

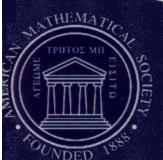
American Mathematical Society

Colloquium Publications

Volume 38

Theory of Graphs

Oystein Ore



Theory of Graphs

This page intentionally left blank

American Mathematical Society
**COLLOQUIUM
PUBLICATIONS**

Volume 38

Theory of Graphs

Oystein Ore



American Mathematical Society
Providence, Rhode Island

2000 *Mathematics Subject Classification*. Primary 05-XX.

Library of Congress Cataloging-in-Publication Data

Ore, Øystein, 1899–1968.

Theory of graphs.

p. cm. — (American Mathematical Society Colloquium publications, ISSN 0065-9258 ; v. 38)

Includes bibliography.

ISBN 978-0-8218-1038-5

1. Graph theory. I. Title.

QA1 .A5225 vol. 38

510

61-15687

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294 USA. Requests can also be made by e-mail to reprint-permission@ams.org.

© 1962 by the American Mathematical Society. All rights reserved.

Third printing, with corrections, 1967.

Printed in the United States of America.

The American Mathematical Society retains all rights
except those granted to the United States Government.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at <http://www.ams.org/>

14 12 12 11 10 9 16 15 14 13 12 11

TABLE OF CONTENTS

FOREWORD

Chapter 1	FUNDAMENTAL CONCEPTS	1
	1.1 Graph definitions	1
	1.2 Local degrees	7
	1.3 Subgraphs	12
	1.4 Binary relations	13
	1.5 Incidence matrices	18
Chapter 2	CONNECTEDNESS	22
	2.1 Sequences, paths and arcs	22
	2.2 Connected components	23
	2.3 One-to-one correspondences	25
	2.4 Distances	27
	2.5 Elongations	31
	2.6 Matrices and paths. Product graphs	33
	2.7 Puzzles	36
Chapter 3	PATH PROBLEMS	38
	3.1 Euler paths	38
	3.2 Euler paths in infinite graphs	42
	3.3 An excursion into labyrinths	47
	3.4 Hamilton circuits	52
Chapter 4	TREES	58
	4.1 Properties of trees	58
	4.2 Centers in trees	62
	4.3 The circuit rank	67
	4.4 Many-to-one correspondences	68
	4.5 Arbitrarily traceable graphs	74
Chapter 5	LEAVES AND LOBES.....	78
	5.1 Edges and vertices of attachment	78
	5.2 Leaves	81
	5.3 Homomorphic graph images	83
	5.4 Lobes	85
	5.5 Maximal circuits	89

Chapter 6	THE AXIOM OF CHOICE	92
	6.1 Well-ordering	92
	6.2 The maximal principles	94
	6.3 Chain sum properties	96
	6.4 Maximal exclusion graphs	99
	6.5 Maximal trees	100
	6.6 Interrelations between maximal graphs	102
Chapter 7	MATCHING THEOREMS	105
	7.1 Bipartite graphs	105
	7.2 Deficiencies	108
	7.3 The matching theorems	110
	7.4 Mutual matchings	113
	7.5 Matchings in special graphs	117
	7.6 Bipartite graphs with positive deficiencies	121
	7.7 Applications to matrices	125
	7.8 Alternating paths and maximal matchings	132
	7.9 Separating sets	138
	7.10 Simultaneous matchings	139
Chapter 8	DIRECTED GRAPHS	145
	8.1 The inclusion relation and accessible sets	145
	8.2 The homomorphism theorem	149
	8.3 Transitive graphs and embedding in order relations	150
	8.4 Basis graphs	152
	8.5 Alternating paths	156
	8.6 Subgraphs of first degree	159
Chapter 9	ACYCLIC GRAPHS	162
	9.1 Basis graphs	162
	9.2 Deformations of paths	163
	9.3 Reproduction graphs	166
Chapter 10	PARTIAL ORDER	170
	10.1 Graphs of partial order	170
	10.2 Representations as sums of ordered sets	170
	10.3 Lattices and lattice operations. Closure relations	175
	10.4 Dimension in partial order	178

TABLE OF CONTENTS

vii

Chapter 11	BINARY RELATIONS AND GALOIS CORRESPONDENCES	183
	11.1 Galois correspondences	183
	11.2 Galois connections for binary relations	187
	11.3 Alternating product relations	191
	11.4 Ferrers relations	193
Chapter 12	CONNECTING PATHS	197
	12.1 The cross-path theorem	197
	12.2 Vertex separation	200
	12.3 Edge separation	202
	12.4 Deficiency	203
Chapter 13	DOMINATING SETS, COVERING SETS AND INDEPENDENT SETS ...	206
	13.1 Dominating sets	206
	13.2 Covering sets and covering graphs	208
	13.3 Independent sets	210
	13.4 The theorem of Turan	214
	13.5 The theorem of Ramsey	216
	13.6 A problem in information theory	220
Chapter 14	CHROMATIC GRAPHS	224
	14.1 The chromatic number	224
	14.2 Sums of chromatic graphs	227
	14.3 Critical graphs	229
	14.4 Coloration polynomials	234
Chapter 15	GROUPS AND GRAPHS	239
	15.1 Groups of automorphisms	239
	15.2 Cayley's color graphs for groups	242
	15.3 Graphs with prescribed groups	244
	15.4 Edge correspondences	245
	BIBLIOGRAPHY	250
	LIST OF CONCEPTS	265
	INDEX OF NAMES	269

This page intentionally left blank

FOREWORD

The present book has grown out of courses on graph theory given from time to time at Yale University. A first set of lectures on binary relations and graphs was presented before the American Mathematical Society at its summer meeting in Chicago, 1942. Since the manuscript to these lectures was not completed for publication at that time due to more urgent tasks it seems appropriate that this book should appear in the Colloquium Lecture Series of the society.

Graph theory as a mathematical discipline was created by Euler in his now famous discussion of the Königsberg Bridge Problem. However, Euler's article of 1736 remained an isolated contribution for nearly a hundred years. About the middle of the last century a resurgence of interest in the problems of graph theory took place, centered mainly in England. There were many causes for this revival of graph studies: The natural sciences had their influence through investigations of electrical networks and models for crystal and molecular structure; the development of formal logic led to the study of binary relations in the form of graphs. A number of popular puzzle problems could be formulated directly in terms of graphs and thus came the realization that many such questions include a mathematical nucleus of general importance. Most celebrated among them is the Four Color Map Conjecture which was first laid before the mathematicians by De Morgan around 1850. No other problem has occasioned as numerous and ingenious contributions to graph theory. Due to its simple formulation and exasperating evasiveness it still remains a powerful incitement to the examination of graph properties.

The present century has witnessed a steady development of graph theory which in the last ten to twenty years has blossomed out into a new period of intense activity. Clearly discernible in this process are the effects of the demands from new fields of applications: game theory and programming, communications theory, electrical networks and switching circuits as well as problems from biology and psychology.

As a consequence of these recent developments the subject of graphs is already so extended that it did not seem feasible to cover all its main ramifications within the framework of a single volume. In the present first volume of an intended two volume work the emphasis has been placed upon basic concepts and the results of particular systematic interest.

There exist very few books on graph theory; the mainstay has been the book by D. König (1936), which for its time gave a most excellent introduction to the subject. Strangely enough, until now there has been no book in English, in spite of the fact that many of the most important contributions to the subject

are due to American and English writers. An effort has been made to present the subject matter in the book in as simple a form as possible. Almost all proofs have been revised; a considerable number of new results are also included. A systematic terminology is introduced which it is hoped may prove acceptable.

For the benefit of the reader a considerable number of problems have been included. Many of these are quite simple; others are more in the nature of proposed research problems; these have been marked with an asterisk.

The second volume will be devoted to more special topics: planar graphs, the four color conjecture, the theory of flow, games, electrical networks, as well as applications to a number of other fields in which graph theory is a principal tool.

Oystein Ore

Yale University
April 1960

BIBLIOGRAPHY

CHAPTER 1

C. BERGE, *Théorie des graphes et ses applications*. Paris, 1958.

D. KÖNIG, *Theorie der endlichen und unendlichen Graphen*. Leipzig, 1936.

A. SAINTE-LAGUE, *Les réseaux (ou graphes)*. Mémor. Sci. Math. v. 18, 1926.

Many collections of mathematical puzzles contain problems which may be formulated in graph terms. We shall mention only:

W. ROUSE-BALL, *Mathematical recreations and problems*. London, 1892. Numerous editions.

E. LUCAS, *Récréations mathématiques*. 3 v. Paris, 1882-1894.

CHAPTER 2

2.7

D.W. CROWE, *The n -dimensional cube and the Tower of Hanoi*. Amer. Math. Monthly, v. 63 (1956) pp. 29-30.

CHAPTER 3

3.1

L.EULER, *Solutio problematis ad geometriam situs pertinentis*. Commentarii Academiae Petropolitanae, v. 8 (1736) pp. 128-140.

N.G. DE BRUIJN, *A combinatorial problem*. Nederl. Akad. Wetensch. Proc., v. 49 (1946) pp. 758-764.

VAN AARDENNE-EHRENFEST AND N.G. de BRUIJN, *Circuits and trees in oriented graphs*. Simon Stevin, v. 28 (1951) pp. 203-217.

3.2

P. ERDÖS, T. GRÜNWARD, AND E. WEISZFELD, *Über Eulersche Linien unendlicher Graphen*. Mat. Fiz. Lapok, v. 43 (1936) pp. 129-140.

P. ERDÖS, T. GRÜNWARD, AND E. VÁZSONYI, *Über Euler-Linien unendlicher Graphen*. J. Math. Phys. Massachusetts Inst. of Technology, v. 17 (1938) pp. 59-75.

E. VÁZSONYI, *Über Gitterpunkte des mehrdimensionalen Raumes*. Acta Litt. Sci. Szeged, v. 9 (1939) pp. 163-173.

G. RINGEL, *Über drei kombinatorische Probleme am n -dimensionalen Würfel und Würfelgitter*. Abh. Math. Sem. Univ. Hamburg, v. 20 (1955) pp. 10-19.

3.3

C. WIENER, *Über eine Aufgabe aus der Geometria situs*. Math. Ann., v. 6 (1873) pp. 29–30.

O. ORE, *An excursion into labyrinths*. Mathematics Teacher, v. 52 (1959) pp. 367–370.

M. BECKMAN, C. B. MCGUIRE, AND C. B. WINSTEN, *Studies in the economics of transportation*. New Haven, Yale University Press, 1956.

L.R. FORD, *Network flow theory*. Rand Corp. Publ. P-23 (1956).

3.4

D. J. NEWMAN, *A problem in graph theory*. Amer. Math. Monthly, v. 65 (1958) p. 611.

O. ORE, *Note on Hamilton circuits*. Amer. Math. Monthly, v. 67 (1960) p. 55.

W. T. TUTTE, *On Hamilton circuits*. J. London Math. Soc., v. 21 (1946) pp. 98–101.

F. SUPNICK, *Extreme Hamilton lines*. Ann. of Math., (2) v. 66 (1957) pp. 179–201.

F. FITTING, *Doppelsymmetrische Rösselsprünge auf Quadraten von ungeraden Felderzahl ohne Mittelfeld*. Jber. Deutsch. Math. Verein, v. 46 (1936) Abt. 1, pp. 38–43.

G. DANTZIG, R. FULKERSON, AND S. JOHNSON, *Solution of a large-scale travelling-salesman problem*. Journal Operational Research Soc. of Amer., v. 2 (1954) pp. 393–410.

I. HELLER, *The travelling-salesman problem* George Washington Univ. Logistics Research Project, 1954.

M. M. FLOOD, *On the travelling salesman's problem*. Journal Operational Research Soc. of Amer., v. 4 (1956) pp. 61–75.

T. S. MOTZKIN, AND E. G. STRAUS, *Some combinatorial extremum problems*. Proc. Amer. Math. Soc., v. 7 (1956) pp. 1014–1021.

G. DANTZIG, R. FULKERSON, AND S. JOHNSON, *On a linear programming combinatorial approach to the travelling-salesman problem*. Journal Operational Research Soc. of Amer. v. 7 (1959) pp. 58–66.

R. BELLMAN, *On a routing problem*. Quart. Appl. Math., v. 16 (1958) pp. 87–90.

CHAPTER 4

4.1

A. CAYLEY, *A theorem on trees*. Quarterly Journal of Pure and Appl. Math., v. 23 (1889) pp. 376–378.

H. PRÜFER, *Neuer Beweis eines Satzes über Permutationen*. Archiv der Math. und Phys., (3) v. 27 (1918) pp. 142–144.

O. DZIOBEK, *Eine Formel der Substitutionstheorie*. Sitzungsberichte der Berliner Math. Gesellschaft, v. 16 (1947) pp. 64–67.

G. BOL, *Über eine kombinatorische Frage*. Abh. Math. Seminar der Hansischen Univ., v. 12 (1938) pp. 242–245.

E. H. NEVILLE, *The codifying of tree-structure*. Proc. Cambridge Philos. Soc., v. 49 (1953) pp. 381–385.

R. BOTT and J. P. MAYBERRY, *Matrices and trees*. Also T. M. WHITIN, *An economic application of "Matrices and Trees"*. Both published in O. MORGENSTERN: *Economic Activity Analysis*, New York, John Wiley and Sons, 1954.

L. VITALBI, *Ricerche sulla teoria dei reticoli*. Giorn. Mat. Battaglini, (5) v. 4 (84) (1956) pp. 93–121.

G. ANDREOLI, *Preliminari topologici su gli alberi*. Giorn. Mat. Battaglini, (5) v. 2 (82) (1954) pp. 237–266.

P. J. KELLY, *A congruence theorem for trees*. Pacific J. Math., v. 7 (1957) pp. 961–968.

G. PÓLYA, *Kombinatorische Anzahlbestimmungen für Gruppen, Graphen und chemische Verbindungen*. Acta Math., v. 68 (1937) pp. 145–254.

J. RIORDAN, *An introduction to combinatorial analysis*. John Wiley and Sons, New York, 1958.

O. BORUVKA, *On a minimal problem*. Práce Moraské Pridovedecké Společnosti, v. 3 (1926).

V. JARNÍK and M. KOSSLER, *Sur les graphes minima contenant n points donnés*. Časopis Mat. Fys., v. 63 (1934) pp. 223–235.

G. CHOQUET, *Etude de certains réseaux de routes*. C. R. Acad. Sci. Paris, v. 206 (1938) pp. 310–313.

J. B. KRUSKAL, *On the shortest spanning subtree of a graph and the traveling salesman problem*. Proc. Amer. Math. Soc., v. 7 (1956) pp. 48–50.

D. BLANUSA, *Über die Anzahl der Bedingungsgleichungen beliebigen geodätischen Netzen*. Z. Vermessungswesen, v. 73 (1944) pp. 54–62.

4.4

O. ORE, *Graphs and correspondences*. Festschrift Andreas Speiser Zurich, 1945 pp. 184–191.

— *incidence matchings in graphs*. Journ. de Math., v. 40 (1961) pp. 123–127.

4.5

O. ORE, *A problem regarding the tracing of graphs*. Elem. Math., v. 6 (1951) pp. 49–53.

F. BÄBLER, *Über eine spezielle Klasse Euler'scher Graphen*. Comment. Math., Helv., v. 27 (1953) pp. 81–100.

F. HARARY, *On arbitrarily traceable graphs and directed graphs*. Scripta Math. v. 23 (1957) pp. 37–41.

Further papers on trees:

A. KOTZIG, *The significance of the skeleton of a graph for the construction of composition bases of some subgraphs*. Mat. Fyz. Casopis Slovensk. Akad. Vied., v. 6 (1956) pp. 68–77.

W. T. TUTTE, *A ring in graph theory*. Proc. Cambridge Philos. Soc., v. 43 (1947) pp. 26–40.

J. S. R. CHISHOLM, *The S-matrix for neutral PS-PV meson-nucleon interaction*. Phil. Mag., (8) v. 1 (1956) pp. 338–344.

W. H. BURGE, *Sorting, trees, and measures of order*. Information and Control, v. 1 (1958) pp. 181–197.

CHAPTER 5

5.1

S. MACLANE, *Some unique separation theorems for graphs*. Amer. J. Math., v. 57 (1935) pp. 805–820.

R. E. NETTLETON, K. GOLDBERG, AND M. S. GREEN, *Dense subgraphs and connectivity*. Canad. J. Math., v. 11 (1959) pp. 262–268.

5.4

H. WHITNEY, *Non-separable and planar graphs*. Trans. Amer. Math. Soc., v. 34 (1932) pp. 339–362.

F. HARARY, *An elementary theorem on graphs*. Amer. Math. Monthly, v. 66 (1959) pp. 405–407.

K. HUSIMI, *Note on Mayers theory of cluster integrals*. Journ. Chem. Phys., v. 18 (1950) pp. 682–684.

F. HARARY AND G. E. UHLENBECK, *On the number of Husimi trees*. Proc. Nat. Acad. Sci. U.S.A., v. 39 (1953) pp. 315–322.

F. HARARY AND R. Z. NORMAN, *The dissimilarity characteristic of Husimi trees*. Ann. of Math. (2) v. 59 (1953), pp. 134–141.

5.5

G. A. DIRAC, *Some theorems on abstract graphs*. Proc. London Math. Soc., (3) v. 2 (1952) pp. 69–81.

P. ERDÖS AND T. GALLAI, *On maximal paths and circuits of graphs*. Acta Math. Acad. Sci. Hungar., v. 10 (1959) pp. 337–356.

CHAPTER 6

6.4.

O. ORE AND T. S. MOTZKIN, *Subsets and subgraphs with maximal properties*. Proc. Amer. Math. Soc., v. 10 (1959) pp. 965–969.

CHAPTER 7

7.1.

R. RADO, *Axiomatic treatment of rank in infinite sets*. Canad. J. Math., v. 1 (1949) pp. 337–343.

W. H. GOTTSCHALK, *Choice functions and Tychonoff's theorem*. Proc. Amer. Math. Soc., v. 2 (1951) p. 172.

7.3.

O. ORE, *Graphs and matching theorems*. Duke Math. J., v. 22 (1955) pp. 625–639.

P. HALL, *On representatives of subsets*. J. London Math. Soc., v. 10 (1935) pp. 26–30.

M. HALL, *Distinct representatives of subsets*. Bull. Amer. Math. Soc., v. 54 (1948) pp. 922–926.

7.4.

N. G. DE BRUIJN, *Gemeenschappelijke representantensystemen van twee klassenindeelingen van een verzameling*. Nieuw Archief voor Wiskunde, (2) v. 22 (1943) pp. 48–52.

G. GRÜNWARD, *Über einen mengentheoretischen Satz*. Mat. Fiz. Lapok, v. 44 (1937) pp. 51–53.

M. HALL, *An algorithm for distinct representatives*. Amer. Math. Monthly, v. 63 (1956) pp. 716–717.

P. R. HALMOS AND H. E. VAUGHAN, *The marriage problem*. Amer. J. Math., v. 72 (1950) pp. 214–215.

L. HENKIN, *Some interconnections between modern algebra and mathematical logic*. Trans. Amer. Math. Soc., v. 74 (1953). pp. 410–427.

P. J. HIGGINS, *Disjoint transversals of subsets*. Canad. J. Math., v. 11 (1959) pp. 280–285.

A. J. HOFFMAN AND H. W. KUHN, *Systems of distinct representatives and linear programming*. Amer. Math. Monthly, v. 63 (1956) pp. 455–460.

— *On systems of distinct representatives*. Annals of Math. Studies, No. 38 (1956).

G. KREWERAS, *Extension d'un théorème sur les répartitions en classes*. C. R. Acad. Sci. Paris, v. 222 (1946) pp. 431–432.

W. MAAK, *Ein Problem der Kombinatorik in seiner Formulierung von H. Weyl*. Math.-Phys. Semesterber. Göttingen, v. 22 (1952) pp. 251–256.

H. B. MANN AND H. J. RYSER, *Systems of distinct representatives*. Amer. Math. Monthly, v. 60 (1953) pp. 397–401.

N. S. MENDELSON AND A. L. DULMAGE, *Some generalizations of the problem of distinct representatives*. Canad. J. Math., v. 10 (1958) pp. 230–241.

— *Coverings of bipartite graphs*. Canad. J. Math., v. 10 (1958) pp. 517–534.

R. RADO, *Factorization of even graphs*. Quart. J. Math., Oxford, v. 20 (1949) pp. 95–104.

V. SHMUSHKOVITCH, *On a combinatorial theorem of the theory of sets*. Mat. Sb., v. 6 (1939) pp. 139–146.

B. L. VAN DER WAERDEN, *Ein Satz über Klasseneinteilungen von endlichen Mengen*. Hamburger Abh., v. 5 (1927) pp. 185–187.

H. WEYL, *Almost periodic invariant vector sets in a metric vector space*. Amer. J. Math., v. 71 (1949) pp. 178–205.

7.5

G. SCORZA, *A proposito di un teorema del Chapman*. Boll. Un. Mat. Ital., v. 6 (1927) pp. 1–6.

H. W. CHAPMAN, *A note on the elementary theory of groups of finite order*. Messenger of Math., v. 42 (1913) pp. 132–134; v. 43 p. 85.

G. A. MILLER, *On a method due to Galois*. Quarterly J. Math., v. 41 (1910) pp. 382–384.

S. SHŪ, *On the common representative system of residue classes of infinite groups*. J. London Math. Soc., v. 16 (1941) pp. 101–104.

O. ORE, *On coset representatives in groups*. Proc. Amer. Math. Soc., v. 9 (1958) pp. 665–670.

— *Conditions for subgraphs of directed graphs*. J. Math. Pures Appl., v. 37 (1958) pp. 321–328.

7.7

A. L. DULMAGE AND N. S. MENDELSON, *The term and stochastic rank of a matrix*. Canad. J. Math., v. 11 (1959) pp. 269–279.

M. HALL, *An existence theorem for Latin squares*. Bull. Amer. Math. Soc., v. 51 (1945) pp. 387–388.

H. J. RYSER, *A combinatorial theorem with an application to Latin rectangles*. Proc. Amer. Math. Soc., v. 2 (1951) pp. 550–552.

— *The term rank of a matrix*. Canad. J. Math., v. 10 (1958) pp. 57–65.

M. MARCUS, *Some properties and applications of doubly stochastic matrices*. Amer. Math. Monthly, v. 67 (1960) pp. 215–221.

J. SINGER, *A class of groups associated with Latin squares*. Amer. Math. Monthly, v. 67 (1960) pp. 235–240.

7.9.

D. KÖNIG, *Graphen und matrices*. Mat. Fiz. Lapok., v. 38 (1931) pp. 116–119.
 — *Über trennende Knotenpunkte in Graphen*. Acta Litt. ac Scient. Szeged, v. 6 (1933) pp. 155–179.

D. KÖNIG AND S. VALKÓ, *Über mehrdeutige Abbildungen von Mengen*. Math. Ann., v. 95 (1926) pp. 135–138.

R. Z. NORMAN AND M. O. RABIN, *An algorithm for a minimum cover of a graph*. Proc. Amer. Math. Soc., v. 10 (1959) pp. 315–319.

C. BERGE, *Two theorems in graph theory*. Proc. Nat. Acad. Sci. U.S.A., v. 43 (1957) pp. 842–844.

7.10.

P. J. HIGGINS, *Disjoint transversals of subsets*. Canad. J. Math., v. 11 (1959) pp. 280–285.

H. J. RYSER, *Combinatorial properties of matrices of zeros and ones*. Canad. J. Math., v. 9 (1957) pp. 371–377.

D. GALE, *A theorem on flows in networks*. Pacific J. Math., v. 7 (1957) pp. 1073–1082.

CHAPTER 8

8.1.

H. E. ROBBINS, *A theorem on graphs with an application to a problem of traffic control*. Amer. Math. Monthly, v. 46 (1939) pp. 281–283.

L. EGYED, *Über die wohlgerichteten unendlichen Graphen*. Mat. Fiz. Lapok., v. 48 (1941) pp. 505–509.

L. REDEI, *Ein kombinatorischer Satz*. Acta. Litt. Szeged, v. 7 (1934) pp. 39–43.

M. FIEDLER, AND J. SEDLACEK, *Über Wurzelbasen von gerichteten Graphen*. Casopis Pěst Mat., v. 83 (1958) pp. 214–225.

8.2.

R. D. LUCE, *Two decomposition theorems for a class of finite oriented graphs*. Amer. J. Math. v. 74 (1952) pp. 701–722.

— *Networks satisfying minimality conditions*. Amer. J. Math., v. 75 (1953) pp. 825–838.

K. CULIK, *Zur Theorie der Graphen*. Casopis Pěst. Mat., v. 83 (1958) pp. 133–155.

8.4.

L. REDEI, *Über die Kantenbasen für endliche vollständige gerichtete Graphen*. Acta. Math. Sci. Hungar., v. 5 (1954) pp. 17–25.

8.6

O. ORE, *Studies on directed graphs*. I, Ann. of Math., v. 63 (1956) pp. 383–406; II, v. 64 (1956) pp. 142–153.

W. T. TUTTE, *The 1-factors of oriented graphs*. Proc. Amer. Math. Soc., v. 4 (1953) pp. 922–931.

CHAPTER 9

9.2.

O. ORE, *Chains in partially ordered sets*. Bull. Amer. Math. Soc., v. 49 (1943) pp. 558–566.

S. MACLANE, *A conjecture of Ore in partially ordered sets*. Bull. Amer. Math. Soc. v. 49 (1943) pp. 567–568.

M. BENADO, *Les ensembles partiellement ordonnés et le théorème de raffinement de O. Schreier*. Acad. R. P. Romîne Bul. Şti. Şecţ. Şti. Mat. Fiz., v. 4 (1952) pp. 585–591.

— *Bemerkungen zu einer Arbeit von Oystein Ore*. Rev. Math. Pures Appl., v. 1 (1956) pp. 5–12.

M. KOLIBIAR, *Bemerkung über die Ketten in teilweise geordneten Mengen*. Acta Fac. Nat. Univ. Comenian, v. 3 (1958) pp. 17–22.

9.3

O. ORE, *Sex in graphs*. Proc. Amer. Math. Soc., v. 11 (1960) pp. 533–539.

CHAPTER 10

10.2

R. P. DILWORTH, *A decomposition theorem for partially ordered sets*. Ann. of Math., (2) v. 51 (1950) pp. 161–166.

G. A. MOREIRA, *Decomposition of partially ordered systems*. Revista Científica, v. 1 (1950) pp. 12–18.

D. T. FULKERSON, *Note on Dilworth's decomposition theorem for partially ordered sets*. Proc. Amer. Math. Soc., v. 7 (1956) pp. 701–702.

G. B. DANTZIG, AND A. J. HOFFMAN, *Dilworth's theorem on partially ordered sets*. Annals of Math. Studies no. 38 (1956), pp. 207–214.

10.3

G. BIRKHOFF, *Lattice theory*. Amer. Math. Soc. Colloquium Publ., v. 25.. Rev. ed. 1948.

10.4.

E. SZPILRAJN, *Sur l'extension de l'ordre partiel*. Fund. Math., v. 16 (1930) pp. 386–389.

B. DUSHNIK AND E. W. MILLER, *Partially ordered sets*. Amer. J. Math., v. 63 (1941) pp. 600–610.

B. DUSHNIK, *Concerning a certain set of arrangements*. Proc. Amer. Math. Soc., v. 1 (1950) pp. 788–796.

H. KOMM, *On the dimension of partially ordered sets*. Amer. J. Math., v. 70 (1948) pp. 507–520.

V. SEDMAK, *Dimension des ensembles partiellement ordonnés associés aux polygones et polyèdres*. Hrvatsko, Prirodoslovno Društvo Glasnik Mat.-Fiz. Astr. Ser II, v. 7 (1952) pp. 169–182.

— *Quelques applications des ensembles partiellement ordonnés*. C. R. Acad. Sci. Paris, v. 236 (1953) pp. 2139–2140.

T. HIRAGUCHI, *On the dimension of partially ordered sets*. Sci. Rep. Kanazawa University, v. 1 (1951) pp. 77–94.

— *A note on Mr. Komm's theorems*. Ibid., v. 2 (1953) No. 1 pp. 1–3.

T. HIRAGUCHI, *On the dimension of orders*. Ibid., v. 4 (1955) No. 1.

— *On the λ -dimension of the product of orders*. Ibid., v. 5 (1956) pp. 1–5.

CHAPTER 11

11.1

O. ORE, *Galois connexions*. Trans. Amer. Math. Soc., v. 55 (1944) pp. 493–513.

G. BIRKHOFF, *Lattice theory*. 2nd ed. 1948, Chapter IV.

C. J. EVERETT, *Closure operators and Galois theory in lattices*. Trans. Amer. Math. Soc., v. 55 (1944) pp. 514–525.

G. PICKERT, *Bemerkungen über Galois-Verbindungen*. Arch. Math., v. 3 (1952) pp. 285–289.

G. AUMANN, *Bemerkungen über Galois-Verbindungen*. Bayer. Akad. Wiss. M. N. Kl. (1955) pp. 281–284.

11.3

J. RIGUET, *Relations binaires, fermetures, correspondences de Galois*. Bull. Soc. Math. France, v. 76 (1948) pp. 114–155.

— *Quelques propriétés des relations difonctionnelles*. C. R. Acad. Sci. Paris, v. 230 (1950) pp. 1999–2000.

J. RIGUET, *Sur les ensembles réguliers de relations binaires*. C. R. Acad. Sci. Paris, v. 231 (1950) pp. 936–937.

M. L. DUBREIL-JACOTIN, *Quelques propriétés des applications multiformes*. C. R. Acad. Sci. Paris, v. 230 (1950) pp. 806–808.

G. Y. RAINICH, *Involution and equivalence*. Michigan Math. J., v. 2 (1954) pp. 33–34.

11.4.

J. RIGUET, *Les relations de Ferrers*. C. R. Acad. Sci. Paris, v. 232 (1951) pp. 1729–1730.

P. DUBREIL, *Relations binaires et applications*. C. R. Acad. Sci. Paris, v. 230 (1950) pp. 1028–1030.

— *Comportement des relations binaires dans une application multiforme*. C. R. Acad. Sci. Paris, v. 230 (1950) pp. 1242–1243.

R. M. THRALL, *A combinatorial problem*. Michigan Math. J., v. 1 (1952) pp. 81–88.

J. LAMBEK, *Goursat's theorem and the Zassenhaus lemma*. Canad. J. Math., v. 10 (1958) pp. 45–56.

K. ONO, *On some properties of binary relations*. Nagoya Math. J., v. 12 (1957) pp. 161–170.

CHAPTER 12

12.2.

D. KÖNIG, *Über trennende Knotenpunkte in Graphen*. Acta. Litt. Sci. Szeged v. 6 (1933) pp. 155–179.

G. HAJÓS, *Zum Mengerschen Graphensatz*. Acta. Litt. Sci. Szeged, v. 7 (1934) pp. 44–47.

T. GRÜNWARD, *Ein neuer Beweis eines Mengerschen Satzes*. J. London Math. Soc., v. 13 (1938) pp. 188–192.

G. A. DIRAC, *Connectivity theorems for graphs*. Quarterly J. Math., Oxford Ser. (2) v. 3 (1952) pp. 171–174.

CHAPTER 13

13.1.

F. SCHEID, *Some packing problems*. Amer. Math. Monthly, v. 67 (1960) pp. 231–235.

13.3

E. NETTO, *Lehrbuch der Combinatorik*, 2nd ed. 1927.

W. SIERPINSKI, *Sur un problème de la théorie des relations*. Fund. Math., v. 28 (1937) pp. 71–74.

S. PICCARD, *Solution d'un problème de la théorie des relations*. Fund. Math. v. 28 (1936) pp. 197–202.

— *Sur un problème de la théorie des relations*. Mathematica, v. 13 (1937) pp. 55–58.

S. MARCUS, *Sur les ensembles indépendants dans la théorie des relations*. Monatsh. Math., v. 63 (1959) pp. 244–255.

G. FODOR, *On two problems concerning the theory of binary relations*. Publ. Math. Debrecen, v. 1 (1950) pp. 199–200.

— *On a theorem in the theory of binary relations*. Compositio Math., v. 8 (1951) p. 250.

— *On a problem concerning the theory of binary relations*. Nieuw Archiv voor Wiskunde, v. 23 (1951) pp. 247–248.

D. LÁZÁR, *On a problem in the theory of aggregates*. Compositio Math. v. 3 (1936) p. 304.

G. GRÜNWARD, *Über einen mengentheoretischen Satz*. Math. Fiz. Lapok, v. 44 (1937) pp. 51–53.

F. BAGEMIHLE, *The Baire category of independent sets*. Compositio Math., v. 13 (1956) pp. 71–75.

P. ERDÖS AND G. FODOR, *Some remarks on set theory*. V. Acta. Sci. Math Szeged, v. 17 (1956) pp. 250–260; VI. v. 18 (1957), pp. 243–260.

T. GALLAI, *Über extreme Punkt- und Kantenmengen*. Annales. Un. Sci. Budapest, v. 2 (1959) pp. 133–138.

13.4

P. TURÁN, *Eine Extremalaufgabe aus der Graphentheorie*. Math. Fiz. Lapok, v. 48 (1941) pp. 436–452.

— *On the theory of graphs*. Colloq. Math., v. 3 (1954) pp. 19–30.

K. ZARANKIEWICZ, *Sur les relations symétriques dans l'ensemble fini*. Colloq. Math., v. 1 (1947) pp. 10–14.

P. ERDÖS AND A. H. STONE, *On the structure of linear graphs*. Bull. Amer. Math. Soc., v. 52 (1946) pp. 1087–1091.

P. ERDÖS, *Some theorems on graphs*. Riveon Lematematika, v. 9 (1955) pp. 13–17.

13.5.

F. P. RAMSEY, *On a problem of formal logic*. Proc. London Math. Soc., (2) v. 30 (1930) pp. 264–286.

T. SKOLEM, *Ein kombinatorischer Satz mit Anwendung auf ein logisches Entscheidungsproblem*. Fund. Math., v. 20 (1933) pp. 254–261.

P. ERDÖS AND R. RADO, *A combinatorial theorem*. J. London Math. Soc., v. 25 (1950) pp. 249–255.

P. ERDÖS AND G. SZEKERES, *A combinatorial problem in geometry*. *Compositio Math.*, v. 2 (1935) pp. 463–470.

R. RADO, *The distributive law for products of infinite series*. *Quart. J. Math.*, Oxford Ser. v. 11 (1940) pp. 229–242.

A. W. GOODMAN, *On set of acquaintances and strangers at any party*. *Amer. Math. Monthly.*, v. 66 (1959) pp. 778–783.

A. M. GLEASON AND R. E. GREENWOOD, *Combinatorial relations and chromatic graphs*. *Canad. J. Math.*, v. 7 (1955) pp. 1–7.

P. ERDÖS, *Some remarks on the theory of graphs*. *Bull. Amer. Math. Soc.*, v. 53 (1947) pp. 292–294.

P. ERDÖS, AND R. RADO, *Combinatorial theorems on classifications of subsets of a given set*. *Proc. London Math. Soc.*, v. 2 (1951) pp. 417–439.

— *A partition calculus in set theory*. *Bull. Amer. Math. Soc.*, v. 62 (1956) pp. 427–489.

13.6.

C. E. SHANNON, *The zero error capacity of a noisy channel*. *Transactions 1956 Symposium Information Theory, Institute of Radio Engineers*. v. IT.-2, pp. 8–19.

CHAPTER 14

14.1.

P. ERDÖS, AND N. G. DE BRUIJN, *A colour problem for infinite graphs and a problem in the theory of relations*. *Indagationes Math.*, v. 13 (1951) pp. 369–373.

P. ERDÖS AND R. RADO, *Partition relations connected with the chromatic number of graphs*, *J. London Math. Soc.*, v. 34 (1959) pp. 63–72.

R. L. BROOKS, *On colouring the nodes of a network*. *Cambridge Philos. Soc.*, v. 37 (1941) pp. 194–197.

J. MYCIELSKI, *Sur le coloriage des graphes*. *Colloq. Math.*, v. 3 (1955) pp. 161–162.

14.2.

E. A. NORDHAUS AND J. W. GADDUM, *On complementary graphs*. *Amer. Math. Monthly.*, v. 63 (1956) pp. 175–177.

A. A. ZYKOV, *On some properties of linear complexes*. *Mat. Sb.*, v. 24 (66) (1949) pp. 163–188. *Amer. Math. Soc. Translations No. 79*.

14.3.

G. A. DIRAC, *Note on the colouring of graphs*. *Math. Z.*, v. 54 (1951) pp. 347–353.

— *Some theorems on abstract graphs*. *Proc. London Math. Soc.*, (3) v. 2 (1952) pp. 69–81.

— *A property of 4-chromatic graphs and some remarks on critical graphs.* J. London Math. Soc., v. 27 (1952) pp. 85–92.

— *The structure of k -chromatic graphs.* Fund. Math., v. 40 (1953) pp. 42–55.

— *Circuits in critical graphs.* Monatsh. Math., v. 59 (1955) pp. 178–187.

— *A theorem of R. L. Brooks and a conjecture of H. Hadwiger.* Proc. London Math. Soc., (3) v. 7 (1957) pp. 161–195.

J. B. KELLY AND L. M. KELLY, *Paths and circuits in critical graphs.* Amer. J. Math., v. 76 (1954) pp. 786–792.

R. C. READ, *Maximal circuits in critical graphs.* J. London Math. Soc., v. 32 (1957) pp. 456–462.

B. ZEIDL, *Über 4- und 5-chrome Graphen.* Monatsh. Math., v. 62 (1958) pp. 212–218.

14.4.

G. D. BIRKHOFF, *A determinant formula for the number of ways of colouring a map.* Ann. of Math., (2) v. 14 (1912) pp. 42–46.

— *On the number of ways of coloring a map.* Proc. Edinburgh Math. Soc., (2) v. 2 (1930) pp. 83–91.

H. WHITNEY, *A logical expansion in mathematics.* Bull. Amer. Math. Soc., v. 38 (1932) pp. 572–579.

— *The coloring of graphs.* Ann. of Math., (2) v. 33 (1932) pp. 688–718.

— *A set of topological invariants for graphs.* Amer. J. Math., v. 55 (1933) pp. 231–235.

CHAPTER 15

15.1.

R. FRUCHT, *Die Gruppe des Petersenschen Graphen und der Kantensysteme der regulären Polyeder.* Comment. Math. Helv., v. 9 (1937) pp. 217–223.

I. N. KAGNO, *Desargues' and Pappus' graphs and their groups.* Amer. J. Math., v. 69 (1947) pp. 859–862.

— *Linear graphs of degree ≤ 6 and their groups.* Corrections *ibid.*, v. 69 (1947) p. 872; v. 77 (1955) p. 392.

G. PÓLYA, *Kombinatorische Anzahlbestimmungen für Gruppen, Graphen und chemische Verbindungen.* Acta. Math., v. 68 (1937) pp. 145–254.

15.2.

A. CAYLEY, *The theory of groups, graphical representation.* Math. Papers, v. 10 pp. 403–405.

— *On the theory of groups.* *Ibid.*, v. 10 pp. 323–330; v. 11 pp. 365–367.

R. A. RANKIN, *A campanological problem in group theory*. Proc. Cambridge Philos. Soc., v. 44 (1948) pp. 17–25.

T. J. FLETCHER, *Campanological groups*. Amer. Math. Monthly, v. 63 (1956) pp. 619–626.

D. J. DICKINSON, *On Fletcher's paper: "Campanological groups"*. Amer. Math. Monthly, v. 64 (1957) pp. 331–332.

E. S. RAPAPORT, *Cayley color groups and Hamilton lines*. Scripta Math., v. 24 (1959) pp. 51–58.

15.3.

R. FRUCHT, *Herstellung von Graphen mit vorgegebener abstrakter Gruppe*. Compositio Math., v. 6 (1938) pp. 239–250.

— *Graphs of degree three with a given abstract group*. Canad. J. Math., v. 1 (1949) pp. 365–378.

— *On groups of repeated graphs*. Bull. Amer. Math. Soc., v. 55 (1949) pp. 418–420.

— *On the construction of partially ordered systems with a given group of automorphisms*. Rev. Un. Mat. Argentina, v. 13 (1948) pp. 12–18.

G. T. TRANQUE, *The type in cubic graphs*. Gaceta Mat., (1) v. 5 (1953) pp. 11–23.

H. IZBICKI, *Reguläre Graphen 3. 4. und 5. Grades mit vorgegebenen abstrakten Automorphismengruppen, Farbenzahlen und Zusammenhängen*. Monatsh. Math., v. 61 (1957) pp. 42–50.

— *Unendliche Graphen endlichen Grades mit vorgegebenen Eigenschaften*. Ibid., v. 63 (1959) pp. 298–307.

— *Reguläre Graphen beliebigen Grades mit vorgegebenen Eigenschaften*. Ibid., v. 64 (1960) pp. 15–21.

G. SABIDUSSI, *Graphs with given group and given graph-theoretical properties*. Canad. J. Math., v. 9 (1957) pp. 515–525.

— *On the minimum order of graphs with given automorphism group*. Monatsh. Math., v. 63 (1959) pp. 124–127.

— *Graphs with given infinite groups*. Ibid., v. 64 (1960) pp. 64–67.

— *Graph multiplication*. Math. Z., v. 72 (1960) pp. 446–457.

— *On a class of fixed-point-free graphs*. Proc. Amer. Math. Soc., v. 9 (1958) pp. 800–804.

— *The composition of graphs*. Duke Math. J., v. 26 (1959) pp. 693–696.

F. HARARY, *On the group of the composition of two graphs*. Duke Math. J., v. 26 (1959) pp. 29–34.

G. BIRKHOFF, *On groups of automorphisms*. Rev. Un. Mat. Argentina, v. 11 (1946) pp. 155–157.

W. T. TUTTE, *A family of cubical graphs*. Proc. Cambridge Philos. Soc., v. 43 (1947) pp. 459–474.

H. S. M. COXETER, *Self-dual configurations and regular graphs*. Bull. Amer. Math. Soc., v. 56 (1950) pp. 413–455.

R. FRUCHT, *A one-regular graph of degree three*. Canad. J. Math., v. 4 (1952) pp. 240–247.

15.4.

H. WHITNEY, *Congruent graphs and the connectivity of graphs*. Amer. J. Math., v. 54 (1932) pp. 150–168.

— *On the classification of graphs*. Ibid., v. 55 (1933) pp. 236–244.

— *2-isomorphic graphs*. Ibid., v. 55 (1933) pp. 245–254.

R. M. FOSTER, *Geometrical circuits of electrical networks*. Bell Tel. Syst. Techn. Publ. B-653; also Transactions Amer. Inst. Elec. Eng., v. 51 (1932) pp. 309–317.

J. KRAUSZ, *Démonstration nouvelle d'un théorème de Whitney sur les réseaux*. Mat. Fiz. Lapok, v. 50 (1943) pp. 75–85.

LIST OF CONCEPTS

- Alternating augmentation 132.
Arc 23, circuit type 54, completed 55, connecting 200, diametral 29, diametral elongation 31, Hamilton 53, longest 31, radial elongation 31.
Ariadne's thread 49, Assignment problem 115, Automorphism 239.
Axiom of choice 92, metric 27.
- Bernstein theorem 114, *Branch* 59, 64, weight 65, Bridge 81.
- Capacity 19, Cartesian sum, product 35.
Center 29, elongation 31, gravity 30, Hamilton 56, interlocking chain 217, mass 64, 65.
Chain 92, interlocking 217, sum property 96.
Choice function 92, Chord 233.
Chromatic number 224, decomposition 224.
Circuit 23, broken 235, edge 81, 82, edge connected 82, end 90, isomorphic 248, rank 67, type 55.
Closed set 177, circuit 82, 149, strongly cyclically 158.
Closure relation 177, transitive 150.
Color function 224.
Coloration, polynomial 235, k -, 224.
Compact edge separation 154, Comparable 151.
Component, complete 135, connected 23, deficient 135, forbidden 103, index 230.
Connected, cyclic edge 150, k -edge 79, 1-vertex 80.
Connective index 89.
Correspondence, inverse 26, many-to-one 68, one-to-one 25.
Covering, progressive 50, set 208.
Cross-out 175, Cross-path theorem 197.
Cycle, directed 26, generalized 68, order 26.
Cyclic interchange 104.
- Dancing problem 115.
Deficiency 109, 159, 203, bounded 109, converse 159, d -, 140, edge 205, maximal 109.
Deformation 164, cyclic 133, equivalent 164.
Degree 10, local 7, 10.
Descendent 167.
Deviation, edge 30, mean vertex 30.
Diameter, 29.
Distance 27, measured 50.
Domination number 207, Duplication 6.
- Edge* 1. a , α^* , 156, acyclic 146, arc designated 199, attachment (number) 78, circuit 81, connecting 78, connectivity 78, covering 40, covering family, number 208, cross 197, cut 81, cyclic 146, deviation 30, directed 1, end points 1, entering 2, essential 155, exterior 78, incident 2, incoming 2, interior 78, isomorphic 245, issuing 2, multiple 5, outgoing 2, separating 81, separation theorem 202, sequence 23, singular exchange, related 104, strongly circuit connected 85, superfluous 152, terminal 58, touching 78, undirected 1.
Elongation, center, diameter, radius, number 31.
Equivalence block, singular, non-singular 16.
Exclusion property 97, Exit 48.
- Factor 13, Ferrers' diagram 195, Fixpoint 27, 69, Five Queens Problem 207, Forest 58.
- Galois*, closure, closed elements 184, connection, correspondence 183, involutory, orthogonal, polar, self-dual 187, perfect 185.

- Graph* 1, acyclic 149, almost first degree 160, alternating composition 158, arbitrarily traceable 74, associated undirected 5, basis 153, bipartite 106, Cayley color 242, Cartesian sum, product 35, circuit closed 149, circuit free 58, colorable 224, complement 12, complete 3, complete, directed 4, converse 5, covering 208, critical 229, Desargues 241, descendent 167, direct sum 13, directed 2, 145, directed leaf 149, edge disjoint 13, Euler 39, exclusion 100, finite 7, generating 152, infinite 7, interchange 19, intersection 12, isomorphic 3, leaf 82, leaf composition 84, 149, mixed 3, mutually connected 147, nearly regular 118, null 3, Pappus 241, permutation 25, Petersen 240, planar 6, Platonic 8–10, product 33, regular 10, 11, regular bipartite 117, reproduction 167, section 12, signal relation 220, singular, singularly related 104, star 12, sum 12, term 126, transitive closure 150, transmission 220, undirected 2, vertex disjoint 13.
- Group*, automorphism 239, complete monomial 240.
- Grundy function 224.
- Hadwiger's conjecture 233.
- Hamilton*, arc 53, circuit 52, center 56.
- Height 69, 70, Hexagonal condition 195.
- Homomorphism* 83, 149, connected, disjoint 85, independent 85, simple 83, multiplicity 83.
- Homomorphic image 83, Husimi tree 89.
- Immediate successor, predecessor 162. Imply 14, Incidence matching 71, 72, Incidence matrix 19, Inclusion property 97, Incomparable 151, Independent edge family 213, Independent order decomposition 174, Independence number 211, 213, Index reduction 50, Intersection ring 177.
- Jordan-Hölder theorem 163–166, Junction 2.
- Kirkman School Girl Problem 212.
- Knight Problem 52.
- Königsberg Bridge Problem 38.
- Labyrinth*, 47, problem 48, Hampton Court 48.
- Lattice* 176, operations, axioms 175, complemented 187.
- Latin square* 130, partial 131.
- Leaf* 82, 149, singular 82.
- Lobe* 86, 150.
- Loop* 4, count 7.
- Lower section 172.
- Marginal sum 129, Marriage Problem 115.
- Matching* 111, 204, incidence 71, maximal 132, partial 111, proper 111, simultaneous 140, theorem 112.
- Maximal principle*, Hausdorff-Kuratowski 94, Zorn 95.
- Measure* 35, 50, matrix 35.
- Minimal Connector Problem 61.
- Mohammed's scimitars 39.
- Multiplicity 7, 10.
- Negation 14, Network 2, Node 2.
- Order (of correspondence) 26, 69.
- Order* 17, decomposition 174, partial 17, 170, product 179, quasi 151, weak 151.
- Path* 23, α , α^* , 156, alternating 132, cross 198, cyclic 23, deficiency 133, Euler 38, finiteness 205, maximal 162, simple 23, simply related 162.
- Partition* 16, dual 142.
- Parent 167, Peninsula 78, Permanent 128, Permutation 25, Polyhedra, regular 10, Predecessor 167.
- Product* space 1, dimension 179.
- Quasi-group 131, Quasi-order 151.
- Radius* 29, elongation 31.
- Relation*, alternating product 193, antireflexive 15, binary 14, bipartite 15, closed 190, commutative 195, complementary 14, converse 14, difunctional 193, equivalence 16, inclusion 17, 145,

- intersection 15, mutually transitive 192, null 14, order 17, product 191, progeny 166, proper inclusion 18, reflexive 15, self-transitive 193, sum 15, symmetric 14, transitive 15, universal 14, weakly symmetric 190, weakly transitive 193.
 Root 59.
 Scaffolding 101.
 Section 23, lower, upper 172.
 Sequence 22, cyclic 23, directed, undirected 23, one-way, two-way infinite 22, proper 168, reverse cross- 200, 203, section 23.
 Set, conformal 138, covering 208, critical 109, 204, deficient 134, dependent, completely dependent, independent, related, unrelated 210, distinct representatives 116, dividing 93, dominating 206, finitely minimal 139, G-12, generating 146, 172, inclusion 17, matching 132, maximal independent 211, maximal ordered 95, M-image 135, minimal critical 109, 136, minimal δ -135, ordered 17, representatives 113, separating 138, 200, separating edge 202, strongly dependent 173, vertex 1, without deficiency 110, 204.
 Sex dichotomy 167, Skeleton 101, Skein 59, Stochastic matrix 129.
 Subgraph 12, proper 210 edge disjoint 12, vertex disjoint 12.
 Term rank 126.
 Transfinite induction 92, construction 95.
 Transposition 27, 60, basis 60.
 Traveling Salesman Problem 53.
 Tree 58, rooted 59, maximal 101.
 Union 175.
 Variance 30.
 Vertex 1, accessible 145, 156, 200 203, accessibly equivalent 145, 158, attachment 79, 80, attachment number 80, connectivity 80, covering number 210, cut 87, cyclic edge connected 146, deficient 133, 160, deviation 30, disjoint 13, immediate predecessor, successor 162, incident 2, independence number 211, initial 1, 22, inner 22, intermediate 22, 162, isolated 3, median 30, mutually connected 145, separating 87, separation theorem 200, strongly circuit connected 86, terminal 1, 22, 58.
 Well-ordering 92.
 Weight 65, branch 65.
 Zermelo's theorem 92.
 Zorn's lemma 95.

This page intentionally left blank

INDEX OF NAMES

van Aardenne-Ehrenfest, 41, 250
 Andreoli, 252
 Aumann, 258

Baebler, 77, 253
 Bagemihl, 213, 260
 Beckman, 50, 251
 Bellman, 251
 Benado, 257
 Berge, 250, 256
 Birkhoff, G., 245, 257, 258, 267
 Birkhoff, G.D., 235, 262
 Blanusa, 252
 Bol, 61, 252
 Boruvka 61, 252
 Bostwick, 220
 Bott, 61, 252
 Brooks, 226, 261
 de Bruijn, 41, 226, 250, 254, 261
 Burge, 253

Cayley, 59–61, 242–244, 251, 262
 Chapman, 118, 255
 Chisholm, 253,
 Choquet, 61, 252
 Coxeter, 245, 264
 Crowe, 250
 Culik, 256

Dantzig, 53, 251, 257
 De Morgan, ix
 Desargues, 241
 Dickinson, 263
 Dickson, 220
 Dilworth, 173, 174, 182, 257
 Dirac, 56, 89, 91, 229, 231–233, 253, 259, 261
 Dubreil, 195, 259
 Dubreil-Jacotin, 195, 259
 Dulmage, 127, 130, 255
 Dushnik, 178, 258
 Dziobek, 61, 252

Egyed, 147, 256
 Eisenhower, 73
 Erdős, 43, 56, 91, 213, 219, 220, 226, 250, 253,
 260, 261,
 Euler V., 38–40, 131, 250
 Everett, 258

Ferrers, 193–196
 Fiedler, 256
 Fitting, 251
 Fletcher, 263,
 Flood, 251,

Fodor, 213, 260
 Ford, 251
 Foster, 264
 Frucht, 240, 241, 244, 245, 262–264
 Fulkerson, 53, 251, 257

Gaddum, 227, 261
 Gale, 143, 256
 Gallai, 56, 91, 213, 253, 260
 Gleason, 218, 261
 Goldberg, 81, 253
 Goodman, 218, 261
 Gottschalk, 254
 Green, 81, 253
 Greenwood, 218, 261
 Grünwald, G., 254, 260
 Grünwald, T., 43, 250, 259
 Grundy, 224–226

Hadwiger, 233, 234
 Hajós, 259
 Hall, M., 116, 131, 254, 255
 Hall, P., 113, 254
 Halmos, 254
 Harary, 77, 89, 253, 263
 Hausdorff, 94
 Heller, 251
 Henkin, 254
 Higgins, 140, 142, 254, 256
 Hiraguchi, 258
 Hoffman, 254, 257
 Husimi, 253

Izbicki, 245, 263

Jarnik, 61, 252
 Johnson, 53, 251

Kagno, 241, 262
 Kelly, J.B., 227, 231, 252, 262
 Kelly, L.M., 227, 231, 262
 Kirkman, 212
 Klein, 219, 220
 Kolibiar, 257
 Komm, 181, 258
 König V., 49, 139, 203, 250, 256, 259
 Kossler, 61, 252
 Kotzig, 253
 Krausz, 264
 Kreweras, 254
 Kruskal, 61, 252
 Kuhn, 254
 Kuratowski, 94

- Lambek, 259
 Lázár, 260
 Lucas, 41, 49, 250
 Luce, 256

 Maak, 254
 MacLane, 81, 165, 253, 257
 Makai, 220
 Mann, 255
 Marcus, M., 255
 Marcus, S., 213, 260
 Mayberry, 61, 252
 McGuire, 50, 251
 Mendelsohn, 127, 130, 255
 Menger, 200, 201, 203, 205
 Miller, E.W., 178, 258
 Miller, G.A., 118, 255
 Minotaur, 47
 Moreira, 257
 Morgenstern, 252
 Motzkin, 251, 254
 Mycielski, 226, 261

 Nettleton, 81, 253
 Netto, 212, 259
 Neville, 252
 Newman, 251
 Nordhaus, 227, 261
 Norman, 89, 253, 256

 Ono, 259
 Ore, 50, 71, 77, 118, 119, 127, 165, 251, 252,
 254, 255, 257, 258

 Pappus, 241
 Petersen, 132, 240, 241
 Piccard, 212, 260
 Pickert, 258
 Polya, 61, 242, 252, 262
 Prüfer, 59, 251

 Rabin, 256
 Rado, 107, 220, 226, 254, 255, 260, 261
 Rainich, 193, 259.
 Ramsey, 216, 218, 219, 220, 260
 Rankin, 263
 Rapaport, 243, 263
 Read, 232, 262
 Redei, 148, 149, 256
 Riguét, 193, 194, 258, 259
 Ringel, 250
 Riordan, 61, 252
 Robbins, 147, 256

 Rouse-Ball, 49, 207, 250
 Ryser, 143, 255, 256

 Sabidussi, 245, 263
 Sainte-Lague, 250
 Scheid, 259
 Schur, 219, 220
 Scorza, 118, 255
 Sedlacek, 256
 Sedmak, 258
 Shannon, 220, 261
 Shmushkovitch, 255
 Shü, 118, 255
 Sierpinski, 212, 259
 Singer, 255
 Skolem, 260
 Steiner, 212
 Stone, 260
 Straus, 251
 Supnick, 251
 Szekeres, 219, 261
 Szpilrajn, 258

 Tarry, 41, 49, 52
 Theseus, 47
 Thrall, 259
 Tranque, 263
 Tremaux, 49
 Turan, 214, 216, 260
 Tutte, 227, 245, 251, 253, 257, 263

 Uhlenbeck, 89, 253

 Valkó, 256
 Vaughan, 254
 Vázonyi, 43, 47, 250
 Vitalbi, 252

 van der Waerden, 255
 Weiszfeld, 43, 250
 Weyl, 255
 Whittin, 61, 252
 Whitney, 235, 236, 238, 245, 247, 248, 249, 253,
 262, 264
 Wiener, C., 49, 251
 Wiener, N., 194
 Winsten, 50, 251

 Zarankiewicz, 216, 260
 Zeidl, 262
 Zermelo, 92
 Zorn, 95
 Zykov, 229, 261

ISBN 978-0-8218-1038-5



9 780821 810385

COLL/38

AMS on the Web
www.ams.org