COLLOQUIUM PUBLICATIONS

American Mathematical Society
Colloquium Publications
Volume 51

Chiral Algebras

Alexander Beilinson
Vladimir Drinfeld

http://dx.doi.org/10.1090/coll/051

American Mathematical Society

Colloquium Publications
Volume 51

Chiral Algebras

Alexander Beilinson
Vladimir Drinfeld

American Mathematical Society
Providence, Rhode Island

2000 *Mathematics Subject Classification.* Primary 17Bxx; Secondary 14Fxx.

For additional information and updates on this book, visit
www.ams.org/bookpages/coll-51

Library of Congress Cataloging-in-Publication Data

Beilinson, Alexander, 1957–
Chiral algebras / Alexander Beilinson, Vladimir Drinfeld.
p. cm. — (Colloquium publications, ISSN 0065-9258 ; v. 51)
Includes bibliographical references and index.
ISBN 0-8218-3528-9 (alk. paper)
1. Geometry, Algebraic. 2. Conformal geometry. 3. Field theory (Physics) 4. Mathematical physics. I. Drinfeld, V. G. II. Title. III. Colloquium publications (American Mathematical Society) ; v. 51.

QC20.7.A37B45 2004
530.15′635—dc22 2003063872

♾ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 09 08 07 06 05 04

To our parents

This page intentionally left blank

Contents

Introduction 1

Chapter 1. Axiomatic Patterns 11

1.1. Pseudo-tensor categories 11

1.2. Complements 18

1.3. Compound tensor categories 24

1.4. Rudiments of compound geometry 32

Chapter 2. Geometry of $\mathcal{D}$-schemes 53

2.1. $\mathcal{D}$-modules: Recollections and notation 53

2.2. The compound tensor structure 67

2.3. $\mathcal{D}_X$-schemes 79

2.4. The spaces of horizontal sections 89

2.5. Lie* algebras and algebroids 95

2.6. Coisson algebras 113

2.7. The Tate extension 117

2.8. Tate structures and characteristic classes 129

2.9. The Harish-Chandra setting and the setting of c-stacks 143

Chapter 3. Local Theory: Chiral Basics 157

3.1. Chiral operations 157

3.2. Relation to "classical" operations 163

3.3. Chiral algebras and modules 164

3.4. Factorization 172

3.5. Operator product expansions 194

3.6. From chiral algebras to associative algebras 200

3.7. From Lie* algebras to chiral algebras 212

3.8. BRST, alias semi-infinite, homology 227

3.9. Chiral differential operators 238

3.10. Lattice chiral algebras and chiral monoids 257

Chapter 4. Global Theory: Chiral Homology 275
4.1. The cookware 275
4.2. The construction and first properties 296
4.3. The BV structure and products 313
4.4. Correlators and coinvariants 323
4.5. Rigidity and flat projective connections 330
4.6. The case of commutative $\mathcal{D}_X$algebras 341
4.7. Chiral homology of the de Rham-Chevalley algebras 348
4.8. Chiral homology of chiral envelopes 353
4.9. Chiral homology of lattice chiral algebras 357
Bibliography 363
Index and Notation 369

Bibliography

[A] F. Akman, *A characterization of the differential in semi-infinite cohomology*, J. Algebra **162** (1993), no. 1, 194–209.

[Ar] S. Arkhipov, *Semiinfinite cohomology of quantum groups*, Comm. Math. Phys. **188** (1997), no. 2, 379–405.

[AG] S. Arkhipov and D. Gaitsgory, *Differential operators on the loop group via chiral algebras*, Int. Math. Res. Notes (2002), no. 4, 165–210.

[BAK] B. Bakalov, A. d'Andrea, and V. Kac, *Theory of finite pseudoalgebras*, Adv. Math. **162** (2001), 1–140.

[BaK] B. Bakalov and V.Kac, *Twisted modules over lattice vertex algebras*, math.QA/0402315.

[BKV] B. Bakalov, V. Kac, and A. Voronov, *Cohomology of conformal algebras*, Comm. Math. Phys. **200** (1999), no. 3, 561–598.

[BGK] V. Baranovsky, V Ginzburg, and A. Kuznetsov, *Wilson's Grassmannian and a noncommutative quadric*, Int. Math. Res. Notes (2003), no. 21, 1155–1197.

[BB] A. Beilinson and J. Bernstein, *A proof of Jantzen conjectures*, I. M. Gelfand Seminar, Advances in Soviet Mathematics, vol. 16, part 1, Amer. Math. Soc., Providence, RI, 1993, pp. 1–50.

[BBE] A. Beilinson, S. Bloch, and H. Esnault, *$\mathcal{E}$-factors for Gauß-Manin determinants*, Moscow Math. Journal **2** (2002), 477–532.

[BD] A. Beilinson and V. Drinfeld, *Quantization of Hitchin's hamiltonians and Hecke eigensheaves*, http://www.math.uchicago.edu/~arinkin/langlands/ (1999).

[BS] A. Beilinson and V. Schechtman, *Determinant bundles and Virasoro algebras*, Commun. Math. Phys. **118** (1988), 651–701.

[BPZ] A. Belavin, A. Polyakov, and A. Zamolodchikov, *Infinite conformal symmetries in two-dimensional quantum field theory*, Nucl. Phys. **B241** (1984), 333–380.

[Ber] J. Bernstein, *Lectures on D-modules*, http://www.math.uchicago.edu/~arinkin/ langlands/ (1984).

[BFS] R. Bezrukavnikov, M. Finkelberg, and V. Schechtman, *Factorizable sheaves and quantum groups*, Lecture Notes in Math., vol. 1691, Springer-Verlag, Berlin–Heidelberg–New York, 1998.

[BoKa] A. Bondal and M. Kapranov, *Enhanced triangulated categories*, Math. USSR Sb. **70** (1991), no. 1, 93–107.

[B1] R. Borcherds, *Vertex algebras, Kac-Moody algebras and the Monster*, Proc. Natl. Acad. Sci. USA **83** (1986), 3068–3071.

[B2] ———, *Vertex algebras*, Topological field theory, primitive forms and related topics (M. Kashiwara, A. Matsuo, K. Saito, and I. Satake, eds.), Birkhäuser, Boston, 1998, pp. 35–77.

[Ba] A. Borel et al., *Aglebraic D-modules*, Academic Press, New York, 1987.

[BU] K. Borsuk and S. Ulam, *On symmetric products of topological spaces*, Bull. Amer. Math. Soc. **37** (1931), 875–882.

[Bo] R. Bott, *On the third symmetric potency of S^1*, Fund. Math. **39** (1952), 264–268.

[BK] A. K. Bousfield and D. Kan, *Homotopy limits, completions and localizations*, Lecture Notes in Math., vol. 304, Springer-Verlag, Berlin–Heidelberg–New York, 1987.

[Bre1] P. Bressler, *Vertex algebroids.* I, math. AG/0202185.

[Bre2] ———, *Vertex algebroids.* II, math. AG/0304115.

[CH] R. C. Cannings and M. P. Holland, *Right ideals of rings of differential operators*, J. Algebra **167** (1994), no. 1, 116–141.

[CFK] I. Ciocan-Fontanine and M. Kapranov, *Derived Quot schemes*, Ann. Sci. ENS, 4[e] série, **34** (2001), 403–440.

[CC] C. E. Contou-Carrère, *Jacobienne locale, groupe debivecteurs de Witt universel et symbole localmodéré*, C. R. Acad. Sci. Paris **318, Série I** (1994), 743–746.

[C] F. R. Cohen, *The homology of C_{n+1}-spaces, $n \geq 0$*, Lecture Notes in Math., vol. 533, Springer-Verlag, Berlin–Heidelberg–New York, 1976, pp. 207–351.

[CN] D. W. Curtis and N. T. Nhu, *Hyperspaces of finite subsets which are homeomorphic to $\aleph_0$-dimensional linear metric spaces*, Topology and Appl. **19** (1985), 251–260.

[DK] A. D'Andrea and V. Kac, *Structure theory of finite conformal algebras*, Selecta Math. **4** (1998), 377–418.

[DJKM] E. Date, M. Jimbo, M. Kashiwara, and T. Miwa, *Transformation groups for soliton equations*, Nonlinear integrable systems – classical theory and quantum theory (Kyoto, 1981) (M. Jimbo and T. Miwa, eds.), World Scientific, Singapore, 1983, pp. 39–120.

[DJM] E. Date, M. Jimbo, and T. Miwa, *Solitons, differential equations, symmetries, and infinite-dimensional algebras*, Cambridge Tracts Math., vol. 135, Cambridge Univ. Press, 2000.

[D1] P. Deligne, *Categories Tanakiennes*, The Grothendieck Festschrift, vol. 2, Progress in Math., vol. 87, Birkhauser, Boston, 1990, pp. 111–195.

[D2] ———, *Le symbole modéré*, Publ. Math. IHES **73** (1991), 147–181.

[DF] P. Deligne and D. Freed, *Classical field theory*, Quantum fields and Strings: a course for mathematicians, vol. 1, Amer. Math. Soc., Providence, RI, 1999, pp. 137–225.

[DM] P. Deligne and J. Morgan, *Notes on Supersymmetry (following Joseph Bernstein)*, Quantum fields and Strings: a course for mathematicians, vol. 1, Amer. Math. Soc., Providence, RI, 1999, pp. 41–97.

[Dem] M. Demazure, *Lectures on p-divisible groups*, Lecture Notes in Math., vol. 302, Springer -Verlag, Berlin-Heidelberg-New York, 1972.

[Di] L. Dickey, *Soliton equations and Hamiltonian systems, second edition*, Advanced Series in Mathematical Physics, vol. 26, World Scientific, River Edge, NJ, 2003.

[Don] C. Dong, *Vertex algebras associated with even lattices*, J. Algebra **161** (1993), 245–265.

[DL] C. Dong and J. Lepowski, *Generalized vertex algebras and relative vertex operators*, Progress in Math., vol. 112, Birkhauser, Boston, 1993.

[DLM] C. Dong, H. Li, and G. Mason, *Vertex Lie algebras, vertex Poisson algebras and vertex algebras*, Recent developments in infinite-dimensional Lie algebras and conformal field theory (Charlottesville, VA, 2000), Contemp. Math., vol. 297, Amer. Math. Soc., Providence, RI, 2002, pp. 69–96.

[DG1] I. Dorfman and I. Gelfand, *Hamiltonian operators and related algebraic structures*, Funct. Anal. Appl. **13** (1979), no. 4, 248–262.

[DG2] ———, *Hamiltonian operators and infinite-dimensional Lie aglebras*, Funct. Anal. Appl. **15** (1981), no. 3, 173–187.

[DoF] V. Dotsenko and V. Fateev, *Conformal algebra and multipoint correlator functions in 2D statistical models*, Nucl. Phys. **B240** (1984), 312–348.

[Dr1] V. Drinfeld, *DG quotients of DG categories*, Jour. of Algebra **272** (2004), 643–691.

[Dr2] ———, *Infinite-dimensional vector bundles in algebraic geometry*, math. AG/0309155; to appear in Proceedings of the International "Unity of Mathematics"conference in honor of I. M. Gelfand, Birkhäuser, Boston, MA.

[DS] V. Drinfeld and V. Sokolov, *Lie algebras and KdV type equations*, J. Soviet Math. **30** (1985), 1975–2036.

[EGA IV] A. Grothendieck and J. Dieudonné, *Élements de géométrie algébrique IV. Étude locale des schémas et des morphismes de schémas (seconde partie)*, Publ. Math. IHES **24** (1965).

[EFK] P. Etingof, I. Frenkel, and A. Kirillov, Jr., *Lectures on representation theory and Knizhnik-Zamolodchikov equations*, Mathematical Surveys and Monographs, vol. 58, Amer. Math. Soc., Providence, RI, 1998.

[F] B. Feigin, *The semi-infinite cohomology of Kac-Moody and Virasoro algebras*, Russian Math. Surveys **39** (1984), no. 2, 155–156.

[FF1] B. Feigin and E. Frenkel, *A family of representations of affine Lie algebras*, Russian Math. Surveys **43** (1988), no. 5, 221–222.

[FF2] ———, *Affine Kac-Moody algebras and semi-infinite flag manifolds*, Commun. Math. Phys. **128** (1990), 161–189.

[FS] B. Feifin and A. Stoyanovsky, *Realization of a modular functor in the space of differentials and geometric approximation of the manifold of moduli of G-bundles*, Funct. Anal. Appl. **28** (1995), no. 4, 257–275.

[Fr] E. Frenkel, *Lectures on Wakimoto modules, opers and the center at the critical level*, math. QA/0210029.

[FBZ] E. Frenkel and D. Ben-Zvi, *Vertex algebras and algebraic curves*, Mathematical Surveys and Monographs, vol. 88, Amer. Math. Soc., Providence, RI, 2001.

[FrG] E. Frenkel and D. Gaitsgory, *D-modules on the affine Grassmannian and representations of affine Kac-Moody algebras*, math. AG/0303173; to appear in Duke Math. J.

[FKRW] E. Frenkel, V. Kac, A. Radul, and W. Wang, *$\mathcal{W}_{1+\infty}$ and $\mathcal{W}(\mathfrak{gl}_N)$ with central charge N*, Commun. Math. Phys. **170** (1995), 337–357.

[FGZ] I. Frenkel, H. Garland, and G. Zuckerman, *Semi-infinite cohomology and string theory*, Proc. Nat. Acad. Sci. USA **83** (1986), no. 22, 8442–8446.

[FLM] I. Frenkel, J. Lepowski, and A. Meurman, *Vertex operator algebras and the monster*, Academic Press, Boston, 1988.

[FHL] I. Frenkel, Y. Huang, and J. Lepowski, *On Axiomatic Approaches to Vertex Operator Algebras and Modules*, Mem. Amer. Math. Soc., 494, vol. 104, 1993.

[FK] I. Frenkel and V. Kac, *Basic representations of affine Lie algebras and dual resonance models*, Invent. Math. **62** (1980), 23–66.

[Ga] D. Gaitsgory, *Notes on 2d conformal field theory and string theory*, Quantum fields and Strings: a course for mathematicians, vol. 2, Amer. Math. Soc., Providence, RI, 1999, pp. 1017–1090.

[GeK] I. Gelfand and D. Kazhdan, *Some problems of differential geometry and the calculation of cohomologies of Lie algebras of vector fields*, Soviet Math. Dokl. **12** (1971), no. 5, 1367–1370.

[Ge] E. Getzler, *Batalin-Vilkovisky algebras and two-dimensional topological field theories*, Commun. Math. Phys. **159** (1994), 265–285.

[GK] V. Ginzburg and M. Kapranov, *Koszul duality for operads*, Duke Math. J. **76** (1994), 203–272.

[GMS1] V. Gorbounov, F. Malikov, and V. Schechtman, *Gerbes of chiral differential operators* I, Math. Res. Lett. **7 no. 1** (2000), 55–66.

[GMS2] ———, *Gerbes of chiral differential operators* II. Vertex algebroids, Inventiones Math. **155** (2004), no. 3, 605–680; math. AG/0003170.

[GMS3] ———, *On chiral differential operators over homogeneous spaces.*, Int. J. Math. Sci. **26** (2001), no. 1, 83–106.

[G] M. Gromov, *Partial differential relations*, Springer-Verlag, Berlin–New York–Heidelberg, 1986.

[Gr1] A. Grothendieck, *Catégories cofibrées additives et complexe cotangent relatif*, Lecture Notes in Math., vol. 79, Springer-Verlag, Berlin–Heidelberg–New York, 1968.

[Gr2] ———, *Crystals and the de Rham cohomology of schemes*, Dix exposés sur la cohomologie des schémas, North-Holland, Amsterdam, 1968, pp. 306–358.

[Ha] R. Hartshorne, *Residues and duality*, Lecture Notes in Math., vol. 20, Springer-Verlag, Berlin–Heidelberg–New York, 1966.

[H] V. Hinich, *Homological algebra of homotopy algebras*, Commun. in Algebra **25(10)** (1997), 3291–3323.

[HS] V. Hinich and V. Schechtman, *Homotopy limits of homotopy algebras*, K-theory, Arithmetic and Geometry, Yu. Manin ed., Lecture Notes in Math., vol. 1289, Springer-Verlag, Berlin–Heidelberg–New York, 1987, pp. 240–264.

[Hu] Y.-Z. Huang, *Two-dimensional conformal geometry and vertex operator algebras*, Progr. Math., vol. 148, Birkhäuser, Boston, MA, 1997.

[HL1] Y.-Z. Huang and J. Lepowsky, *Tensor products of modules for a vertex operator algebra and vertex tensor categories*, Lie theory and geometry, Progr. Math., vol. 123, Birkhäuser, Boston, MA, 1994, pp. 349–383.

[HL2] Y.-Z. Huang and J. Lepowsky, *On the $\mathcal{D}$-module and formal variable approaches to vertex algebras*, Topics in Geometry, Progr. Nonlinear Differential Equations Appl., vol. 20, Birkhäuser, Boston, MA, 1996, pp. 175–202.

[Hue] J. Huebschmann, *Lie-Rinehart algebras, Gerstenhaber algebras and Batalin-Vilkovisky algebras*, Ann. Inst. Fourier (Grenoble) **48** (1998), no. 2, 425–440.

[Il] L. Illusie, *Complexe cotangent et déformations.* I, Lecture Notes in Math., vol. 239, Springer-Verlag, Berlin–Heidelberg–New York, 1971.

[J] J. D. S. Jones, *Lectures on operads*, Quantization, Poisson brackets and beyond, Th. Voronov ed., Contemp. Math., vol. 315, Amer. Math. Soc., Providence, RI, 2002, pp. 89–130.

[K] V. Kac, *Vertex algebras for beginners, second edition*, Univ. Lecture Series, vol. 10, Amer. Math. Soc., Providence, RI, 1998.

[KP] V. Kac and D. Peterson, *Spin and wedge representations of infinite-dimensional Lie algebras and groups*, Proc. Nat. Acad. Sci. USA **78** (1981), 3308–3312.

[KR] V. Kac and A. Radul, *Representation theory of the vertex algebra* $W_{1+\infty}$, Transform. Groups **1** (1996), 41–70.

[Ka] I. Kaplansky, *Projective modules*, Ann. Math. **68** (1958), 372-377; *Selected Papers by I. Kaplansky*, Springer-Verlag, Berlin–Heidelberg–New York, 1995.

[Kap1] M. Kapranov, *Operads and algebraic geometry*, Documenta Math., ICM 1998, vol II, 277–286.

[Kap2] ———, *Semiinfinite symmetric powers*, math. QA/0107089.

[KV] M. Kapranov and E. Vasserot, *Vertex algebras and the formal loop space*, math. AG/01 07143.

[Kas1] M. Kashiwara, *Representation theory and D-modules on flag varieties*, Astérisque **173–174** (1989), 55–109.

[Kas2] ———, *D-modules and microlocal calculus*, Amer. Math. Soc., Providence, RI, 2003.

[KM] N. Katz and B. Mazur, *Arithmetic moduli of elliptic curves*, Princeton Univ. Press, Princeton, NJ, 1985.

[KL] D. Kazhdan and G. Lusztig, *Tensor structures arising from affine Lie algebras. I, II*, J. Amer. Math. Soc. **6** (1993), no. 4, 905–947, 949–1011.

[KZ] V. Knizhnik and A. Zamolodchikov, *Current algebra and Wess-Zumino model in two dimensions*, Nucl. Phys. **B247** (1984), 83–103.

[KS] B. Kostant and S. Sternberg, *Symplectic reduction, BRS cohomology, and infinite-dimensional Clifford algebras*, Ann. Phys. **176** (1987), no. 1, 49–113.

[Kos] J-L. Koszul, *Crochet de Schouten-Nijenhuis et cohomologie*, Astérisque, vol Hors-Serie: Elie Cartan et les Mathématiques d'Aujourd'hui, Soc. Math. France, 1985, pp. 257–271.

[KrM] I. Kriz and J. P. May, *Operads, algebras, modules and motives*, Astérisque **233** (1995).

[Ku] B. Kupershmidt, *Geometry of jet bundles and the structure of Lagrangian and Hamiltonian formalisms*, Lecture Notes in Math., vol. 775, Springer-Verlag, Berlin–New York–Heidelberg, 1980, pp. 162–218.

[KuM] B. Kupershmidt and Yu. Manin, *Equations of long waves with a free surface I. Conservation laws and solutions*, Funkcional. Anal. I Prilozh. **11** (1977), no. 3, 31–42; English transl. Func. Anal. Appl. **11** (1977).

[La] J. Lambek, *Deductive systems and categories*, Lecture Notes in Math., vol. 86, Springer-Verlag, Berlin–New York–Heidelberg, 1969, pp. 76–122.

[LMB] G. Laumon and L. Moret-Bailly, *Champs algébriques*, Ergebnisse der Mathematik und ihrer Grenzgebiete, 3 Folge, vol. 39, Springer-Verlag, Berlin, 2000.

[Lef] S. Lefschetz, *Algebraic topology*, Colloq. Publ., vol. 27, Amer. Math. Soc., Providence, RI, 1942.

[Li] F. E. J. Linton, *The multilinear Yoneda lemmas: Toccata, fugue, and fantasia on themes by Eilenberg-Kelly and Yoneda*, Lecture Notes in Math., vol. 195, Springer-Verlag, Berlin–New York–Heidelberg, 1971, pp. 209–229.

[L] J.-L. Loday, *La renaissance des opérades*, Sém. Bourbaki 1994/95, exp. 792, Astérisque, vol. 237, 1996, pp. 47–74.

[M] K. Mackenzie, *Lie groupoids and Lie algebroids in differential geometry*, London Math. Soc. Lecture Notes, vol. 124, Cambridge Univ. Press, Cambridge, 1987.

[MSV] F. Malikov, V. Schechtman, and A. Vaintrob, *Chiral de Rham complex*, Commun. Math. Phys. **204** (1999), 439–473.

[Ma] Yu. Manin, *Algebraic aspects of non-linear differential equations*, Itogi Nauki, Sovremennye Problemy Matematiki, vol. 11, 1978, pp. 5–152; English transl., J. Sov. Math. **11** (1979), 1–122.

[MSS] M. Markl, S. Shnider, and J. Stasheff, *Operads in algebra, topology and physics*, Mathematical Surveys and Monographs, vol. 96, Amer. Math. Soc., Providence, RI, 2002.

[May] P. May, *Geometry of iterated loop spaces*, Lecture Notes in Math., vol. 271, Springer-Verlag, Berlin–Heidelberg–New York, 1972.

[OS] P. Orlik and L. Solomon, *Combinatorics and topology of complements of hyperplanes*, Inv. Math. **56** (1980), 167–189.

[Po] A. Polishchuk, *Abelian varieties, theta functions and the Fourier transform*, Cambridge Tracts in Mathematics, vol. 153, Cambridge University Press, Cambridge, 2003.

[PS] A. Pressley and G. Segal, *Loop groups*, Clarendon Press, Oxford, 1986.

[P] M. Primc, *Vertex algebras generated by Lie algebras*, J. Pure Appl. Algebra **135** (1999), no. 3, 253–293.

[Q1] D. Quillen, *Rational homotopy theory*, Ann. Math. **90** (1968), no. 2, 205–295.

[Q2] ———, *On the (co-)homology of commutative rings*, Proc. Symp. Pure Math., vol. 17, Amer. Math. Soc., Providence, RI, 1970, pp. 65–87.

[Q3] ———, *Higher algebraic K-theory.* I **341** (1973), Springer-Verlag, Berlin–New York–Heidelberg, 85–147.

[R1] Z. Ran, *Derivatives of moduli*, Intern. Math. Res. Notices (1993), no. 4, 93–106.

[R2] ———, *Canonical infinitesimal deformations*, J. Alg. Geometry **9** (2000), no. 1, 43–69.

[RG] M. Raynaud and L. Gruson, *Critères de platitude et de projectivité*, Invent. Math. **13** (1971), 1–89.

[Re] A. Retakh, *Associative conformal algebras of linear growth*, J. Algebra **237** (2001), no. 2, 769–788.

[Rin] G. S. Rinehart, *Differential forms on general Lie algebras*, Trans. Amer. Math. Soc. **108** (1963), 195–222.

[Ro] M. Roitman, *Combinatorics of free vertex algebras*, J. of Algebra **255** (2002), no. 2, 297–323.

[S1] M. Saito, *Induced D-modules and differential complexes*, Bull. Soc. Math. France **117** (1989), 361–387.

[S2] ———, *D-modules on analytic spaces*, Publ. Res. Inst. Math. Studies **27** (1991), no. 2, 291–332.

[Sa] M. Sato, *The KP hierarchy and infinite-dimensional Grassmann manifolds*, Proc. Sym. Pure Math., vol. 49, part 1, 1989, pp. 51–66.

[Sch1] V. Schechtman, *Sur les algèbres vertex attachées aux variétés algébriques*, Proc. ICM 2002, Beijing, vol. 2, Higher Education Press, pp. 525–532.

[Sch2] ———, *Definitio nova algebroidis verticiani*, Prépublicationno. 271, Laboratoire de Mathématiques Emile Picard, Toulouse (2003), http://picard.ups-tlse.fr.

[Schw] A. Schwarz, *Topological quantum field theories*, XIIIth International Congress on Mathematical Physics (London, 2000), Int. Press, Boston, 2001, pp. 123–142.

[Se] G. Segal, *Classifying spaces and spectral sequences*, Publ. Math. IHES **34** (1968), 105–112.

[SW] G. Segal and G. Wilson, *Loop groups and equations of KdV type*, Publ. Math. IHES **61** (1985), 5–65.

[SGA 4] *Théorie des topos et cohomologie étale des schémas*, Séminaire de Géométrie Algébrique du Bois-Marie 1963–1964. Dirigé par M. Artin, A. Grothendieck et J.-L. Verdier, Lecture Notes in Math., vol. 269, 270, 305, Springer-Verlag, Berlin-Heidelberg-New York, 1972–1973.

[Sp] N. Spaltenstein, *Resolutions of unbounded complexes*, Compositio Math. **65** (1988), 121–154.

[Tam1] D. Tamarkin, *Deformations of chiral algebras*, Proc. ICM 2002, Beijing, vol. 2, Higher Education Press, pp. 105–118.

[Tam2] ———, *A formalism for the renormalization procedure*, math. QA/0312219.

[T] J. Tate, *Residues of differentials on curves*, Ann. Sci. ENS **4 ser., t. 1** (1968), 149–159.

[V] A. Vinogradov, *Cohomological analysis of partial differential equations and secondary calculus*, Transl. Math. Monographs, vol. 204, Amer. Math. Soc., Providence, RI, 2001.

[Vo] A. Voronov, *Semi-infinite homological algebra*, Inventiones Math. **113** (1993), no. 1, 103–146.

[W] J. Wiennfield, *Formal geometry*, preprint (1999).

[X] P. Xu, *Gerstenhaber algebras and BV algebras in Poisson geometry*, Comm. Math. Phys. **200** (1999), no. 3, 545–560.

[Z] G. Zuckerman, *Action principles and global geometry*, Mathematical Aspects of String Theory (S. T. Yau, ed.), World Scientific, 1987, pp. 259–284.

Index and Notation

action of a Lie* algebra on a commutative$^!$ algebra 1.4.9
action of a Lie* algebra on a chiral algebra 3.3.3
action of a Lie algebroid on a chiral algebra 4.5.4, twisted 4.5.6
action of a pseudo-tensor category on a category 1.2.11
action of a tensor category on a pseudo-tensor category 1.1.6(v)
admissible complex of sheaves on $X^{\mathcal{S}}$ 4.2.1
admissible $\mathcal{D}$-complex on $X^{\mathcal{S}}$ 4.2.6
algebraic $\mathcal{D}_X$-space 2.3.1
annihilator 3.3.7
augmentation functor, non-degenerate 1.2.5, reliable 1.4.7
augmentation functor in compound setting 1.3.10, in $\mathcal{D}$-module setting 2.2.7
augmented compound tensor category, functor 1.3.16
augmented operad 1.2.4
augmented pseudo-tensor category 1.2.4
augmented pseudo-tensor functor, unit 1.2.8

Batalin-Vilkovisky (BV) algebras 4.1.6
BV extension of a Lie algebroid 4.1.9
BV quantization of an odd Poisson algebra 4.1.6
BV structure on the chiral chain complex 4.3.1
BRST reduction, charge, differential: classical 1.4.23, quantum 3.8.9, 3.8.20
BRST property 1.4.24, 3.8.10, 3.8.21

calculus of variations 2.3.20
cdo 3.9.5
centralizer 3.3.7
center of a chiral algebra 3.3.7
central chiral modules 3.3.7
Chern classes $ch_n^{\mathcal{D}}$ 2.8.10
Chevalley-Cousin complex of a chiral algebra 3.4.11, relative version 4.4.9
Chevalley complex of a Lie* algebra, inner 1.4.5, 1.4.10
chiral action of a Lie* algebra 3.7.16, of a chiral Lie algebroid 3.9.24
chiral action of a chiral monoid 3.10.17
chiral algebras 3.3.3, commutative 3.3.3, non-unital 3.3.2, universal 3.3.14
chiral algebra freely generated by (N, P) 3.4.14,
chiral $R_{\mathcal{D}if}$-algebras 3.9.4
chiral enveloping algebra of a Lie* algebra 3.7.1, of a chiral Lie algebroid 3.9.11
chiral extension of a Lie* algebroid 3.9.6, rigidified 3.9.8
chiral homology 4.2.11
chiral lattice algebras 3.10.1

chiral Lie algebroids 3.9.6
chiral modules 3.3.4
chiral L-modules 3.7.16
chiral $R_{\mathcal{D}if}$-modules 3.9.1
chiral monoid 3.10.17
chiral operations 3.1.1, for $(\mathfrak{g}, K)$-modules 3.1.16
chiral A-operations 3.3.4
chiral L^{ch}-operations 3.7.16
chiral product 3.3.2
chiral pseudo-tensor structure 3.1.2
Clifford algebra, coisson 1.4.21, chiral 3.8.6, linear algebra version 3.8.17
coisson algebras 1.4.18, modules 1.4.20, $\mathcal{D}$-module setting 2.6.1, elliptic 2.6.6
commutative$^!$ algebras, modules 1.4.6
commutative $\mathcal{D}_X$-algebras 2.3.1
complementary quotients 1.3.1
compound operad 1.3.18
compound pseudo-tensor category 1.3.7, augmented 1.3.10
compound pseudo-tensor functor 1.3.9
compound tensor category, functor 1.3.14
compound tensor product maps 1.3.12, binary 1.3.13
connections for Lie* algebroids 1.4.17
connections on chiral homology 4.5
Contou-Carrère symbol 3.10.13
convenient $\mathcal{D}_X$-algebra 4.6.1
convenient R^ℓ-modules 4.6.3
coordinate system on a $\mathcal{D}_X$-scheme 2.3.17
c operations 1.4.27, in $\mathcal{D}$-module setting 3.2.5
correlators 4.4.1
cotangent complex 2.3.15, 4.1.5, 4.6.6
cotorsor 3.4.16, 3.10.12
Cousin $\mathcal{D}$-complex 4.2.9
Cousin filtration 4.2.1, 4.2.19
Cousin spectral sequence 4.2.3, 4.2.11, 4.2.19
c-stack, c-rank 2.9.10

$\mathcal{D}_X$-algebras 2.3.1
$\mathcal{D}$-algebra point 2.3.1
$\mathcal{D}$-modules: left and right 2.1.1, functoriality 2.1.2, induced 2.1.8
$\mathcal{D}$-modules: quasi-induced 2.1.11, maximal constant quotient 2.1.12
$\mathcal{D}$-modules: topology at a point x 2.1.13, universal 2.9.9
$\mathcal{D}$-modules on $\mathcal{R}(X)$, left 3.4.2
$\mathcal{D}$-modules on $X^{\mathcal{S}}$, right 3.4.10
de Rham-Chevalley complex of a Lie* algebroid, inner 1.4.14
de Rham complex of a $\mathcal{D}$-module 2.1.7
de Rham homology 2.1.12, cohomology 2.1.7
$\mathcal{D}_X$-scheme 2.3.1, formally smooth, smooth, very smooth 2.3.15
$\mathcal{D}_X$-scheme: the global space of horizontal sections 2.4.1
$\mathcal{D}_X$-scheme: the local space of horizontal sections 2.4.8
Dolbeault algebras 4.1.3

Dolbeault $\mathcal{D}_{\mathcal{R}(X)}$-algebra 4.2.7
Dolbeault resolutions 4.1.4
Dolbeault-style algebra 4.1.4
Dolbeault-style $\mathcal{D}_{\mathcal{R}(X)}$-algebra 4.2.16
duality 1.3.11, 2.2.16
duality for de Rham cohomology, global 2.2.17, local 2.7.10

elliptic morphism, Lie* algebroid 2.5.22, coisson algebra 2.6.6
enveloping algebra of an operadic algebra 1.2.16, of a Lie algebroid 2.9.2
enveloping BV algebra of a BV algebroid 4.1.8. 4.1.9
enveloping chiral algebra of a chiral Lie* algebroid 3.9.11
Euler-Lagrange equations 2.3.20

factorization algebras 3.4.1, $\mathcal{D}$-module setting 3.4.4, truncated 3.4.13
factorization algebras: canonical $\mathcal{D}$-module structure 3.4.7, commutative 3.4.20
factorization algebra freely generated by (N, P) 3.4.14
factorization B-modules 3.4.18
factorization structure 3.4.4
filtration on a chiral algebra, commutative, unital 3.3.12
flabby complex of sheaves 4.2.2
formal groupoid 1.4.15
formally smooth/étale morphism of $\mathcal{D}_X$-schemes 2.3.16
Fourier-Mukai trnasform 4.9.9

$(\mathfrak{g}, K)$-modules 2.9.7, chiral structure 3.1.16
$(\mathfrak{g}, K)$-structure 2.9.8
Gelfand-Dikii coisson algebra 2.6.8
Gelfand-Kazhdan structure 2.9.9
group action on a chiral algebra 3.4.17

hamiltonian reduction 1.4.19
handsome complexes of !-sheaves on $X^{\mathcal{S}}$ 4.2.2
Harish-Chandra pair, module 2.9.7
Heisenberg Lie* algebra 2.5.9
Heisenberg group 3.10.13
homotopically $\mathcal{O}_X$- and $\mathcal{D}_X$-flat complexes 2.1.1
homotopy unit commutative algebra 4.1.14
homotopy unital commutative algebra 4.1.14, BV algebra 4.1.15
Hopf chiral algebras 3.4.16

ind-scheme 2.4.1
induced modules 3.7.15, 3.9.24
inner Hom, inner P objects 1.2.1, in augmented sense 1.2.7

jet scheme 2.3.2–2.3.3

Kac-Moody extension 2.5.9
Kashiwara's lemma 2.1.3
Knizhnik-Zamolodchikov (KZ) equations 4.4.6

lattices, c- and d- 2.7.7
Lie* algebras and modules 1.4.4, $\mathcal{D}$-module setting 2.5.3–2.5.4
Lie algebroid 2.9.1

Lie* algebroid 1.4.11, $\mathcal{D}$-module setting 2.5.16, elliptic 2.6.6
Lie coalgebroid 1.4.14

matrix $*$ algebra 1.4.2
middle de Rham cohomology sheaf h 2.1.6
Miura torsor 2.8.17
module operads 1.2.11
modules over operadic algebras 1.2.13
morphisms of DG $\mathcal{D}_X$-algebras: semi-free, elementary 4.3.7
multijet scheme 3.4.21
mutually commuting morphisms of chiral algebras 3.4.15

n-coisson algebra 1.4.18
n-Poisson algebra 1.4.18
nice complexes of !-sheaves on $X^{\mathcal{S}}$ 4.2.1
non-degenerate pairs $(\mathfrak{F}, \nabla^h)$ 2.5.23
normally ordered tensor product 3.6.1

$\mathcal{O}$-modules on $\mathcal{R}(X)$ 3.4.2
odd coisson algebra 1.4.18
odd Poisson algebra 1.4.18
ope algebra, associative, commutative 3.5.9
oper 2.6.8
operad 1.1.4, of Jacobi type 3.7.1
operadic algebra 1.1.6(iii), augmented 1.2.8
operator product expansion 3.5.8

perfect BV algebra 4.1.18
perfect commutative DG algebra 4.1.17
perfect complexes 4.1.16
Poincaré-Birkhoff-Witt theorem for Lie* algebras 3.7.14, twisted case 3.7.20
PBW theorem for usual Lie algebroids 2.9.2, 3.9.12
PBW theorem for chiral Lie algebroids 3.9.11
polydifferential $*$ operations 1.4.8, $\mathcal{D}$-module setting 2.3.12
pre-factorization algebra 3.4.14
projective connections 2.5.10
pseudo-tensor category 1.1.1
pseudo-tensor category: $\mathcal{A}$-, k-, additive, abelian 1.1.7
pseudo-tensor functors, adjointness 1.1.5
pseudo-tensor k-category 1.1.7
pseudo-tensor product 1.1.3
pseudo-tensor structure 1.1.2
pseudo-tensor subcategory, full 1.1.6(i)

quantization of a coisson algebra 3.3.11, mod t^2 3.9.10
quasi-factorization algebra 3.4.14

Ran's space 3.4.1
reasonable topological algebra 2.4.8
representable pseudo-tensor structure 1.1.3
resolutions of commutative $\mathcal{D}_X$-algebras 4.3.7
rigidification of a Lie* algebroid 1.4.13, of a Lie algebroid 2.9.1

rigidified B-extension 3.9.8

$\mathcal{A}$-structure 1.2.11
$*$ algebras 1.4.1, $\mathcal{D}$-module setting 2.5.1
$*$ pairing, non-degenerate 1.4.2
$*$ operations for $\mathcal{D}$-modules 2.2.3, induced case 2.2.4(i)
Schouten-Nijenhuis bracket 1.4.18
semi-free $\mathcal{D}_X$-algebras 4.3.7, 4.6.1
semi-free modules 4.1.5, 4.6.3
smooth $\mathcal{D}_X$-algebras 2.3.15
special $\mathcal{D}_{X^I}$-module 3.1.6
stress-energy tensor 3.7.25
Sugawara's construction 3.7.25
super complex 1.1.16
super conventions 1.1.16

I-topology 2.1.17
Ξ-topologies: Ξ_x- 2.1.13, Ξ_x^{Lie} 2.5.12, $\Xi_x^{R\mathcal{L}}$ 2.5.18, Ξ_x^{sp} 2.7.11
Ξ-topologies: Ξ_x^{cois} 2.6.3, Ξ_x^{as} 3.6.4
tangent Lie* algebroid 1.4.16, for a $\mathcal{D}$-scheme: Remark (ii) in 2.3.12, 2.3.15
Tate extension: $\mathcal{D}$-module setting 2.7.2, 2.7.3, on a $\mathcal{D}_X$-scheme 2.7.6
Tate extension: linear algebra setting 2.7.8, chiral approach 3.8.5
Tate extension of a Lie$*$ algebra 2.8.15, 3.8.7
Tate structure on a vector $\mathcal{D}$-bundle 2.8.1
Tate vector space, compact, discrete 2.7.7
tensor product of chiral algebras 3.4.15
tensor product of pseudo-tensor categories 1.1.9
topological associative algebra 3.6.1
topological commutative algebra 2.4.1
topological Lie algebroid 2.5.18
transversal quotients 1.3.1
twists of chiral algebras 3.4.17

unit object in a compound tensor category 1.3.16, strong 1.3.17

vector $\mathcal{D}$-bundles 2.1.5, on a $\mathcal{D}_X$-scheme 2.3.10
vector $\mathcal{D}_X$-scheme 2.3.19
vertex operator 3.5.14
very smooth $\mathcal{D}_X$-algebras 2.3.15
Virasoro extension 2.5.10
Virasoro vector 3.7.25

W-algebras 3.8.16
Weyl algebra, chiral and coisson 3.8.1, linear algebra version 3.8.17
Wick algebra 3.6.11, global 3.6.20

θ-datum 3.10.3

Notation

$\langle A \rangle$ 2.4.1, 4.2.16
A_x^{as} 2.4.8, 3.6.2, 3.6.4, 3.6.13

A^{as} 3.6.14
A^{Lie} 3.3.3
$A(P)$ 3.4.17
A^w_x 3.6.11
$A^w(X)$ 3.6.20
$B(M)$ 3.4.18
$\mathcal{BV}$, $\mathcal{BV}_u$, $\bar{\mathcal{B}}\mathcal{V}$ 4.1.6
$C(A)$ 3.4.11
$C(B, A)$ 4.4.9
$C^{ch}(X, A)$ 4.2.11
$\tilde{C}^{ch}(X, A)_{\mathcal{Q}}$, $C^{ch}(X, A)_{\mathcal{PQ}}$ 4.2.12
$C^{ch}(X, A, \{M_s\})$, $C^{ch}(X, A, \{M_s\})_{\mathcal{PQ}}$, $\mathcal{C}^{ch}(X, A, \{M_s\})_{\mathcal{PQ}}$ 4.2.19
$\mathcal{C}^{ch}(X, A, M)_{\mathcal{PQ}}$, $\mathcal{C}^{ch}(X, A, M_I)_{\mathcal{PQ}}$ 4.2.19
$C^{ch}(X, B, A)_{\mathcal{PQ}}$ 4.4.9
$\mathcal{CA}(X)$ 3.3.3
$\mathcal{CA}(X)^{F}$ 3.4.16
$\mathcal{C}^{\ell}_R(\mathcal{L}, \cdot)$, $\mathcal{C}^r_R(\mathcal{L}, \cdot)$, $\mathcal{C}_R(\mathcal{L})$ 2.9.1 $\mathcal{C}^{pois}(R)$ 2.9.1
$C\mathcal{M}(\mathcal{R}(X))$ 4.2.6
$\mathcal{C}^{cois}(R)$ 1.4.18
$\mathcal{C}omu$ 1.4.6, 4.1.6
$\mathcal{C}omu_{\mathcal{D}}(X)$ 2.3.1
$\Delta^{(\mathcal{S})}_*$ 3.4.10
$\mathcal{D}_{DR}$ 3.9.18
$D\mathcal{M}(\mathcal{R}(X))$ 4.2.6
$\mathcal{E}nd^*(V)$ 2.2.15, 2.5.6(a)
$\mathcal{E}xp(X)$ 3.4.1
$\mathfrak{gl}(V)$ 2.5.6(a)
$\mathfrak{gl}(V)^\flat$ 2.7.4, 2.7.8
$H^{ch}_\cdot(X, A)$ 4.2.11
$H^\cdot_{DR}(X, M)$ 2.1.7
$H^{DR}_\cdot(X, M)$ 2.1.12
$\mathcal{H}o\mathcal{C}$ 4.1.7
$\mathcal{I}Z$ 2.3.2
$L_\dagger$ 1.1.16
$L^\diamond$ 1.4.16, 2.9.1
M^ℓ, N^r 2.1.1
M_{const} 2.1.12
M_X 2.8.17
$\mathcal{M}^r(X)$, $\mathcal{M}^\ell(X)$, $\mathcal{M}(X)$ 2.1.1
$\mathcal{M}(X)^{*!}$ 2.2.6
$\mathcal{M}(X)^{ch}$ 3.1.2,
$\mathcal{M}(X)^{cl}$ 3.2.5
$\mathcal{M}(X, R^\ell)$ 2.3.5
$\mathcal{M}(X, A)$, $\mathcal{M}(X, A)^{ch}$ 3.3.4
$\mathcal{M}(X^{\mathcal{S}})$ 3.4.10
$\mathcal{M}^\ell(\mathcal{R}(X))$ 3.4.2
O_I 3.5.8
$\mathcal{P}^{ch}(A, L)$ 3.9.6

$\mathcal{P}^{ch}(\mathcal{L})$ 3.9.6
$\mathcal{P}^{cl}(\mathcal{L})$ 2.8.2
$\mathcal{P}(F)$ 3.4.16
$Q(I)$ 1.3.1
$Q(I, m)$ 1.4.27
$\mathcal{R}(X)$ 3.4.1
$\mathcal{S}$ 1.1.1
$\hat{\mathcal{S}}$ 1.2.4
$\mathcal{S}ch_{\mathcal{D}}(X)$ 2.3.1
Spec $\langle R, \mathcal{L}\rangle$ 2.5.18
Spf Q 2.4.1
$\mathcal{T}ate(V)$, $\mathcal{T}ate(\mathcal{Y})$ 2.8.1
$U^{(I)}$ 3.1.1
$U^{[J/I]}$ 3.4.4
$U(L)$ 3.7.1
$\mathcal{V}(a)$ 3.6.14
$Z(A)$ 3.3.7
λ_I 2.2.2, 3.1.4
$\otimes^*$, $\otimes^{ch}$ 3.4.10
$\vec{\otimes}$ 3.6.1
Ξ_x^{as} 2.4.8, 3.6.4
Ξ_x 2.1.13
Ξ_x^{Lie} 2.5.12
Ξ_x^R 2.4.11
$\Xi_x^{R\mathcal{L}}$ 2.5.19
$1^{ch} = 1_A^{ch}$ 4.2.16.

This page intentionally left blank

Titles in This Series

51 **Alexander Beilinson and Vladimir Drinfeld,** Chiral algebras, 2004
50 **E. B. Dynkin,** Diffusions, superdiffusions and partial differential equations, 2002
49 **Vladimir V. Chepyzhov and Mark I. Vishik,** Attractors for equations of mathematical physics, 2002
48 **Yoav Benyamini and Joram Lindenstrauss,** Geometric nonlinear functional analysis, Volume 1, 2000
47 **Yuri I. Manin,** Frobenius manifolds, quantum cohomology, and moduli spaces, 1999
46 **J. Bourgain,** Global solutions of nonlinear Schrödinger equations, 1999
45 **Nicholas M. Katz and Peter Sarnak,** Random matrices, Frobenius eigenvalues, and monodromy, 1999
44 **Max-Albert Knus, Alexander Merkurjev, and Markus Rost,** The book of involutions, 1998
43 **Luis A. Caffarelli and Xavier Cabré,** Fully nonlinear elliptic equations, 1995
42 **Victor Guillemin and Shlomo Sternberg,** Variations on a theme by Kepler, 1990
41 **Alfred Tarski and Steven Givant,** A formalization of set theory without variables, 1987
40 **R. H. Bing,** The geometric topology of 3-manifolds, 1983
39 **N. Jacobson,** Structure and representations of Jordan algebras, 1968
38 **O. Ore,** Theory of graphs, 1962
37 **N. Jacobson,** Structure of rings, 1956
36 **W. H. Gottschalk and G. A. Hedlund,** Topological dynamics, 1955
35 **A. C. Schaeffer and D. C. Spencer,** Coefficient regions for Schlicht functions, 1950
34 **J. L. Walsh,** The location of critical points of analytic and harmonic functions, 1950
33 **J. F. Ritt,** Differential algebra, 1950
32 **R. L. Wilder,** Topology of manifolds, 1949
31 **E. Hille and R. S. Phillips,** Functional analysis and semigroups, 1957
30 **T. Radó,** Length and area, 1948
29 **A. Weil,** Foundations of algebraic geometry, 1946
28 **G. T. Whyburn,** Analytic topology, 1942
27 **S. Lefschetz,** Algebraic topology, 1942
26 **N. Levinson,** Gap and density theorems, 1940
25 **Garrett Birkhoff,** Lattice theory, 1940
24 **A. A. Albert,** Structure of algebras, 1939
23 **G. Szegö,** Orthogonal polynomials, 1939
22 **C. N. Moore,** Summable series and convergence factors, 1938
21 **J. M. Thomas,** Differential systems, 1937
20 **J. L. Walsh,** Interpolation and approximation by rational functions in the complex domain, 1935
19 **R. E. A. C. Paley and N. Wiener,** Fourier transforms in the complex domain, 1934
18 **M. Morse,** The calculus of variations in the large, 1934
17 **J. M. Wedderburn,** Lectures on matrices, 1934
16 **G. A. Bliss,** Algebraic functions, 1933
15 **M. H. Stone,** Linear transformations in Hilbert space and their applications to analysis, 1932
14 **J. F. Ritt,** Differential equations from the algebraic standpoint, 1932
13 **R. L. Moore,** Foundations of point set theory, 1932
12 **S. Lefschetz,** Topology, 1930
11 **D. Jackson,** The theory of approximation, 1930

TITLES IN THIS SERIES

10 **A. B. Coble,** Algebraic geometry and theta functions, 1929

9 **G. D. Birkhoff,** Dynamical systems, 1927

8 **L. P. Eisenhart,** Non-Riemannian geometry, 1927

7 **E. T. Bell,** Algebraic arithmetic, 1927

6 **G. C. Evans,** The logarithmic potential, discontinuous Dirichlet and Neumann problems, 1927

5.1 **G. C. Evans,** Functionals and their applications; selected topics, including integral equations, 1918

5.2 **O. Veblen,** Analysis situs, 1922

4 **L. E. Dickson,** On invariants and the theory of numbers

W. F. Osgood, Topics in the theory of functions of several complex variables, 1914

3.1 **G. A. Bliss,** Fundamental existence theorems, 1913

3.2 **E. Kasner,** Differential-geometric aspects of dynamics, 1913

2 **E. H. Moore,** Introduction to a form of general analysis

M. Mason, Selected topics in the theory of boundary value problems of differential equations

E. J. Wilczyński, Projective differential geometry, 1910

1 **H. S. White,** Linear systems of curves on algebraic surfaces

F. S. Woods, Forms on noneuclidean space

E. B. Van Vleck, Selected topics in the theory of divergent series and of continued fractions, 1905

Chiral algebras form the primary algebraic structure of modern conformal field theory. Each chiral algebra lives on an algebraic curve, and in the special case where this curve is the affine line, chiral algebras invariant under translations are the same as well-known and widely used vertex algebras.

The exposition of this book covers the following topics:

- the "classical" counterpart of the theory, which is an algebraic theory of non-linear differential equations and their symmetries;
- the local aspects of the theory of chiral algebras, including the study of some basic examples, such as the chiral algebras of differential operators;
- the formalism of chiral homology treating "the space of conformal blocks" of the conformal field theory, which is a "quantum" counterpart of the space of the global solutions of a differential equation.

The book will be of interest to researchers working in algebraic geometry and its applications to mathematical physics and representation theory.

ISBN 0-8218-3528-9

9 780821 835289

COLL/51