kanter also rendered valuable assistance in the preparation of the manuscript.

My gratitude for the encouragement and help of these friends, colleagues, and institutions can hardly be measured by any formal acknowledgment. Lastly, I wish to express to the American Mathematical Society my great appreciation for the inclusion of the present book in its Colloquium Series.

G. Szegö

Washington University, 1938.

[2] Cf. the bibliography in Jackson **8**, p. 423.

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PREFACE TO THE REVISED EDITION

The first printing of this book published in 1939 was about exhausted in 1948. Reprinting was arranged then but for various reasons no change in the text was made. During the past twenty years since the preparation of the original edition was completed, considerable progess was made in this field. A glance at the pertinent section of the Mathematical Reviews suggests that the interest in this topic is still very much alive. Systematic treatment of orthogonal polynomials has been incorporated in various modern texts published in the meantime. We refer only to the *Higher Transcendental Functions* published by the Bateman Manuscript Project Staff (cf. in particular, vol. 2, Chapter X, edited by Professor A. Erdélyi), and to the book of F. Tricomi, *Vorlesungen über Orthogonalreihen* (Chapters IV–VI).

Recently the council of the American Mathematical Society has authorized the author to prepare a revised edition of the book, adding a moderate amount of material in order to bring it up to date. Naturally, limitations of space and time did not allow including all new results (or, for that matter, the old ones which were missing from the original edition). Only a few particularly interesting new items have been added as well as some details which deserve attention because of elegance of the method or originality of ideas. We mention here in particular the important Pollaczek polynomials; they are treated in an Appendix. Further new material was incorporated in the form of Problems and Exercises. New bibliographic items have been included, again in a rather selective way. Finally, misprints have been corrected and numerous minor improvements and additions made.

The author recollects again, as was stated in the Preface of 1938, that the preparation of this book was suggested to him by the late Professor J. D. Tamarkin. Since his untimely death in 1945 his name is not too frequently mentioned. It is justified and probably necessary to remind the younger mathematical generation, in the rush of modern developments, how much American mathematics owes to his great energy and far-sighted intelligence.

STANFORD UNIVERSITY, 1958 G. SZEGÖ

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PREFACE TO THE THIRD EDITION

The interest of the mathematical community for orthogonal polynomials, classical and non-classical, is still not entirely exhausted. During the past years I lectured about this subject several times at Stanford. The attendants of the course were upper division and graduate students, specializing in mathematics, mathematical statistics, calculus of probability, etc.

Only minor changes have been made in the text. I owe numerous improvements and corrections to various friends and colleagues. I mention particularly Professor Paul Turan (Budapest, Hungary) and Professor Lee Lorch (Edmonton, Canada). New references, published in the time interval 1958-1966, have been included.

STANFORD UNIVERSITY, 1966 G. SZEGÖ

PREFACE TO THE FOURTH EDITION

Again the American Mathematical Society has taken the initiative to reprint the present book, allowing some minor changes and new material. Among the persons interested in the field of orthogonal polynomials who have contributed to these changes and additions, I mention with particular indebtedness my friend and colleague Professor Richard Askey (Madison, Wisconsin) and the very active and original group of mathematicians around him. A very important set of lectures by Askey entitled, "Orthogonal Polynomials and Special Functions," reached me too late to be incorporated in the present edition.

Further material has been furnished by Professor Paul Turán (Budapest, Hungary) and Professor Lee Lorch (Toronto, Canada). New problems and exercises have also been included. Peter Szego (Redwood City, California) gave me valuable assistance in preparing the present manuscript.

My gratitude goes to all these friends and colleagues.

STANFORD UNIVERSITY, 1975 G. SZEGÖ

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The general theory of orthogonal polynomials was developed in the late 19th century from a study of conti ued fractions by P. L. Chebyshev, even though special cases were introduced earlier by Legendre, Hermit Jacobi, Laguerre, and Chebyshev himself. It was further developed by A. A. Markov, T. J. Stieltjes, and ma other mathematicians. The book by Szegő, originally published in 1939, is the first monograph devoted to t theory of orthogonal polynomials and its applications in many areas, including analysis, differential equ tions, probability and mathematical physics. Even after all the years that have passed since the book fi appeared, and with many other books on the subject published since then, this classic monograph by Sze remains an indispensable resource both as a textbook and as a reference book. It can be recommended anyone who wants to be acquainted with this central topic of mathematical analysis.

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