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TO THE MEMORY OF

J. D. TAMARKIN

WHO ENCOURAGED AND INSPIRED

US BOTH

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FOREWORD TO THE REVISED EDITION

Seven years have now elapsed since the publication of the first edition of this treatise in October 1948. The friendly interest which the mathematical community has shown for this work has been most gratifying; in fact, the edition was out of print by 1954. We can also report the appearance of a Russian translation of the book in 1951 [*Funkcional'nyj Analiz i Polugruppy*, Izdatel'stvo Innostrannoj Literatury, Moskva, 1951, 636 pages].

Since 1948 both the analytical theory of semi-groups and its applications have made vigorous progress. K. Yosida found the basic generation theorem independently of Hille in 1948 and proceeded to apply it to the diffusion equation in a series of important papers. Inspired by Yosida's work, Hille made a new attack on Cauchy's problem with the aid of semi-group theory, starting in 1949. Soon thereafter, W. Feller became interested in the possibilities of the new approach and he and his students have contributed much to the theory; we mention in particular his penetrating investigation of the singular boundary value problem for the diffusion equation. Another of the early workers in the general theory of semi-groups of linear operators was R. S. Phillips who filled in many of the gaps which Hille had left behind and then went on to broaden the theory using representation theory for semi-group algebras, perturbation methods, extended classes of semi-groups, and adjoint semi-groups. In a different direction, in 1948 Hille laid the foundations of a Lie theory of semi-groups.

When early in 1952 it became obvious that a new printing of the treatise would be needed and it was clear that the new advances in the theory called for extensive revision, Hille asked Phillips to collaborate with him on a new edition. The resulting treatise is now offered to the public. The original has been completely rewritten, mostly by Phillips; the old framework is still there together with most of the old results, but much has been added, very much indeed. The changes are partly a matter of exposition, and partly a matter of methods and results. Thus in keeping with the spirit of the times the algebraic tools now play a major role and are introduced early in the book; they lead to a more satisfactory operational calculus and spectral theory in Chapters XV and XVI. On the other hand, the Laplace-Stieltjes transform methods, used by Hille for such purposes, have not been replaced but rather supplemented by the new tools.

Part I on Functional Analysis has been augmented and rearranged. The old Chapter IV (Functions on Vectors to Vectors) has been relegated to the end of the book except for certain parts essential for the body of the work which have been incorporated with Chapter III. The new chapter IV contains a treatment of the Gelfand representation theory for commutative Banach algebras together with substantial portions of the old Chapters V (Analysis in a Banach Algebra) and XXII (Notes on Banach Algebras). The new Chapter V contains a modified version of the operational calculus for Banach algebras which was presented in the old Chapter V and, in addition, it contains an operational calculus for closed unbounded linear operators. The discussion of Laplace integrals has been moved from Chapter X to Chapter VI. Part II is now called Basic Properties of Semi-Groups but contains most of the material to be found in the first half of the old Part II. However important additions have been made; we note Phillips' classification of semi-groups in Chapters X and XI and his solution of the generation problem in Chapter XII. The new part III, Advanced Analytic Theory of Semi-Groups, is based on the latter half of the old Part II. Nevertheless except for the last two chapters, the material here is largely new. It contains Phillips's main contributions to the theory (perturbation theory, adjoint theory, operational calculus, and spectral theory). Part IV, Special Semi-Groups and Applications,

corresponds to the old Part III. Here the main change, sad to report, is the omission of the discussion of the applications to partial differential equations. These applications had grown so tremendously that an adequate treatment now requires a treatise of its own. All we could do was to insert a discussion of the abstract Cauchy problem in Chapter XXIII. Finally the old Appendix has become a fifth part entitled Extensions of the Theory. Here the main addition is Hille's development of the Lie theory of semi-groups.

As usual the revision has taken more work and more time than originally planned. Both authors have been liberally supported by grants from public funds, Hille through the Office of Scientific Research of the Air Research and Development Command, United States Air Force, Contract No. AF 18 (600)-469, Phillips through the Office of Ordnance Research, United States Army, Contract No. DA-04-495-Ord-406. This support is gratefully acknowledged. On the personal side it is a great pleasure for us to express our gratitude to many friends who have aided us in preparing the manuscript of the revised edition. Advice, aid, and valuable suggestions have been received from R. P. Agnew, A. V. Balakrishnan, J. Brooks, N. Dunford, H. A. Dye, N. Jacobson, S. Kakutani, T. Kato, D. G. Kendall, R. A. Moore, J. Schwartz, and K. Yosida. To all helpers, named and unnamed, we extend our warmest thanks.

New Haven, Conn. and Los Angeles, Calif. EINAR HILLE, R. S. PHILLIPS
September 1955

CONVENTIONS

Each Part of the book starts with a Summary, each Chapter with an Orientation. The chapters are divided into sections and the sections, except for orientations, are grouped into paragraphs. Cross references are normally to sections, rarely to paragraphs. Section 3.10 is the tenth section in Chapter III; it belongs to §2 which is referred to as §3.2 when necessary. The page headings show the numbers of current section and paragraph, the integral part of the former is the number of the chapter. Definitions, formulas, lemmas, and theorems are numbered separately within each section; thus Theorem 9.4.2 is the second theorem in section 9.4. References to the literature give the author's name followed by numerals in brackets referring to his book or paper by that number in the Bibliography at the close of the book. Such references are given in the text when needed; collected references for a whole chapter occur after the orientation to the chapter in question except for chapters with heterogeneous subject matter in which case they are given at the end of each paragraph.

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FUNCTIONAL ANALYSIS

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