

American Mathematical Society

Colloquium Publications

Volume 46

# Global Solutions of Nonlinear Schrödinger Equations

J. Bourgain



## Selected Titles in This Series

- 46 **J. Bourgain**, Global solutions of nonlinear Schrödinger equations, 1999
- 45 **Nicholas M. Katz and Peter Sarnak**, Random matrices, Frobenius eigenvalues, and monodromy, 1999
- 44 **Max-Albert Knus, Alexander Merkurjev, and Markus Rost**, The book of involutions, 1998
- 43 **Luis A. Caffarelli and Xavier Cabré**, Fully nonlinear elliptic equations, 1995
- 42 **Victor Guillemin and Shlomo Sternberg**, Variations on a theme by Kepler, 1990
- 41 **Alfred Tarski and Steven Givant**, A formalization of set theory without variables, 1987
- 40 **R. H. Bing**, The geometric topology of 3-manifolds, 1983
- 39 **N. Jacobson**, Structure and representations of Jordan algebras, 1968
- 38 **O. Ore**, Theory of graphs, 1962
- 37 **N. Jacobson**, Structure of rings, 1956
- 36 **W. H. Gottschalk and G. A. Hedlund**, Topological dynamics, 1955
- 35 **A. C. Schaeffer and D. C. Spencer**, Coefficient regions for Schlicht functions, 1950
- 34 **J. L. Walsh**, The location of critical points of analytic and harmonic functions, 1950
- 33 **J. F. Ritt**, Differential algebra, 1950
- 32 **R. L. Wilder**, Topology of manifolds, 1949
- 31 **E. Hille and R. S. Phillips**, Functional analysis and semigroups, 1957
- 30 **T. Radó**, Length and area, 1948
- 29 **A. Weil**, Foundations of algebraic geometry, 1946
- 28 **G. T. Whyburn**, Analytic topology, 1942
- 27 **S. Lefschetz**, Algebraic topology, 1942
- 26 **N. Levinson**, Gap and density theorems, 1940
- 25 **Garrett Birkhoff**, Lattice theory, 1940
- 24 **A. A. Albert**, Structure of algebras, 1939
- 23 **G. Szegő**, Orthogonal polynomials, 1939
- 22 **C. N. Moore**, Summable series and convergence factors, 1938
- 21 **J. M. Thomas**, Differential systems, 1937
- 20 **J. L. Walsh**, Interpolation and approximation by rational functions in the complex domain, 1935
- 19 **R. E. A. C. Paley and N. Wiener**, Fourier transforms in the complex domain, 1934
- 18 **M. Morse**, The calculus of variations in the large, 1934
- 17 **J. M. Wedderburn**, Lectures on matrices, 1934
- 16 **G. A. Bliss**, Algebraic functions, 1933
- 15 **M. H. Stone**, Linear transformations in Hilbert space and their applications to analysis, 1932
- 14 **J. F. Ritt**, Differential equations from the algebraic standpoint, 1932
- 13 **R. L. Moore**, Foundations of point set theory, 1932
- 12 **S. Lefschetz**, Topology, 1930
- 11 **D. Jackson**, The theory of approximation, 1930
- 10 **A. B. Coble**, Algebraic geometry and theta functions, 1929
- 9 **G. D. Birkhoff**, Dynamical systems, 1927
- 8 **L. P. Eisenhart**, Non-Riemannian geometry, 1927
- 7 **E. T. Bell**, Algebraic arithmetic, 1927
- 6 **G. C. Evans**, The logarithmic potential, discontinuous Dirichlet and Neumann problems, 1927

*(Continued in the back of this publication)*

*This page intentionally left blank*

# Global Solutions of Nonlinear Schrödinger Equations

*This page intentionally left blank*

American Mathematical Society

---

Colloquium Publications

Volume 46

# Global Solutions of Nonlinear Schrödinger Equations

J. Bourgain



American Mathematical Society  
Providence, Rhode Island

## Editorial Board

Joan S. Birman  
Susan J. Friedlander, Chair  
Stephen Lichtenbaum

1991 *Mathematics Subject Classification*. Primary 35Q55.

ABSTRACT. The aim of this book is to describe recent progress on various issues in the theory of nonlinear dispersive equations, primarily the nonlinear Schrödinger equation (NLS). In particular, the Cauchy problem for the defocusing critical NLS with radial data is discussed. New techniques and results are described on global existence of large data solutions below the energy norm. Current research in Harmonic Analysis around Strichartz' inequalities and its relevance to nonlinear PDE is presented. Also several topics in NLS theory on bounded domains are reviewed. In this respect, a partial survey is given of the theory of invariant Gibbs measures and recent developments in KAM theory for PDE's.

---

### Library of Congress Cataloging-in-Publication Data

Bourgain, Jean, 1954–

Global solutions of nonlinear Schrödinger equations / Jean Bourgain.

p. cm. — (American Mathematical Society colloquium publications, ISSN 0065-9258 ; v. 46)

Includes bibliographical references and index.

ISBN 0-8218-1919-4

1. Schrödinger equation. 2. Differential equations, Partial—Numerical solutions. 3. Nonlinear theories. I. Title. II. Series: Colloquium publications (American Mathematical Society) ; v. 46.

QC174.26.W28B68 1999

515'.353—dc21

99-13066

CIP

---

**Copying and reprinting.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Assistant to the Publisher, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to [reprint-permission@ams.org](mailto:reprint-permission@ams.org).

© 1999 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights  
except those granted to the United States Government.

Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines  
established to ensure permanence and durability.

Visit the AMS home page at URL: <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 04 03 02 01 00 99

# Contents

Introduction and summary	1
I. An overview of results on the Cauchy problem for NLS	5
1. Equations	5
2. Wellposedness of the Cauchy problem	6
3. Scattering results	10
4. Estimates on the linear group	11
5. Solving the Cauchy problem	14
6. Derivative nonlinear Schrödinger equations	14
II. Further comments	17
1. Construction of blowup solutions for conformal NLS from the ground state	17
2. Behaviour of higher Sobolev norms	28
3. Fourier restriction theory beyond $L^2$	33
4. $L^2$ -concentration phenomenon	37
5. The Schrödinger maximal function	45
6. Derivative nonlinear Schrödinger equation	48
III. 3D $H^1$ -critical defocusing NLS	51
1. Consider 3D NLS	51
2. Fix a time interval $I = [0, T]$	51
3. Sketch of the argument	54
4. A concentration property	59
5. A version of Morawetz inequality	61
6. Construction of an appropriate time interval	63
7. Details on the perturbative analysis	66
8. A variant of the method	70
IV. Global wellposedness below energy norm	79
1. Description of the method	79
2. The example of the NLW	79
3. The case of the nonlinear Schrödinger equation	83
4. Symplectic capacities and symplectic Hilbert spaces	93
5. Global wellposedness of the NLW (4.41)	97
V. Nonlinear Schrödinger equation with periodic boundary conditions	105
1. Introduction	105
2. Results on the Cauchy problem	106
3. Periodic Strichartz inequalities	108



4. Sketch of proof of Theorems 2.1 and 2.7	109
5. Invariant Gibbs measures (1D)	122
6. Invariant Gibbs measures ( $D > 1$ )	129
7. Invariant Gibbs measures (unbounded domains)	135
8. Quasi-periodic solutions	143
Appendix 1. Growth of Sobolev norms in linear Schrödinger equations with smooth time dependent potential	155
Appendix 2. Zakharov systems	173
References	176
Index	181

## References

- [A-A]. H. Added, S. Added, *Existence globale de solutions fortes pour les equations de la turbulence de Langmuir en dimension 2*, CR Acad. Sci. Paris 299 (1984), 551-554.
- [B1]. J. Bourgain, *Global wellposedness of defocusing 3D critical NLS in the radial case*, to appear in J. AMS.
- [B2]. J. Bourgain, *Refinements of Strichartz' inequality and applications to 2D-NLS with critical nonlinearity*, IMRN 1998, N5, 253-283.
- [B3]. J. Bourgain, *Scattering in the energy space and below for 3D NLS*, Analyse, Vol. 70 (1998), 267-298.
- [B4]. J. Bourgain, *Fourier restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations*, Part I, Geometric and Functional Analysis 3 (1993), 107-156.  
*Fourier restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations*, Part II, Geometric and Functional Analysis (1993), 209-262.
- [B5]. J. Bourgain, *Aspects of longtime behaviour of solutions of nonlinear Hamiltonian evolution equations*, GAFA, Vol. 5, N2 (1995), 105-140.
- [B6]. J. Bourgain, *Nonlinear Schrödinger equations*, Park City Lectures, July 1995.
- [B7]. J. Bourgain, *On the growth in time of higher Sobolev norms of smooth solutions of Hamiltonian PDE*, IMRN, 1996, N6, 277-304.
- [B8]. J. Bourgain, *Periodic Nonlinear Schrödinger equation and invariant measures*, Comm. Math. Phys. 160 (1994), 1-26.
- [B9]. J. Bourgain, *Invariant measures for the 2D-defocusing nonlinear Schrödinger equation*, CMP 176 (1996), 421-445.
- [B10]. J. Bourgain, *Invariant measures for the Gross-Piatevskii equation*, J. Math. Pures et Appliquées, T76, Fase 8, 649-702 (1997).
- [B11]. J. Bourgain, *Quasi-periodic solutions of Hamiltonian perturbations of 2D linear Schrödinger equations*, Annals of Math. 148 (1998), 363-439.
- [B12]. J. Bourgain, *On Melnikov's persistency problem*, Math. Research Letters 4, (1997), 445-458.
- [B13]. J. Bourgain, *Construction of approximative and almost-periodic solutions of perturbed linear Schrödinger and wave equations*, GAFA, Vol. 6, N2 (1996), 201-230.
- [B14]. J. Bourgain, *On the growth of Sobolev norms in linear Schrödinger equations with smooth time dependent potential*, to appear in J. Analyse Math.
- [B15]. J. Bourgain, *On growth in time of Sobolev norms of smooth solutions of nonlinear Schrödinger equations in  $\mathbb{R}^d$* , J. Analyse Mathématique, Vol. 72 (1997), 299-310.
- [B16]. J. Bourgain, *Besicovitch type maximal operators and applications to Fourier Analysis*, Geometric and Functional Analysis 1 (1991), 147-187.
- [B17]. J. Bourgain, *On the dimension of Kakeya sets and related maximal inequalities*, preprint, to appear in GAFA.
- [B18]. J. Bourgain, *A remark on Schrödinger operators*, Israel J. Math. Vol. 77, N 1-2 (1992), 1-16.
- [B19]. J. Bourgain, *Invariant measures for NLS in infinite volume*, preprint 98.
- [B20]. J. Bourgain, *Growth of Sobolev norms in linear Schrödinger equations with quasi-periodic potential*, to appear in CMP.
- [B21]. J. Bourgain, *On the Cauchy and invariant measure problem for the periodic Zakharov system*, Duke Math. J. 76 (1994), 175-202.

- [B-C]. J. Bourgain, J. Colliander, *On wellposedness of Zakharov systems*, IMRN 11 (1996), 515-546.
- [B-F-G]. G. Benettin, J. Fröhlich, A. Giorgilli, *A Nekhoroshev-type theorem for Hamiltonian systems with infinitely many degrees of freedom*, CMP 119 (1989), 95-108.
- [B-G]. H. Brezis, T. Gallouet, *Nonlinear Schrödinger evolution equations*, Nonlinear Anal. 4 (1980), 677-681.
- [B-L]. H. Brascamp, E. Lieb, *On extensions of the Brunn-Minkowski and Prikopa-Leinder theorems, including inequalities for log concave functions, and with an application to the diffusion equation*, JFA 22 (1976), 366-389.
- [B-S]. D. Brydges, G. Slade, *Statistical mechanics of the 2-dimensional focusing nonlinear Schrödinger equation*, CMP, 1996, 182, 485-504.
- [B-W]. J. Bourgain, W. Wang, *Construction of blowup solutions for the nonlinear Schrödinger equation with critical nonlinearity*, preprint 97, to appear in Annali Scuola Ecole Normale, Pisa (Vol. dedicated to De Giorgio).
- [C]. L. Carleson, *Some analytical problems related to statistical mechanics*, Euclidean Harmonic Analysis, LNM 779 (1979), 5-45.
- [Caz]. T. Cazenave, *An introduction to nonlinear Schrödinger equations*, Textos de Metodes Mathematicos 26 (Rio de Janeiro).
- [CH]. H. Chihara, *Local existence for semilinear Schrödinger equations*, Math. Japonica 42, N1 (1995), 35-52.
- [C]. J. Colliander, *Wellposedness of Zakharov systems with generalized nonlinearity*, J. Diff. Equations 145, 351-363 (1998), 351-363.
- [Co]. B. Connes, *Sur les coefficients des séries trigonomatiques convergents sphériquement*, CRA SC Paris t 283 (1976), Ser, A, 159-161.
- [C-P]. L. Chierchia, P. Peretti, *Maximal almost-periodic solutions for Lagrangian equations on infinite dimension al tori*, in 'Seminar on Dynamical Systems', Progress in Nonlinear Differential Equations and Their Applications, Vol. 12, Birkhäuser 1994.
- [C-St]. J. Colliander, G. Staffilani, *Global wellposedness of KdV below  $L^2$* , preprint 98.
- [C-W]. T. Cazenave, F. Weissler, *The Cauchy problem for the critical nonlinear Schrödinger equation in  $H^s$* , Nonlinear Anal., TMA 14 (1990), 807-836.
- [Cr-Wa]. W. Craig, C. Wayne, *Newton's method and periodic solutions of nonlinear wave equations*, Comm. Pure and Appl. Math. 46(1993), 1409-1501.
- [Dav]. R.O. Davies, *Some remarks on the Kakeya problem*, Proc. Cambridge Phil. Soc. 69 (1971), 417-421.
- [E]. L.H. Eliasson, *Perturbations of stable invariant tori*, Ann. Sc. Super. Pisa, CP Sci., IV, Ser. 15, 115-147 (1988).
- [Fal]. F. Falconer, *The geometry of fractal sets*, Cambridge UP, 1985.
- [Fef]. C. Fefferman, *The multiplier problem for the ball*, Ann. Math. 94 (1971), 330-336.
- [F-K-Tr]. J. Feldman, H. Knörrer, E. Trubowitz, *Perturbatively unstable eigenvalues of a periodic Schrödinger operator*, Comment, Math. Helv. 66 (1991), n4, 557-579.
- [F-S-W]. J. Fröhlich, T. Spencer, E. Wayne, *Localization in disordered, nonlinear dynamical systems*, J. Stat. Ph., Vol. 42, 247-274 (1986).
- [G]. R. Glassey, *On the blowing up of solutions to the Cauchy problem for nonlinear Schrödinger operators*, J. Math Phys. 8 (1977), 1794-1797.
- [Gr]. M. Grillakis, *Regularity and asymptotic behavior of the wave equation with a critical nonlinearity*, Annals Math. 132 (1990), 485-509.

- [G-J]. J. Glimm, A. Jaffe, *Quantum Physics*, Springer-Verlag (1987).
- [G-T-V]. J. Ginibre, Y. Tsutsumi, G. Velo, *On the Cauchy problem for the Zakharov system*, JFA 151, No2 (1997), 384-436.
- [G-V]. J. Ginibre, G. Velo, *Scattering theory in the energy space for a class of nonlinear Schrödinger equations*, J. Math Pure Appl. 64 (1985), 363-401.
- [H-O]. N. Hayashi, T. Ozawa, *Remarks on nonlinear Schrödinger equations in one space dimension*, Diff. and Integral Eqs, Vol. 7, N2 (1994), 453-461.
- [J-S-S]. J.L. Journé, A. Soffer, C. Sogge,  $L^p - L^{p'}$  estimates for time dependent Schrödinger Equations, Bull AMS, 23, N2 (1990).
- [K-P-V]. C. Kenig, G. Ponce, L. Vega, *On the IVP for the nonlinear Schrödinger equations*, Harmonic Analysis and operator theory (Caracas, 1994), 353-367, Contemporary Math, 189 AMS, Providence, RI, 1995.
- [K-P-V2]. C. Kenig, G. Ponce, L. Vega, *Small solutions to nonlinear Schrödinger equations*, Ann. Inst. H. Poincaré, Vol. 10, N3 (1993), 255-288.
- [K-P-V3]. C. Kenig, G. Ponce, L. Vega, *On the initial value problem for the Ishimori system*, preprint.
- [K-P-V4]. C. Kenig, G. Ponce, L. Vega, *Smoothing effects and local existence theory for the generalized nonlinear Schrödinger equations*, Inventiones Math., Vol. 134 (1998), 489-545.
- [K-P-V5]. C. Kenig, G. Ponce, L. Vega, *A bilinear estimate with applications to the KdV equation*, J.AMS, 9: 573-603, 1996.
- [Kuk1]. S. Kuksin, *Infinite-dimensional symplectic capacities and a squeezing theorem for Hamiltonian PDE's*, CMP 167 (1995), 531-552.
- [Kuk2]. S. Kuksin, *Nearly integrable infinite-dimensional Hamiltonian systems LNM*, 1556, Springer.
- [Ku-Po]. S. Kuksin, J. Pöschel, *Invariant Cantor manifolds of quasi-periodic oscillations for a nonlinear Schrödinger equation*, Annals of Math., Vol 143, N1(1996), 149-179.
- [Kw]. M. Kwong, *Uniqueness of positive solutions of  $\Delta u - u + u^p = 0$  in  $\mathbb{R}^N$* , Arch. Rat. Mech. Anal. 105 (1989), 243-266.
- [L-R-S]. J. Lebowitz, R. Rose, E. Speer, *Statistical mechanics of the nonlinear Schrödinger equation*, J. Stat. Phys. V. 50(1998), 657-687.
- [L-S]. J. Lin, W. Strauss, *Decay and scattering of solutions of a nonlinear Schrödinger equation*, JFA Vol. 30, N2 (1978), 245-263.
- [McK]. H. McKean, *Statistical mechanics of nonlinear wave equations (4): Cubic Schrödinger*, CMP 168 (1995), 479-491 (see also CMP 173 (1995), 675).
- [McK-V]. H. McKean, K. Vaninski, *Statistical mechanics of nonlinear wave equations and Brownian motion with restoring drift: The petit and microcanonical ensembles*, Comm. Math. Phys. 160 (1994), 615-630.
- [McK-V]. H. McKean, K. Vaninski, *Statistical mechanics of nonlinear wave equations, in 'Stochastic Analysis'*, Proc. Symp. Pure. Math., Vol. 57 (1996), 457-463.
- [M1]. F. Merle, *Determination of blow-up solutions with minimal mass for nonlinear Schrödinger equation with critical power*, Duke Math. J. 69 (1993), 427-453.
- [M2]. F. Merle, *Construction of solutions with exact  $k$  blow-up points for the Schrödinger equation with critical power nonlinearity*, Comm. Math. Phys. 149 (1992), 205-214.
- [M-T]. F. Merle, Y. Tsutsumi,  *$L^2$ -concentration of blowup solutions for the nonlinear Schrödinger equation with critical power nonlinearity*, J. Diff. Eq. 84 (1990), 205-214.

- [M-V]. F. Merle, L. Vega, *Compactness at blow-up time for  $L^2$ -solutions of the critical nonlinear Schrödinger equation in 2D*, IMRN 1998, N8, 399-425.
- [M-V-V]. A. Moyua, A. Vargas, L. Vega, *Schrödinger maximal function and restriction properties of the Fourier transform*, IMRN 16 (1996).
- [N]. H. Nawa, *Asymptotic profiles of blowup solutions of the nonlinear Schrödinger equation with critical power nonlinearity*, J. Math. Soc. Japan 46 (1994), 557-586.
- [Pos]. J. Pöschel.
- [Sp]. T. Spencer, Private communication.
- [S]. R. Strichartz, *Restrictions of Fourier transforms to quadratic surfaces and decay of solutions of wave equations*, Duke Math. J. 44 (1977), 705-714.
- [St]. G. Staffilani, *Quadratic forms for a 2D semilinear Schrödinger equation*, Duke Math. J., Vol. 86, N1 (1997), 79-108.
- [Str]. M. Struwe, *Globally regular solutions to the  $u^5$ -Klein-Gordon equation*, Ann. Scuola Norm Sup. Pisa, Ser. 4, 15 (1988), 495-513.
- [S-S]. J. Shatah, M. Struwe, *Regularity results for nonlinear wave equations*, Annals of Math. 138 (1993), 503-518.
- [S-S]. C. Sulem, R.L. Sulem, *Quelques résultats de régularité por les équations de la turbulence de Langmuir*, CR Acad. Se. Paris 289 (1979), 173-176.
- [S-W]. A. Soffer, M. Weinstein, *Resonances, Radiation Dumping and Instability in Hamiltonian Nonlinear Wave Equations*, preprint 1997.
- [T-V-V]. T. Tao, A. Vargas, L. Vega, *A bilinear approach to the restriction and Kakeya conjectures*, J. AMS 11 (1998), 967-1000.
- [Ve]. L. Vega, *Schrödinger equations: pointwise convergence to the critical data*, Proc. AMS 102 (1998), 874-878.
- [Wa]. C.E. Wayne, *Periodic and quasi-periodic solutions of nonlinear wave equations via KAM theory*, CMP 127, 479-528 (1990).
- [Wein]. M. Weinstein, *On the structure and formation of singularities in solutions to the nonlinear dispersive evolution equation*, Comm. PDE 11 (1986), 545-565.
- [Wein1]. M. Weinstein, *Modulational stability of ground states of nonlinear Schrödinger equations*, Siam J. Math. Anal. 16 (1985), 25-40.
- [Wein2]. M. Weinstein, *The nonlinear Schrödinger equation: singularity formation, stability and dispersion*, AAS-SIAM conference on the connection between infinite dimensional and finite dimensional dynamical systems (1987), 25-40.
- [W1]. T. Wolff, *An improved bound for Kakeya type maximal functions*, Revista Math. Iberoamericana 11 (1995), 651-674.
- [W2]. T. Wolff, *Recent work connected with the Kakeya problem*, in Prospects in Mathematics, Invited talks on the occasion of the 250th anniversary of Princeton University, March 17-21, 1996. Editor Hugo Rossi, AMS 1999.
- [Zh]. P. Zhidkov, *On the invariant measure for the nonlinear Schrödinger equation*, Doklady Akad. Nauk. SSSR 317, N3 543, (1991).

*This page intentionally left blank*

# Index

- admissible, 13
- almost periodic solutions, 143
  
- ball multiplier, 35
- Bloch waves, 155
- blowup solutions, 17
- Brascamp-Lieb inequality, 136
  
- canonical coordinates, 5
- compactness property, 44
- conformal NLS, 17
- critical, 6
  
- Decay estimate, 11
- defocusing, 5
- Derivative nonlinear Schrödinger equations, 14
- Dirichlet, 144
  
- energy estimates, 50
- exponential pair, 13
- exponential sum, 108
  
- Fermi-golden rule, 27
- focusing, 5
  
- gauge transformation, 15
- Gibbs measure, 106
- Gibbs measures, 4
- Glassey's viriel inequality, 8
- Global scattering in the energy space, 11
- Global scattering with decay, 10
- groundstate, 8
  
- $H^1$ -critical case, 10
- Hamiltonian, 106
- Hamiltonian perturbation, 143
- Hamiltonian vector field, 93
- Hartree, 133
- Hausdorff-Young inequality, 116
- Hermith polynomial, 130
  
- initial value problem (IVP), 1
- integral equation, 10
- interactions, 153
- Invariant Gibbs measures, 122
- invariant tori, 143
  
- Keakeya set, 33
- KAM (Kolmogorov-Arnold-Moser), 4
- KdV-equation, 79
  
- $L^2$ -concentration, 37
- $L^2$ -critical case, 9
- $L^2$ -cutoff, 124
- lattice points, 146
- linear scattering, 19
- linear Schrödinger equations, 29
- linear Schrödinger group, 33
  
- $m$ -admissible, 93
- maximal existence time, 7
- Maximal inequalities, 12
- Melnikov's theorem, 152
- Morawetz' inequality*, 11
  
- nonlinear coupling, 173
- nonlinear Schrödinger equation  
(NLS for short), 1
- nonlinear string equation, 93
- nonsqueezing theory, 83
- normal frequencies, 144
  
- oscillatory integral, 108
  
- periodic boundary conditions, 105, 144
- periodic solutions, 143
- Periodic Strichartz inequalities, 108
- pseudo-conformal conservation law, 6
  
- Quasi-periodic solutions, 143
- quasi-periodic solutions, 143
  
- random fields, 129
- resonance relation, 149
- restriction conjecture, 12, 33
- restriction norm, 109
  
- ( $S_0 =$  scaling exponent), 6
- Scattering, 10
- secular modes, 20
- "separated-cluster structure, 156
- short range, 153
- Small data scattering, 10
- Smoothing inequalities, 12
- space-time norms, 109
- Strichartz' inequality, 11

Sturm-Liouville operator, 143  
subcritical, 6  
Symplectic capacities, 93  
symplectic capacity, 83  
symplectic Hilbert space, 5, 80  
symplectic structure, 93  
symplectic transformations, 148  
  
tangential frequencies, 144  
the nonlinear wave equation (NLW), 2  
Tomas-Stein restriction theorem, 12  
truncated Gibbs measures, 175  
  
wave equation, 10  
weighted Wiener measure, 123  
Wiener measure, 106  
*Wick-ordering*, 130  
  
Young's inequality, 40  
  
Zakharov system, 4  
Zakharov systems, 173



## Selected Titles in This Series

*(Continued from the front of this publication)*

- 5.1 **G. C. Evans**, Functionals and their applications; selected topics, including integral equations, 1918
- 5.2 **O. Veblen**, Analysis situs, 1922
  - 4 **L. E. Dickson**, On invariants and the theory of numbers
  - W. F. Osgood**, Topics in the theory of functions of several complex variables, 1914
- 3.1 **G. A. Bliss**, Fundamental existence theorems, 1913
- 3.2 **E. Kasner**, Differential-geometric aspects of dynamics, 1913
  - 2 **E. H. Moore**, Introduction to a form of general analysis
  - M. Mason**, Selected topics in the theory of boundary value problems of differential equations
  - E. J. Wilczyński**, Projective differential geometry, 1910
- 1 **H. S. White**, Linear systems of curves on algebraic surfaces
  - F. S. Woods**, Forms on noneuclidean space
  - E. B. Van Vleck**, Selected topics in the theory of divergent series and of continued fractions, 1905

ISBN 0-8218-1919-4



9 780821 819197

COLL/46

AMS *on the Web*  
[www.ams.org](http://www.ams.org)