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Volume 48

# Geometric Nonlinear Functional Analysis

Volume 1

Yoav Benyamini

Joram Lindenstrauss



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# Geometric Nonlinear Functional Analysis

Volume 1

Yoav Benyamini  
Joram Lindenstrauss



American Mathematical Society  
Providence, Rhode Island

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ABSTRACT. This volume is devoted to the study of uniformly continuous and, in particular, Lipschitz functions between Banach spaces. Also studied is the related question of classifying Banach spaces and their subsets (mainly spheres) with respect to uniform or Lipschitz classification.

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Dedicated to A. Dvoretzky, our teacher, colleague and friend  
Founder of the functional analysis school in Israel



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