American Mathematical Society
Colloquium Publications
Volume 38

COLLOQUIUM PUBLICATIONS

Theory of Graphs

Oystein Ore

Theory of Graphs

This page intentionally left blank

http://dx.doi.org/10.1090/coll/038

American Mathematical Society
COLLOQUIUM PUBLICATIONS

Volume 38

Theory of Graphs

Oystein Ore

American Mathematical Society
Providence, Rhode Island

2000 *Mathematics Subject Classification.* Primary 05-XX.

Library of Congress Cataloging-in-Publication Data

Ore, Øystein, 1899–1968.
Theory of graphs.
p. cm. — (American Mathematical Society Colloquium publications, ISSN 0065-9258 ; v. 38)
Includes bibliography.
ISBN 978-0-8218-1038-5
1. Graph theory. I. Title.

QA1 .A5225 vol. 38
510

61-15687

---

14 12 12 11 10 9 16 15 14 13 12 11

TABLE OF CONTENTS

This page intentionally left blank

FOREWORD

The present book has grown out of courses on graph theory given from time to time at Yale University. A first set of lectures on binary relations and graphs was presented before the American Mathematical Society at its summer meeting in Chicago, 1942. Since the manuscript to these lectures was not completed for publication at that time due to more urgent tasks it seems appropriate that this book should appear in the Colloquium Lecture Series of the society.

Graph theory as a mathematical discipline was created by Euler in his now famous discussion of the Königsberg Bridge Problem. However, Euler's article of 1736 remained an isolated contribution for nearly a hundred years. About the middle of the last century a resurgence of interest in the problems of graph theory took place, centered mainly in England. There were many causes for this revival of graph studies: The natural sciences had their influence through investigations of electrical networks and models for crystal and molecular structure; the development of formal logic led to the study of binary relations in the form of graphs. A number of popular puzzle problems could be formulated directly in terms of graphs and thus came the realization that many such questions include a mathematical nucleus of general importance. Most celebrated among them is the Four Color Map Conjecture which was first laid before the mathematicians by De Morgan around 1850. No other problem has occasioned as numerous and ingenious contributions to graph theory. Due to its simple formulation and exasperating evasiveness it still remains a powerful incitement to the examination of graph properties.

The present century has witnessed a steady development of graph theory which in the last ten to twenty years has blossomed out into a new period of intense activity. Clearly discernible in this process are the effects of the demands from new fields of applications: game theory and programming, communications theory, electrical networks and switching circuits as well as problems from biology and psychology.

As a consequence of these recent developments the subject of graphs is already so extended that it did not seem feasible to cover all its main ramifications within the framework of a single volume. In the present first volume of an intended two volume work the emphasis has been placed upon basic concepts and the results of particular systematic interest.

There exist very few books on graph theory; the mainstay has been the book by D. König (1936), which for its time gave a most excellent introduction to the subject. Strangely enough, until now there has been no book in English, in spite of the fact that many of the most important contributions to the subject

are due to American and English writers. An effort has been made to present the subject matter in the book in as simple a form as possible. Almost all proofs have been revised; a considerable number of new results are also included. A systematic terminology is introduced which it is hoped may prove acceptable.

For the benefit of the reader a considerable number of problems have been included. Many of these are quite simple; others are more in the nature of proposed research problems; these have been marked with an asterisk.

The second volume will be devoted to more special topics: planar graphs, the four color conjecture, the theory of flow, games, electrical networks, as well as applications to a number of other fields in which graph theory is a principal tool.

Oystein Ore

Yale University
April 1960

BIBLIOGRAPHY

CHAPTER 1

C. BERGE, *Théorie des graphes et ses applications.* Paris, 1958.

D. KÖNIG, *Theorie der endlichen und unendlichen Graphen.* Leipzig, 1936.

A. SAINTE-LAGUË, *Les réseaux (ou graphes).* Mémor. Sci. Math. v. 18, 1926.

Many collections of mathematical puzzles contain problems which may be formulated in graph terms. We shall mention only:

W. ROUSE-BALL, *Mathematical recreations and problems.* London, 1892. Numerous editions.

E. LUCAS, *Récréations mathématiques.* 3 v. Paris, 1882-1894.

CHAPTER 2

2.7

D.W. CROWE, *The n-dimensional cube and the Tower of Hanoi.* Amer. Math. Monthly, v. 63 (1956) pp. 29–30.

CHAPTER 3

3.1

L.EULER, *Solutio problematis ad geometriam situs pertinentis.* Commentarii Academiae Petropolitanae, v. 8 (1736) pp. 128–140.

N.G. DE BRUIJN, A *combinatorial problem.* Nederl. Akad. Wetensch. Proc., v. 49 (1946) pp. 758–764.

VAN AARDENNE-EHRENFEST AND N.G. de BRUIJN, *Circuits and trees in oriented graphs.* Simon Stevin, v. 28 (1951) pp. 203–217.

3.2

P. ERDÖS, T. GRÜNWALD, AND E. WEISZFELD, *Über Eulersche Linien unendlicher Graphen.* Mat. Fiz. Lapok, v. 43 (1936) pp. 129-140.

P. ERDÖS, T. GRÜNWALD, AND E. VÁZSONYI, *Über Euler-Linien unendlicher Graphen.* J. Math. Phys. Massachussets Inst. of Technology, v. 17 (1938) pp. 59–75.

E. VÁZSONYI, *Über Gitterpunkte des mehrdimensionalen Raumes.* Acta Litt. Sci. Szeged, v. 9 (1939) pp. 163–173.

G. RINGEL, *Über drei kombinatorische Probleme am n-dimensionalen Würfel und Würfelgitter.* Abh. Math. Sem. Univ. Hamburg, v. 20 (1955) pp. 10–19.

3.3

C. WIENER, *Über eine Aufgabe aus der Geometria situs.* Math. Ann., v. 6 (1873) pp. 29–30.

O. ORE, *An excursion into labyrinths.* Mathematics Teacher, v. 52 (1959) pp. 367–370.

M. BECKMAN, C. B. MCGUIRE, AND C. B. WINSTEN, *Studies in the economics of transportation.* New Haven, Yale University Press, 1956.

L.R. FORD, *Network flow theory.* Rand Corp. Publ. P-23 (1956).

3.4

D. J. NEWMAN, *A problem in graph theory.* Amer. Math. Monthly, v. 65 (1958) p. 611.

O. ORE, *Note on Hamilton circuits.* Amer. Math. Monthly, v. 67 (1960) p. 55.

W. T. TUTTE, *On Hamilton circuits.* J. London Math. Soc., v. 21 (1946) pp. 98–101.

F. SUPNICK, *Extreme Hamilton lines.* Ann. of Math., (2) v. 66 (1957) pp. 179–201.

F. FITTING, *Doppeltsymmetrische Rösselsprünge auf Quadraten von ungeraden Felderzahl ohne Mittelfeld.* Jber. Deutsch. Math. Verein, v. 46 (1936) Abt. 1, pp. 38–43.

G. DANTZIG, R. FULKERSON, AND S. JOHNSON, *Solution of a large-scale travelling-salesman problem.* Journal Operational Research Soc. of Amer., v. 2 (1954) pp. 393–410.

I. HELLER, *The travelling-salesman problem* George Washington Univ. Logistics Research Project, 1954.

M. M. FLOOD, *On the travelling salesman's problem.* Journal Operational Research Soc. of Amer., v. 4 (1956) pp. 61–75.

T. S. MOTZKIN, AND E. G. STRAUS, *Some combinatorial extremum problems.* Proc. Amer. Math. Soc., v. 7 (1956) pp. 1014–1021.

G. DANTZIG, R. FULKERSON, AND S. JOHNSON, *On a linear programming combinatorial approach to the travelling-salesman problem.* Journal Operational Research Soc. of Amer. v. 7 (1959) pp. 58–66.

R. BELLMAN, *On a routing problem.* Quart. Appl. Math., v. 16 (1958) pp. 87–90.

CHAPTER 4

4.1

A. CAYLEY, *A theorem on trees.* Quarterly Journal of Pure and Appl. Math., v. 23 (1889) pp. 376–378.

H. PRÜFER, *Neuer Beweis eines Satzes über Permutationen.* Archiv der Math. und Phys., (3) v. 27 (1918) pp. 142–144.

O. DZIOBEK, *Eine Formel der Substitutionstheorie.* Sitzungsberichte der Berliner Math. Gesellchaft, v. 16 (1947) pp. 64–67.

G. BOL, *Über eine kombinatorische Frage.* Abh. Math. Seminar der Hansischen Univ., v. 12 (1938) pp. 242–245.

E. H. NEVILLE, *The codifying of tree-structure.* Proc. Cambridge Philos. Soc., v. 49 (1953) pp. 381–385.

R. BOTT and J. P. MAYBERRY, *Matrices and trees.* Also T. M. WHITIN, *An economic application of "Matrices and Trees".* Both published in O. MORGENSTERN: Economic Activity Analysis, New York, John Wiley and Sons, 1954.

L. VITALBI, *Ricerche sulla teoria dei reticoli.* Giorn. Mat. Battaglini, (5) v. 4 (84) (1956) pp. 93–121.

G. ANDREOLI, *Preliminari topologici su gli alberi.* Giorn. Mat. Battaglini, (5) v. 2 (82) (1954) pp. 237–266.

P. J. KELLY, *A congruence theorem for trees.* Pacific J. Math., v. 7 (1957) pp. 961–968.

G. PÓLYA, *Kombinatorische Anzahlbestimmungen für Gruppen, Graphen und chemische Verbindungen.* Acta Math., v. 68 (1937) pp. 145–254.

J. RIORDAN, *An introduction to combinatorial analysis.* John Wiley and Sons, New York, 1958.

O. BORUVKA, *On a minimal problem.* Práce Moraské Pridovedecké Spolecnosti, v. 3 (1926).

V. JARNIK AND M. KOSSLER, *Sur les graphes minima contenant n points donnés.* Časopis Mat. Fys., v. 63 (1934) pp. 223–235.

G. CHOQUET, *Etude de certains réseaux de routes.* C. R. Acad. Sci. Paris, v. 206 (1938) pp. 310–313.

J. B. KRUSKAL, *On the shortest spanning subtree of a graph and the traveling salesman problem.* Proc. Amer. Math. Soc., v. 7 (1956) pp. 48–50.

D. BLANUSA, *Über die Anzahl der Bedingungsgleichungen beliebigen geodätischen Netzen.* Z. Vermessungswesen, v. 73 (1944) pp. 54–62.

4.4

O. ORE, *Graphs and corresponaences.* Festschrift Andreas Speiser Zurich, 1945 pp. 184–191.

— *incidence matchings in graphs.* Journ. de Math., v. 40 (1961) pp. 123–127.

4.5

O. ORE, *A problem regarding the tracing of graphs.* Elem. Math., v. 6 (1951) pp. 49–53.

F. BÄBLER, *Über eine spezielle Klasse Euler'scher Graphen.* Comment. Math., Helv., v. 27 (1953) pp. 81–100.

F. HARARY, *On arbitrarily traceable graphs and directed graphs.* Scripta Math. v. 23 (1957) pp. 37–41.

Further papers on trees:

A. KOTZIG, *The significance of the skeleton of a graph for the construction of composition bases of some subgraphs.* Mat. Fyz. Casopis Slovensk. Akad. Vied., v. 6 (1956) pp. 68–77.

W. T. TUTTE, *A ring in graph theory.* Proc. Cambridge Philos. Soc., v. 43 (1947) pp. 26–40.

J. S. R. CHISHOLM, *The S-matrix for neutral PS-PV meson-nucleon interaction.* Phil. Mag., (8) v. 1 (1956) pp. 338–344.

W. H. BURGE, *Sorting, trees, and measures of order.* Information and Control, v. 1 (1958) pp. 181–197.

CHAPTER 5

5.1

S. MACLANE, *Some unique separation theorems for graphs.* Amer. J. Math., v. 57 (1935) pp. 805–820.

R. E. NETTLETON, K. GOLDBERG, AND M. S. GREEN, *Dense subgraphs and connectivity.* Canad. J. Math., v. 11 (1959) pp. 262–268.

5.4

H. WHITNEY, *Non-separable and planar graphs.* Trans. Amer. Math. Soc., v. 34 (1932) pp. 339–362.

F. HARARY, *An elementary theorem on graphs.* Amer. Math. Monthly, v. 66 (1959) pp. 405–407.

K. HUSIMI, *Note on Mayers theory of cluster integrals.* Journ. Chem. Phys., v. 18 (1950) pp. 682–684.

F. HARARY AND G. E. UHLENBECK, *On the number of Husimi trees.* Proc. Nat. Acad. Sci. U.S.A., v. 39 (1953) pp. 315–322.

F. HARARY, AND R. Z. NORMAN, *The dissimilarity characteristic of Husimi trees.* Ann. of Math. (2) v. 59 (1953), pp. 134–141.

5.5

G. A. Dirac, *Some theorems on abstract graphs.* Proc. London Math. Soc., (3) v. 2 (1952) pp. 69–81.

P. ERDÖS AND T. GALLAI, *On maximal paths and circuits of graphs.* Acta Math. Acad. Sci. Hungar., v. 10 (1959) pp. 337–356.

CHAPTER 6

6.4.

O. ORE AND T. S. MOTZKIN, *Subsets and subgraphs with maximal properties.* Proc. Amer. Math. Soc., v. 10 (1959) pp. 965–969.

CHAPTER 7

7.1.

R. RADO, *Axiomatic treatment of rank in infinite sets.* Canad. J. Math., v. 1 (1949) pp. 337–343.

W. H. GOTTSCHALK, *Choice functions and Tychonoff's theorem.* Proc. Amer. Math. Soc., v. 2 (1951) p. 172.

7.3.

O. ORE, *Graphs and matching theorems.* Duke Math. J., v. 22 (1955) pp. 625–639.

P. HALL, *On representatives of subsets.* J. London Math. Soc., v. 10 (1935) pp. 26–30.

M. HALL, *Distinct representatives of subsets.* Bull. Amer. Math. Soc., v. 54 (1948) pp. 922–926.

7.4.

N. G. DE BRUIJN, *Gemeenschapelijke representantensystemen van twee klassenindeelingen van een versameling.* Nieuw Archiev voor Wiskunde, (2) v. 22 (1943) pp. 48–52.

G. GRÜNWALD, *Über einen mengentheoretischen Satz.* Mat. Fiz. Lapok, v. 44 (1937) pp. 51–53.

M. HALL, *An algorithm for distinct representatives.* Amer. Math. Monthly, v. 63 (1956) pp. 716–717.

P. R. HALMOS AND H. E. VAUGHAN, *The marriage problem.* Amer. J. Math., v. 72 (1950) pp. 214–215.

L. HENKIN, *Some interconnections between modern algebra and mathematical logic.* Trans. Amer. Math. Soc., v. 74 (1953). pp. 410–427.

P. J. HIGGINS, *Disjoint transversals of subsets.* Canad. J. Math., v. 11 (1959) pp. 280–285.

A. J. HOFFMAN AND H. W. KUHN, *Systems of distinct representatives and linear programming.* Amer. Math. Monthly, v. 63 (1956) pp. 455–460.

— *On systems of distinct representatives.* Annals of Math. Studies, No. 38 (1956).

G. KREWERAS, *Extension d'un théorème sur les répartitions en classes.* C. R. Acad Sci. Paris, v. 222 (1946) pp. 431–432.

W. MAAK, *Ein Problem der Kombinatorik in seiner Formulierung von H. Weyl.* Math.-Phys. Semesterber. Göttingen, v. 22 (1952) pp. 251–256.

H. B. MANN AND H. J. RYSER, *Systems of distinct representatives.* Amer. Math. Monthly, v. 60 (1953) pp. 397–401.

N. S. MENDELSOHN AND A. L. DULMAGE, *Some generalizations of the problem of distinct representatives.* Canad. J. Math., v. 10 (1958) pp. 230–241.

— *Coverings of bipartite graphs.* Canad. J. Math., v. 10 (1958) pp. 517–534.

R. RADO, *Factorization of even graphs.* Quart. J. Math., Oxford, v. 20 (1949) pp. 95–104.

V. SHMUSHKOVITCH, *On a combinatorial theorem of the theory of sets.* Mat. Sb., v. 6 (1939) pp. 139–146.

B. L. VAN DER WAERDEN, *Ein Satz über Klasseneinteilungen von endlichen Mengen.* Hamburger Abh., v. 5 (1927) pp. 185–187.

H. WEYL, *Almost periodic invariant vector sets in a metric vector space.* Amer. J. Math., v. 71 (1949) pp. 178–205.

7.5

G. SCORZA, *A proposito di un teorema del Chapman.* Boll. Un. Mat. Ital., v. 6 (1927) pp. 1–6.

H. W. CHAPMAN, *A note on the elementary theory of groups of finite order.* Messenger of Math., v. 42 (1913) pp. 132–134; v. 43 p. 85.

G. A. MILLER, *On a method due to Galois.* Quarterly J. Math., v. 41 (1910) pp. 382–384.

S. SHÜ, *On the common representative system of residue classes of infinite groups.* J. London Math. Soc., v. 16 (1941) pp.101–104.

O. ORE, *On coset representatives in groups.* Proc. Amer. Math. Soc., v. 9 (1958) pp. 665–670.

— *Conditions for subgraphs of directed graphs.* J. Math. Pures Appl., v. 37 (1958) pp. 321–328.

7.7

A. L. DULMAGE AND N. S. MENDELSOHN, *The term and stochastic rank of a matrix.* Canad. J. Math., v. 11 (1959) pp. 269–279.

M. HALL, *An existence theorem for Latin squares.* Bull Amer. Math. Soc., v. 51 (1945) pp. 387–388.

H. J. RYSER, *A combinatorial theorem with an application to Latin rectangles.* Proc. Amer. Math. Soc., v. 2 (1951) pp. 550–552.

— *The term rank of a matrix.* Canad. J. Math., v. 10 (1958) pp. 57–65.

M. MARCUS, *Some properties and applications of doubly stochastic matrices.* Amer. Math. Monthly, v. 67 (1960) pp. 215–221.

J. SINGER, *A class of groups associated with Latin squares.* Amer. Math. Monthly, v. 67 (1960) pp. 235–240.

7.9.

D. KÖNIG, *Graphen und matrices.* Mat. Fiz. Lapok., v. 38 (1931) pp. 116–119.

— *Über trennende Knotenpunkte in Graphen.* Acta Litt. ac Scient. Szeged, v. 6 (1933) pp. 155–179.

D. KÖNIG AND S. VALKÓ, *Über mehrdeutige Abbildungen von Mengen.* Math. Ann., v. 95 (1926) pp. 135–138.

R. Z. NORMAN AND M. O. RABIN, *An algorithm for a minimum cover of a graph.* Proc. Amer. Math. Soc., v. 10 (1959) pp. 315–319.

C. BERGE, *Two theorems in graph theory.* Proc. Nat. Acad. Sci. U.S.A., v. 43 (1957) pp. 842–844.

7.10.

P. J. HIGGINS, *Disjoint transversals of subsets.* Canad. J. Math., v. 11 (1959) pp. 280–285.

H. J. RYSER, *Combinatorial properties of matrices of zeros and ones.* Canad. J. Math., v. 9 (1957) pp. 371–377.

D. GALE, *A theorem on flows in networks.* Pacific J. Math., v. 7 (1957) pp. 1073–1082.

CHAPTER 8

8.1.

H. E. ROBBINS, *A theorem on graphs with an application to a problem of traffic control.* Amer. Math. Monthly, v. 46 (1939) pp. 281–283.

L. EGYED, *Über die wohlgerichteten unendlichen Graphen.* Mat. Fiz. Lapok., v. 48 (1941) pp. 505–509.

L. REDEI, *Ein kombinatorischer Satz.* Acta. Litt. Szeged, v. 7 (1934) pp. 39–43.

M. FIEDLER, AND J. SEDLACEK, *Über Wurzelbasen von gerichteten Graphen.* Casopis Pěst Mat., v. 83 (1958) pp. 214–225.

8.2.

R. D. LUCE, *Two decomposition theorems for a class of finite oriented graphs.* Amer. J. Math. v. 74 (1952) pp. 701–722.

— *Networks satisfying minimality conditions.* Amer. J. Math., v. 75 (1953) pp. 825–838.

K. CULIK, *Zur Theorie der Graphen.* Casopis Pěst. Mat., v. 83 (1958) pp. 133–155.

8.4.

L. REDEI, *Über die Kantenbasen für endliche vollständige gerichtete Graphen.* Acta. Math. Sci. Hungar., v. 5 (1954) pp. 17–25.

8.6

O. ORE, *Studies on directed graphs.* I, Ann. of Math., v. 63 (1956) pp. 383–406; II, v. 64 (1956) pp. 142–153.

W. T. TUTTE, *The 1-factors of oriented graphs.* Proc. Amer. Math. Soc., v. 4 (1953) pp. 922–931.

CHAPTER 9

9.2.

O. ORE, *Chains in partially ordered sets.* Bull. Amer. Math. Soc., v. 49 (1943) pp. 558–566.

S. MACLANE, *A conjecture of Ore in partially ordered sets.* Bull. Amer. Math. Soc. v. 49 (1943) pp. 567–568.

M. BENADO, *Les ensembles partiellement ordonnés et le théorème de raffinement de O. Schreier.* Acad. R. P. Romîne Bul. Şti. Şect. Şti. Mat. Fiz., v. 4 (1952) pp. 585–591.

— *Bemerkungen zu einer Arbeit von Oystein Ore.* Rev. Math. Pures Appl., v. 1 (1956) pp. 5–12.

M. KOLIBIAR, *Bemerkung uber die Ketten in teilweise geordneten Mengen.* Acta Fac. Nat. Univ. Comenian, v. 3 (1958) pp. 17–22.

9.3

O. ORE, *Sex in graphs.* Proc. Amer. Math. Soc., v. 11 (1960) pp. 533–539.

CHAPTER 10

10.2

R. P. DILWORTH, *A decomposition theorem for partially ordered sets.* Ann. of Math., (2) v. 51 (1950) pp. 161–166.

G. A. MOREIRA, *Decomposition of partially ordered systems.* Revista Cientifica, v. 1 (1950) pp. 12–18.

D. T. FULKERSON, *Note on Dilworth's decomposition theorem for partially ordered sets.* Proc. Amer. Math. Soc., v. 7 (1956) pp. 701–702.

G. B. DANTZIG, AND A. J. HOFFMAN, *Dilworth's theorem on partially ordered sets.* Annals of Math. Studies no. 38 (1956), pp. 207–214.

10.3

G. BIRKHOFF, *Lattice theory.* Amer. Math. Soc. Colloquium Publ., v. 25.. Rev. ed. 1948.

10.4.

E. SZPILRAJN, *Sur l'extension de l'ordre partiel.* Fund. Math., v. 16 (1930) pp. 386–389.

B. DUSHNIK AND E. W. MILLER, *Partially ordered sets.* Amer. J. Math., v. 63 (1941) pp. 600–610.

B. DUSHNIK, *Concerning a certain set of arrangements.* Proc. Amer. Math. Soc., v. 1 (1950) pp. 788–796.

H. KOMM, *On the dimension of partially ordered sets.* Amer. J. Math., v. 70 (1948) pp. 507–520.

V. SEDMAK, *Dimension des ensembles partiellement ordonnés associés aux polygones et polyèdres.* Hrvatsko, Prirodoslovno Društvo Glasnik Mat.-Fiz. Astr. Ser II, v. 7 (1952) pp. 169–182.

— *Quelques applications des ensembles partiellement ordonnés.* C. R. Acad. Sci. Paris, v. 236 (1953) pp. 2139–2140.

T. HIRAGUCHI, *On the dimension of partially ordered sets.* Sci. Rep. Kanazawa University, v. 1 (1951) pp. 77–94.

— *A note on Mr. Komm's theorems.* Ibid., v. 2 (1953) No. 1 pp. 1–3.

T. HIRAGUCHI, *On the dimension of orders.* Ibid., v. 4 (1955) No. 1.

— *On the λ-dimension of the product of orders.* Ibid., v. 5 (1956) pp. 1–5.

CHAPTER 11

11.1

O. ORE, *Galois connexions.* Trans. Amer. Math. Soc., v. 55 (1944) pp. 493–513.

G. BIRKHOFF, *Lattice theory.* 2nd ed. 1948, Chapter IV.

C. J. EVERETT, *Closure operators and Galois theory in lattices.* Trans. Amer. Math. Soc., v. 55 (1944) pp. 514–525.

G. PICKERT, *Bemerkungen über Galois-Verbindungen.* Arch. Math., v. 3 (1952) pp. 285–289.

G. AUMANN, *Bemerkungen über Galois-Verbindungen.* Bayer. Akad. Wiss. M. N. Kl. (1955) pp. 281–284.

11.3

J. RIGUET, *Relations binaires, fermetures, correspondences de Galois.* Bull. Soc. Math. France, v. 76 (1948) pp. 114–155.

— *Quelques propriétés des relations difonctionelles.* C. R. Acad. Sci. Paris, v. 230 (1950) pp. 1999–2000.

J. RIGUET, *Sur les ensembles réguliers de relations binaires.* C. R. Acad. Sci. Paris, v. 231 (1950) pp. 936–937.

M. L. DUBREIL-JACOTIN, *Quelques propriétés des applications multiformes.* C. R. Acad. Sci. Paris, v. 230 (1950) pp. 806–808.

G. Y. RAINICH, *Involution and equivalence.* Michigan Math. J., v. 2 (1954) pp. 33–34.

11.4.

J. RIGUET, *Les relations de Ferrers.* C. R. Acad. Sci. Paris, v. 232 (1951) pp. 1729–1730.

P. DUBREIL, *Relations binaires et applications.* C. R. Acad. Sci. Paris, v. 230 (1950) pp. 1028–1030.

— *Comportement des relations binaires dans une application multiforme.* C. R. Acad. Sci. Paris, v. 230 (1950) pp. 1242–1243.

R. M. THRALL, *A combinatorial problem.* Michigan Math. J., v. 1 (1952) pp. 81–88.

J. LAMBEK, *Goursat's theorem and the Zassenhaus lemma.* Canad. J. Math., v. 10 (1958) pp. 45–56.

K. ONO, *On some properties of binary relations.* Nagoya Math. J., v. 12 (1957) pp. 161–170.

CHAPTER 12

12.2.

D. KÖNIG, *Über trennende Knotenpunkte in Graphen.* Acta. Litt. Sci. Szeged v. 6 (1933) pp. 155–179.

G. HAJÓS, *Zum Mengerschen Graphensatz.* Acta. Litt. Sci. Szeged, v. 7 (1934) pp. 44–47.

T. GRÜNWALD, *Ein neuer Beweis eines Mengerschen Satzes.* J. London Math. Soc., v. 13 (1938) pp. 188–192.

G. A. DIRAC, *Connectivity theorems for graphs.* Quarterly J. Math., Oxford Ser. (2) v. 3 (1952) pp. 171–174.

CHAPTER 13

13.1.

F. SCHEID, *Some packing problems.* Amer. Math. Monthly, v. 67 (1960) pp. 231–235.

13.3

E. NETTO, *Lehrbuch der Combinatorik*, 2nd ed. 1927.

W. SIERPINSKI, *Sur un problème de la théorie des relations.* Fund. Math., v. 28 (1937) pp. 71–74.

S. PICCARD, *Solution d'un problème de la théorie des relations.* Fund. Math. v. 28 (1936) pp. 197–202.

— *Sur un problème de la théorie des relations.* Mathematica, v. 13 (1937) pp. 55–58.

S. MARCUS, *Sur les ensembles indépendants dans la théorie des relations.* Monatsh. Math., v. 63 (1959) pp. 244–255.

G. FODOR, *On two problems concerning the theory of binary relations.* Publ. Math. Debrecen, v. 1 (1950) pp. 199–200.

— *On a theorem in the theory of binary relations.* Compositio Math., v. 8 (1951) p. 250.

— *On a problem concerning the theory of binary relations.* Nieuw Archiv voor Wiskunde, v. 23 (1951) pp. 247–248.

D. LÁZÁR, *On a problem in the theory of aggregates.* Compositio Math. v. 3 (1936) p. 304.

G. GRÜNWALD, *Über einen mengentheoretischen Satz.* Math. Fiz. Lapok, v. 44 (1937) pp. 51–53.

F. BAGEMIHL, *The Baire category of independent sets.* Compositio Math., v. 13 (1956) pp. 71–75.

P. ERDÖS AND G. FODOR, *Some remarks on set theory.* V. Acta. Sci. Math Szeged, v. 17 (1956) pp. 250–260; VI. v. 18 (1957), pp. 243–260.

T. GALLAI, *Über extreme Punkt-und Kantenmengen.* Annales. Un. Sci. Budapest, v. 2 (1959) pp. 133–138.

13.4

P. TURÀN, *Eine Extremalaufgabe aus der Graphentheorie.* Math. Fiz. Lapok, v. 48 (1941) pp. 436–452.

— *On the theory of graphs.* Colloq. Math., v. 3 (1954) pp. 19–30.

K. ZARANKIEWICZ, *Sur les relations symétriques dans l'ensemble fini.* Colloq. Math., v. 1 (1947) pp. 10–14.

P. ERDÖS AND A. H. STONE, *On the structure of linear graphs.* Bull. Amer. Math. Soc., v. 52 (1946) pp. 1087–1091.

P. ERDÖS, *Some theorems on graphs.* Riveon Lematematika, v. 9 (1955) pp. 13–17.

13.5.

F. P. RAMSEY, *On a problem of formal logic.* Proc. London Math. Soc., (2) v. 30 (1930) pp. 264–286.

T. SKOLEM, *Ein kombinatorischer Satz mit Anwendung auf ein logisches Entscheidungsproblem.* Fund. Math., v. 20 (1933) pp. 254–261.

P. ERDÖS AND R. RADO, *A combinatorial theorem.* J. London Math. Soc., v. 25 (1950) pp. 249–255.

P. ERDÖS AND G. SZEKERES, *A combinatorial problem in geometry.* Compositio Math., v. 2 (1935) pp. 463–470.

R. RADO, *The distributive law for products of infinite series.* Quart. J. Math., Oxford Ser. v. 11 (1940) pp. 229–242.

A. W. GOODMAN, *On set of acquaintances and strangers at any party.* Amer. Math. Monthly., v. 66 (1959) pp. 778–783.

A. M. GLEASON AND R. E. GREENWOOD, *Combinatorial relations and chromatic graphs.* Canad. J. Math., v. 7 (1955) pp. 1–7.

P. ERDÖS, *Some remarks on the theory of graphs.* Bull. Amer. Math. Soc., v. 53 (1947) pp. 292–294.

P. ERDÖS, AND R. RADO, *Combinatorial theorems on classifications of subsets of a given set.* Proc. London Math. Soc., v. 2 (1951) pp. 417–439.

— *A partition calculus in set theory.* Bull. Amer. Math. Soc., v. 62 (1956) pp. 427–489.

13.6.

C. E. SHANNON, *The zero error capacity of a noisy channel.* Transactions 1956 Symposium Information Theory, Institute of Radio Engineers. v. IT.-2, pp. 8–19.

CHAPTER 14

14.1.

P. ERDÖS, AND N. G. DE BRUIJN, *A colour problem for infinite graphs and a problem in the theory of relations.* Indagationes Math., v. 13 (1951) pp. 369–373.

P. ERDÖS AND R. RADO, *Partition relations connected with the chromatic number of graphs*, J. London Math. Soc., v. 34 (1959) pp. 63–72.

R. L. BROOKS, *On colouring the nodes of a network.* Cambridge Philos. Soc., v. 37 (1941) pp. 194–197.

J. MYCIELSKI, *Sur le coloriage des graphes.* Colloq. Math., v. 3 (1955) pp. 161–162.

14.2.

E. A. NORDHAUS AND J. W. GADDUM, *On complementary graphs.* Amer. Math. Monthly., v. 63 (1956) pp. 175–177.

A. A. ZYKOV, *On some properties of linear complexes.* Mat. Sb., v. 24 (66) (1949) pp. 163–188. Amer. Math. Soc. Translations No. 79.

14.3.

G. A. DIRAC, *Note on the colouring of graphs.* Math. Z., v. 54 (1951) pp. 347–353.

— *Some theorems on abstract graphs.* Proc. London Math. Soc., (3) v. 2 (1952) pp. 69–81.

— *A property of 4-chromatic graphs and some remarks on critical graphs.* J. London Math. Soc., v. 27 (1952) pp. 85–92.

— *The structure of k-chromatic graphs.* Fund. Math., v. 40 (1953) pp. 42–55.

— *Circuits in critical graphs.* Monatsh. Math., v. 59 (1955) pp. 178–187.

— *A theorem of R. L. Brooks and a conjecture of H. Hadwiger.* Proc. London Math. Soc., (3) v. 7 (1957) pp. 161–195.

J. B. KELLY AND L. M. KELLY, *Paths and circuits in critical graphs.* Amer. J. Math., v. 76 (1954) pp. 786–792.

R. C. READ, *Maximal circuits in critical graphs.* J. London Math. Soc., v. 32 (1957) pp. 456–462.

B. ZEIDL, *Über 4- und 5-chrome Graphen.* Monatsh. Math., v. 62 (1958) pp. 212–218.

14.4.

G. D. BIRKHOFF, *A determinant formula for the number of ways of colouring a map.* Ann. of Math., (2) v. 14 (1912) pp. 42–46.

— *On the number of ways of coloring a map.* Proc. Edinburgh Math. Soc., (2) v. 2 (1930) pp. 83–91.

H. WHITNEY, *A logical expansion in mathematics.* Bull. Amer. Math. Soc., v. 38 (1932) pp. 572–579.

— *The coloring of graphs.* Ann. of Math., (2) v. 33 (1932) pp. 688–718.

— *A set of topological invariants for graphs.* Amer. J. Math., v. 55 (1933) pp. 231–235.

CHAPTER 15

15.1.

R. FRUCHT, *Die Gruppe des Petersenschen Graphen und der Kantensysteme der regulären Polyeder.* Comment. Math. Helv., v. 9 (1937) pp. 217–223.

I. N. KAGNO, *Desargues' and Pappus' graphs and their groups.* Amer. J. Math., v. 69 (1947) pp. 859–862.

— *Linear graphs of degree $\leqq 6$ and their groups.* Corrections ibid., v. 69 (1947) p. 872; v. 77 (1955) p. 392.

G. PÓLYA, *Kombinatorische Anzahlbestimmungen für Gruppen, Graphen und chemische Verbindungen.* Acta. Math., v. 68 (1937) pp. 145–254.

15.2.

A. CAYLEY, *The theory of groups, graphical representation.* Math. Papers, v. 10 pp. 403–405.

— *On the theory of groups.* Ibid., v. 10 pp. 323–330; v. 11 pp. 365–367.

R. A. RANKIN, *A campanological problem in group theory.* Proc. Cambridge Philos. Soc., v. 44 (1948) pp. 17–25.

T. J. FLETCHER, *Campanological groups.* Amer. Math. Monthly, v. 63 (1956) pp. 619–626.

D. J. DICKINSON, *On Fletcher's paper: "Campanological groups".* Amer. Math. Monthly, v. 64 (1957) pp. 331–332.

E. S. RAPAPORT, *Cayley color groups and Hamilton lines.* Scripta Math., v. 24 (1959) pp. 51–58.

15.3.

R. FRUCHT, *Herstellung von Graphen mit vorgegebener abstrakter Gruppe.* Compositio Math., v. 6 (1938) pp. 239–250.

— *Graphs of degree three with a given abstract group.* Canad. J. Math., v. 1 (1949) pp. 365–378.

— *On groups of repeated graphs.* Bull. Amer. Math. Soc., v. 55 (1949) pp. 418–420.

— *On the construction of partially ordered systems with a given group of automorphisms.* Rev. Un. Mat. Argentina, v. 13 (1948) pp. 12–18.

G. T. TRANQUE, *The type in cubic graphs.* Gaceta Mat., (1) v. 5 (1953) pp. 11–23.

H. IZBICKI, *Reguläre Graphen* **3.** **4.** *und* **5.** *Grades mit vorgegebenen abstrakten Automorphismengruppen, Farbenzahlen und Zusammenhängen.* Monatsh. Math., v. 61 (1957) pp. 42–50.

— *Unendliche Graphen endlichen Grades mit vorgegebenen Eigenschaften.* Ibid., v. 63 (1959) pp. 298–307.

— *Reguläre Graphen beliebigen Grades mit vorgegebenen Eigenschaften.* Ibid., v. 64 (1960) pp. 15–21.

G. SABIDUSSI, *Graphs with given group and given graph-theoretical properties.* Canad. J. Math., v. 9 (1957) pp. 515–525.

— *On the minimum order of graphs with given automorphism group.* Monatsh. Math., v. 63 (1959) pp. 124–127.

— *Graphs with given infinite groups.* Ibid., v. 64 (1960) pp. 64–67.

— *Graph multiplication.* Math. Z., v. 72 (1960) pp. 446–457.

— *On a class of fixed-point-free graphs.* Proc. Amer. Math. Soc., v. 9 (1958) pp. 800–804.

— *The composition of graphs.* Duke Math. J., v. 26 (1959) pp. 693–696.

F. HARARY, *On the group of the composition of two graphs.* Duke Math. J., v. 26 (1959) pp. 29–34.

G. BIRKHOFF, *On groups of automorphisms.* Rev. Un. Mat. Argentina, v. 11 (1946) pp. 155–157.

W. T. TUTTE, *A family of cubical graphs.* Proc. Cambridge Philos. Soc., v. 43 (1947) pp. 459–474.

H. S. M. COXETER, *Self-dual configurations and regular graphs.* Bull. Amer. Math. Soc., v. 56 (1950) pp. 413–455.

R. FRUCHT, *A one-regular graph of degree three.* Canad. J. Math., v. 4 (1952) pp. 240–247.

15.4.

H. WHITNEY, *Congruent graphs and the connectivity of graphs.* Amer. J. Math., v. 54 (1932) pp. 150–168.

— *On the classification of graphs.* Ibid., v. 55 (1933) pp. 236–244.

— *2-isomorphic graphs.* Ibid., v. 55 (1933) pp. 245–254.

R. M. FOSTER, *Geometrical circuits of electrical networks.* Bell Tel. Syst. Techn. Publ. B-653; also Transactions Amer. Inst. Elec. Eng., v. 51 (1932) pp. 309–317.

J. KRAUSZ, *Démonstration nouvelle d'un théorème de Whitney sur les réseaux.* Mat. Fiz. Lapok, v. 50 (1943) pp. 75–85.

LIST OF CONCEPTS

This page intentionally left blank

INDEX OF NAMES

ISBN 978-0-8218-1038-5

9 780821 810385

COLL/38