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Frobenius Manifolds, Quantum Cohomology, and Moduli Spaces

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Frobenius Manifolds, Quantum Cohomology, and Moduli Spaces

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ABSTRACT. This volume is a research monograph describing mathematical developments which originated in physics (quantum string theory) and which during the last six years have generated much activity in differential, symplectic, and algebraic geometry. In particular, the book provides an indispensable mathematical background for studying the Mirror Conjecture, which is one of the dualities in quantum string theory, recently discovered by physicists.

The book can be used by researchers and graduate students in algebraic geometry, differential geometry, theory of integrable systems, and mathematical physics; and by seminar leaders on these topics.

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To Xenia, with love and gratitude

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Contents

Preface

The mathematical language of classical physics is based upon real numbers. Configuration spaces and phase spaces of classical systems are differentiable manifolds, and physical laws are expressed by differential equations in the real domain.

The mathematical language of quantum physics is based upon complex numbers, and it would be natural to expect that the complex analytic and the algebraic geometry should replace the differential geometry of the classical period. In a sense, this is what has been happening during the last two or three decades, with the advent of scattering matrices, twistors, strings propagating in the ten–dimensional space–time, quantum cohomology, and M–theory. The mathematical physics of the dawning New Age sets as its ultimate goal construction of the universal quantum theory of all interactions including gravity. In the meantime it distanced itself from the traditional preoccupations of experimental particle physics and cosmology and did not just become heavily mathematicized, but in fact almost merged with mathematics. What made this development so exciting for mathematicians was that physicists brought not only a wealth of fresh insights, ideas, and problems, but also heuristic tools of great power and a certain freedom of expression which supplanted a rather strait–laced mood in the mathematical community of the fifties and sixties.

This book summarizes some of the mathematical developments that took place in the last decade or so and that focus on the notion of Quantum Cohomology, introduced by Cumrun Vafa (see [Va]) and Edward Witten. However, this is a mathematical monograph, and the reader who is interested in physical motivation and history will have to refer to other sources: see [MirS1], [MirS2], and the references therein.

Quantum Cohomology is a construction which endows with an additional highly non–linear structure the usual cohomology space $H = H^*(V)$ with complex coefficients of any projective algebraic (or symplectic) manifold V. The resulting structure, suitably axiomatized by B. Dubrovin, is called the *Frobenius manifold*. Interest in this axiomatization depends on the fact that there exist several general constructions of Frobenius manifolds, seemingly quite different, and unexpected isomorphisms between Frobenius manifolds of various classes (dualities, including Mirror duality). The first part of the book, Chapters I–IV, is dedicated to this notion and its multiple interconnections with geometry, differential equations, operads, and perturbation formalism. A more detailed summary can be found in the Introduction.

Although Quantum Cohomology in the proper sense of the word is invoked in several places in the first part of the book (Introduction, examples in Chapter II, axiomatic exposition in Chapter III), its systematic treatment is postponed until

Chapters V and VI. But whereas Chapters I–IV are reasonably self–contained and provide complete proofs of the main results, the final part of the book is meant as an introduction to the original papers and cannot replace them. In fact, the construction of Quantum Cohomology requires considerable algebraic geometric technique: the machinery of the Deligne–Mumford and Artin stacks, including intersection theory and the deformation theory for them. Already for schemes, this machinery takes hundreds of pages in standard sources: see [Ful] for intersection theory and [Il] for the deformation formalism. A monograph exhaustively treating the algebraic–geometric background for Quantum Cohomology is highly desirable. Hopefully, this book might stimulate its appearance.

A word of warning is in order: although the Mirror Conjecture initially provided the main stimulus for studying Quantum Cohomology, it is not treated in this book. On the one hand, this subject is still in a state of flux and rapid change. On the other hand, the body of firmly established facts, among which Givental's proof of the Mirror Identity of [COGP] for quintics occupies the prominent position (see [Giv2], [BiCPP], [Pa3], and the further development in [LiLY]), still constitutes only a fraction of the extremely varied and fascinating insights into what might be called the Mirror Phenomenon, which is an ambitious collective project bridging the physical and the mathematical communities.

Acknowledgements. Work on this book started in 1992–93, when Iz Singer and I led a seminar on the Mirror Conjecture at MIT. Contacts with Cumrun Vafa and Ed Witten were crucial at this stage.

The book took its present form after several lecture courses given at the Max–Planck–Institut für Mathematik in Bonn in 1994–98, and many shorter lecture courses delivered at various summer schools and conferences.

The vision of Quantum Cohomology expounded here was greatly influenced by Maxim Kontsevich, with whom I collaborated at the Max–Planck–Institut in 1994 and later. A part of the results in this book, including the axiomatic treatment of Gromov–Witten invariants, the theory of operadic tensor products in Chapter III, and the treatment of gravitational descendants in Chapter VI, is based on our joint work. Boris Dubrovin's papers, in particular his lecture notes [D2], provide the basic source of information about Frobenius manifolds, and most of the key definitions and theorems of Chapters I–II are due to him. The notion of weak Frobenius manifolds was introduced in my joint paper with Claus Hertling. Ralph Kaufmann's study of tensor products in the categories of local and global (as opposed to the operadic and formal) Frobenius manifolds is also incorporated in Chapter III. Chapter IV can serve as a brief introduction to operads and perturbation series. Our presentation owes much to the work of Misha Kapranov and Ezra Getzler. The final part of the book prepares and presents the construction of Gromov–Witten invariants which in genus zero are the coefficients of the formal series (potential) embodying Quantum Cohomology, and in higher genus provide a far–reaching extension of this theory in which much work remains to be done. This construction is due to Kai Behrend and Barbara Fantechi: see [Beh] and [BehF]. It was motivated by the earlier construction of the Gromov–Witten invariants in the symplectic and complex–analytic context due to J. Li and G. Tian: see [LiT1] and [LiT2]. The Behrend–Fantechi theory uses in essential ways stacks and their intersection theory, which are reviewed in Chapter V of this book. It is based on the work of Pierre Deligne, David Mumford, Mike Artin, Vistoli, and many others.

During the course of the work, I profited from many enlightening conversations and/or correspondence with my colleagues, friends, and collaborators mentioned above, and with Victor Batyrev, Sergei Barannikov, Alexander Givental, Vadim Schechtman, Sergey Merkulov, Markus Rosellen, and Don Zagier. Their contributions are gratefully acknowledged.

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