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Part 1: Classical Theory

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To my grandchildren and their parents

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Contents

Preface to Part 1

For an overview of the subject of these volumes, see Section 1.1.

Here is how this book came to be. I've worked in part on the spectral theory of Schrödinger operators for all of my career. In the late 1970s, it became clear, especially for those of us trying to understand localization in random systems, that it was technically easier to study the discrete Schrödinger operator, which in one dimension reads

$$(hu)_n = u_{n+1} + u_{n-1} + v_n u_n \tag{1}$$

In about 1985, I met Mourad Ismail who urged me to look at the literature on orthogonal polynomials — and I should have paid more attention to his advice.

At least I did look a little at orthogonal polynomials on the real line (OPRL) and learned that theory was essentially the study of operators of the form

$$(Ju)_n = a_{n+1}u_{n+1} + b_{n+1}u_n + a_n u_{n-1} \tag{2}$$

where $n = 0, 1, 2, \dots$ and $(a_1, a_2, \dots)$ and $(b_1, b_2, \dots)$ are real with $a_n > 0$ for $n \geq 1$ and $a_0 = 0$. The discrete Schrödinger case is $a_n \equiv 1$.

One thing I realized early is that the Jacobi matrix (2) is the natural arena for inverse theory and that it seemed difficult to clarify what spectral measures corresponded to $a_n \equiv 1$ with b_n arbitrary. As inverse problems became an important component of my research, I worked on what was essentially OPRL, even going so far as writing a long review article [**974**] on moment problems that could be thought of as a primer on OPRL. But I didn't systematically look at literature from the OP community.

My first real exposure to orthogonal polynomials on the unit circle (OPUC) came in connection with my role as an editor of Communications in Mathematical Physics (CMP), in particular, with a submission of Golinskii and Nevai [**464**]. My section in CMP had accepted an earlier paper on the subject by Geronimo and Johnson [**398**], but at the time I hadn't paid close attention to it. I personally knew both those authors and the paper seemed to be studying some kind of general difference equation, so I sent it to an appropriate referee and followed the recommendation. The Golinskii-Nevai paper was different. I knew of Nevai's reputation but didn't know either author. The paper was clearly about orthogonal polynomials and I was reluctant, given CMP's perpetual page crunch, to open up the journal to a subject that I worried might be disconnected to either physics or the areas that we normally cover. So I spent some time carefully reading the introduction, thinking about the issues, and skimming the rest. I realized this paper was one on spectral theory related to many papers in CMP, so I was comfortable sending it out to a referee and following a positive recommendation.

The next aspects of the story involve my own interests and those of my research group. A major theme of our work in the 1990s concerned the spectral behavior of discrete Schrödinger operators (1) and the continuum analog in one dimension:

$$(Hu)(x) = -u''(x) + V(x)u(x) \tag{3}$$

It has been known since the 1930s that $V \in L^1$ or

$$|V(x)| \leq C(1 + |x|)^{-a} \tag{4}$$

with $a > 1$ (equivalently, $a_n \equiv 1$ and $b_n \in \ell^1$ or $|b_n| \leq C(1 + |n|)^{-a}$ for the discrete case) is a natural dividing line. If $V \in L^1$, the H given by (3) has purely absolutely continuous spectrum at positive energies and scattering states.

In my own work, it became clear $a = \frac{1}{2}$ was also a natural dividing line. This appeared first in work I did on decaying random potentials [**965**]; there was dense point spectrum if $a < \frac{1}{2}$ and subsequent work [**265, 268, 649**] showed a.c. spectrum if $a > \frac{1}{2}$. I saw it again when I proved that Baire generic potentials decaying slower than $n^{-1/2}$ have purely singular continuous spectrum [**969**].

I then emphasized that the case $1 > a > \frac{1}{2}$ was open, and Kiselev, in his thesis done under me [**637**], and then Christ-Kiselev-Remling [**201**] filled in the region showing there was always a.c. spectrum in this regime. Kiselev-Last-Simon [**639**], motivated by this work, conjectured that the right borderline condition for a.c. spectrum was L^2. Rowan Killip, a graduate student I was supervising at the time, got interested in this question and he asked Percy Deift about it while Percy was visiting. Percy noticed that the KdV sum rule involved L^2 precisely, suggesting the sum rule could be relevant. They then implemented this idea [**248**], not by proving the sum rule in this generality but using the sum rule for nice V's and taking suitable limits.

At the same time, I was encouraging workers in the field to find examples of mixed spectrum in this $1 > a > \frac{1}{2}$ region. It was known one could have point spectrum mixed in the a.c. spectrum [**795, 973**], but the issue was singular continuous spectrum and what kind. In honor of the change of millennium, I produced a list of open problems in 1999 and high in the list was mixed singular continuous spectrum with power decay.

In January 2001, I learned of a paper by a young Russian, Sergey Denisov [**271**]. In it, he showed there existed L^2 potentials V with essentially arbitrary singular part on $[0, E_0]$ for $E_0 < \infty$. Technically, this didn't solve the problem I stated since I asked for a power bound (this was later done by Kiselev [**638**]), but I regarded it as essentially a solution. Nick Makarov and I were so impressed by this result that we arranged a postdoc position for Denisov, of which more later.

That summer, Rowan Killip visited Caltech and we tried to understand what made Denisov's proof work. We weren't able to understand the technical details of his proof because he used Krein systems, which we had never seen and for which the existing literature was spotty. But we understood that he was using some kind of inverse theory construction. Initially, this puzzled me. For years, I'd been told that one should use inverse spectral theory to construct mixed spectra, and my response was to point out that the Marchenko theory wouldn't work in such cases and that the Gel'fand-Levitan theory provided no information about asymptotics at infinity.

Nevertheless, Denisov proved his potential was in L^2. At first, we thought the magic must be in Krein systems, but then we realized he was getting the L^2

property from a sum rule. So we focused on using sum rules going beyond their use in Deift-Killip. In connection with the earlier work and its followup, Killip had learned of the sum rules of Case [**190**] and various review articles of Nevai [**814**]. With these as background, we were able to handle discrete eigenvalues in the Jacobi case and prove a general sum rule that described exactly which spectral measures corresponded to Jacobi matrices with $\sum_n (a_n - 1)^2 + b_n^2 < \infty$ [**633**]. Along the way, I learned that due to work of Szegő and Shohat, this problem had been solved in the early part of the twentieth century under the additional strong condition that $\text{supp}(d\mu) \subset [-2, 2]$. But I put aside understanding how they had done so while we worked out some of the details of our work.

In the fall of 2001, Denisov showed up at Caltech and, with my encouragement, gave a course on Krein systems. Since OPUC are a discrete analog that motivated Krein, Denisov spent the first few weeks presenting their theory, following in part the chapter in Akhiezer's book [**17**] (I had, of course, read this book when I was writing my article on the moment problem for $\mathbb{R}$ but I had skipped this chapter as irrelevant to me!). I was struck by the beauty of the subject and began to try to understand it better. I quickly found the proof I give of the Szegő recursion, wondering why Akhiezer's proof was so complex. I eventually found this proof in Atkinson [**60**] and Landau [**672**], but it was unknown enough that I was able to surprise one expert with it.

At the same time, in connection with my work with Killip, I realized what we'd found was an analog of Szegő's theorem, which I began to study. I also realized that OPUC was an analog of Schrödinger operators, a virtual playground for spectral theorists. By February of 2002, I was convinced that the sensible way to carry over all the Schrödinger operators to OPUC was in a single article, rather than lots of short ones. I also realized though that OPUC had no summary since Geronimus' book forty years before, and even that had focused on only part of the then known subject. The idea began to form of writing two review articles on OPUC, each I estimated to be 120–150 pages long, the first reviewing the classical theory and the second on spectral theory.

As I began planning the articles, I began to wonder who had first written about Szegő's theorem as a sum rule. I asked Paul Nevai, who replied that he didn't know — but it was an interesting question and so he sent out an e-mail blast to about a dozen experts. One from the Russian tradition replied that he wasn't sure and that he hadn't seen the paper, but he'd heard it might be " ..." and he gave the reference to Verblunsky [**1067**]. I got hold of this and the earlier [**1066**] and read them with fascination. It was hard going, but as I absorbed the papers, it became clear that there was an enormous number of ideas in these papers that had become important, but then forgotten and later rediscovered! So added to the agenda was making sure Verblunsky got the credit so long denied him!

I began an e-mail correspondence with Golinskii and Nevai about their paper, and in discussing the GGT representation, Golinskii told me about the paper of Cantero, Moral, and Velázquez [**181**] written in 2000 but only published in 2003. He told me the basic idea that orthonormalizing $\{1, z, z^{-1}, z^2, z^{-2}, \dots\}$ leads to a basis expressible in terms of the usual OPUC and a five-diagonal matrix expressible in terms of the Verblunsky coefficients. Within a few hours, I had rederived the formulae and then over a period of a few weeks, Golinskii and I, via e-mail, worked out the details for general spectral theory that appear as Section 4.3. My project had become larger as I realized that presenting the CMV matrix and exploiting it was also a part of the mix.

As the summer of 2002 finished, I realized I still had a lot to do and the idea of two 150-page articles wouldn't suffice. A book was needed, and when Sergei Gelfand offered to publish it in the same series where Szegő's book [**1026**] appeared, the temptation to accept without consulting other publishers was so great.

Slowly the project took on extra elements that caused it to grow. I had included background sections on Schur functions and on OPRL to make the book accessible to spectral theorists, but Doron Lubinsky convinced me to add a background section on operator theory to make it accessible to workers with a background in OP. The desire to reach both groups produced background material in many places that, I hope, also makes it accessible to graduate students.

Other factors caused manuscript growth. I realized that there were results like the Bello-López extension of Rakhmanov's theorem that I needed to include if I wanted the book to be as comprehensive as I desired. Percy Deift kept pushing me to include all the main proofs of the strong Szegő theorem. Once the book had grown as it had, I found it appropriate to add a chapter on uses of OPUC in the theory of OPRL.

In this way, two 150-page articles grew to one book grew to two volumes.

The table of contents and list of notation cover both volumes. To save trees, the author index, subject index, and the bibliography in this volume are only for this volume, although there is a complete bibliography and complete indices in Part 2. For consistent numbering between volumes, the bibliography in Part 1 has gaps in its numbering. I considered more elegant reference referral with author and year but decided against it. When there are four of five "S" references, it is easy to locate [ST97], but when there are over a hundred "S" references, it is difficult. In the end, functionality won over elegance.

I warn the reader of a personal quirk. I'm told that proper usage requires the addition of a period in a sentence that ends with a set-out equation. But I find extra dots in such equations confusing, so I don't use punctuation in set-out formulae, even if proper grammar says they should be there.

I doubt that these books will have the four editions that Szegő [**1026**] did, but it seems likely there will be later editions. That means I especially welcome comments, corrections, missing topics and references, and information on new papers. For the latter, I much prefer a link to an online archive rather than that you send me attachments. Addenda and major corrections will be posted at `http://www.math.caltech.edu/opuc.html`. My email is `bsimon@caltech.edu`.

One person who learned about this book wondered how I could even TeX so many pages in "only two years." The secret is that I didn't. I wrote out long hand and Cherie Galvez, my superb secretary, TeXed the manuscript. I am indebted for her hard work and competence. Rowan Killip provided advice to her on TeX technical issues, and I'm grateful for his help on this.

Many mathematicians provided useful input, but I should begin with four deserving special mention. Rowan Killip provided insight on many of the issues I studied here. Percy Deift was merciless in pushing me to include material that worked out well and has added to the usefulness of the book. Fritz Gesztesy taught me much about periodic Jacobi matrices. Most of all, Leonid Golinskii spent a huge amount of time going through the manuscript. He not only found typos, but places where the arguments weren't quite right.

I also want to thank Richard Askey, Alexei Borodin, Albrecht Böttcher, Daniel Bump, Alicia Cachafeiro, Danny Calegari, Tiberiu Constantinescu, David Damanik, Sergey Denisov, Nathan Dunfield, Harry Dym, Jeff Geronimo, Dimitri Gioev, Peter Harremoës, Mourad Ismail, Alain Joye, Thomas Kailath, Victor Katsnelson, Sergey Khrushchev, Peter Kuchment, Ari Laptev, Yoram Last, Daniel Lenz, Guillermo López Lagomasino, Doron Lubinsky, Russell Lyons, Alphonse Magnus, Francisco Marcellán, Andrei Martínez-Finkelshtein, Peter Miller, Irina Nenciu, Paul Nevai, Olav Njåstad, Franz Peherstorfer, Yuri Safarov, Mihai Stoiciu, Gunter Stolz, Alexander Teplyaev, John Toland, Vilmos Totik, Doron Zeilberger, and Andrej Zlatoš for useful input.

Finally, my wife Martha, whose love makes it all easier.

Barry Simon
Los Angeles, August 2004

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Notation

This list of notation is broken into the Greek and Roman alphabets and then nonalphabetic. Since there are only a finite number of letters, some symbols are used in different ways!

Greek Alphabet in the following order:
$\alpha, \beta, \gamma, \Gamma, \delta, \Delta, \varepsilon, \zeta, \eta, \theta, \Theta, \kappa, \lambda, \Lambda, \mu, \nu, \xi, \Xi, \pi, \Pi, \rho, \sigma, \Sigma, \tau, \Upsilon, \varphi, \Phi, \chi, \psi, \Psi, \Omega$

α_n	Verblunsky coefficients; see (1.1.8)
$\beta_j^\pm$	solution of $\beta + \beta^{-1} = E_j^\pm$ with $\lvert\beta\rvert > 1$; see (13.8.21)
$\beta(z_0)$	Jitomirskaya-Last ratio; see (10.10.13)
$d\beta_{2n,k}(\theta)$	measure in varying ratio asymptotics; see (9.8.7)
$\gamma(z)$	Lyapunov exponent, $\gamma(z) = \lim_{n\to\infty} \frac{1}{n} \log \lVert T_n(z)\rVert$; see (10.5.14)
$\Gamma_{a,\lambda}$	essential spectrum for single arc, $\Gamma_{a,\lambda} = \{z \in \partial\mathbb{D} \mid \lvert\arg(\lambda z)\rvert \geq 2\arcsin(a)\}$; see (9.9.2)
$\Gamma_e(x)$	function of Damanik-Killip, $\Gamma_e(x) = \frac{1}{2}[u(x) - v(x)]$; see the Notes to Section 13.1
$\Gamma_o(x)$	function of Damanik-Killip, $\Gamma_o(x) = -\frac{1}{2}[u(x) + v(x)]$; see the Notes to Section 13.1
$\Gamma(\tilde{B}_j)$	contour surrounding $\tilde{B}_j$; see (11.7.16)
$\Gamma(G_j)$	contour surrounding G_j; see (11.7.16)
$\Gamma(z)$	analytic function whose real part is the Lyapunov exponent, $\gamma(z)$; see (10.11.18)
$\delta_0 D$	relative Szegő function; see (2.9.6)
$\Delta(z)$	Freud function; see Theorem 2.2.14
$\Delta(z)$	discriminant for periodic Verblunsky coefficients; see (11.1.2)
$\zeta_\beta^{(n+1)}$	point mass approximations; see Theorem 2.2.12
$d\eta_n(z)$	varying measure; see (9.8.1)
$\theta_n(z_0)$	Prüfer angle for OPUC; see (10.12.1)
Θ_j	CMV building block, $\Theta_j = \begin{pmatrix} \bar\alpha_j & \rho_j \\ \rho_j & -\alpha_j \end{pmatrix}$; see (4.2.20)
κ_n	leading coefficient of $\varphi_n(z) = \kappa_n z^n + \cdots$, $\kappa_n = \lVert\Phi_n\rVert^{-1}$; see (1.5.1)
$\lambda_\infty(\zeta)$	Christoffel function, $\lambda_\infty(\zeta) = \inf\{\int \lvert\pi(e^{i\theta})\rvert^2 \, d\mu(\theta) \mid \pi$ a polynomial, $\pi(\zeta) = 1\}$; see (2.2.2)
$\lambda_j(A)$	eigenvalues of a compact operator A; see (1.4.27)
$\lambda_n(\zeta)$	approximate Christoffel function, $\lambda_n(\zeta) = \min\{\int \lvert\pi(e^{i\theta})\rvert^2 \, d\mu(\theta) \mid \deg \pi \leq n,\ \pi(\zeta) = 1\}$; see (2.2.1)
$\lambda_n(\zeta, d\mu; p)$	p-Christoffel function, $\lambda_n(\zeta, d\mu; p) = \min\{\int \lvert\pi(e^{i\theta})\rvert^p \, d\mu \mid \deg \pi \leq n,\ \pi(\zeta) = 1\}$; see (2.5.1)
$f^{(\lambda)}(z)$	Schur function of an Aleksandrov family; see Subsection 1.3.9

$F^{(\lambda)}(z)$	Carathéodory function of an Aleksandrov family; see (1.3.90)
Φ_n^λ	OPs of an Aleksandrov family; see (3.2.1)
$d\mu_\lambda$	measures of an Aleksandrov family; see (1.3.91)
Λ	unitary perturbation, $\Lambda : P\mathcal{H} \to P\mathcal{H}$, $V = U\Lambda P + U(1-P)$; see (4.5.10)
$\mu_n(A)$	singular values of an operator A, i.e., eigenvalues of $\lvert A\rvert$; see (1.4.33)
$d\mu_{\rm ac}$	absolutely continuous part of the measure $d\mu$, i.e., if $d\mu = w\frac{d\theta}{2\pi} + d\mu_{\rm s}$, then $d\mu_{\rm ac} = w\frac{d\theta}{2\pi}$; see Subsection 1.4.6
$d\mu_{\rm pp}$	pure point part of a measure, $d\mu_{\rm pp} = \sum_x \mu(\{x\})\delta_x$; see Subsection 1.4.6
$d\mu_{\rm s}(\theta)$	singular part of a measure, $d\mu_{\rm s} = d\mu - d\mu_{\rm ac}$; see Subsection 1.4.6
$d\mu_{\rm sc}$	singular continuous part of a measure, $d\mu_{\rm sc} = d\mu_{\rm s} - d\mu_{\rm pp}$; see Subsection 1.4.6
$d\mu_N^{\rm GI}(\theta)$	GI approximations to a measure; see (6.1.24)
$\nu(n)$	Beurling weight; see (5.1.38)
$d\nu$	density of zeros measure, $d\nu = \lim d\nu_n$ if it exists; see Section 8.1
$d\nu_n$	finite density of zeros, $d\nu_n = \frac{1}{n}\sum_{j=1}^n \delta_{z_j}$ where z_j are the zeros of $\Phi_n(z)$ counting multiplicity; see Example 1.7.17
$d\nu_{n,\ell}(\theta)$	approximations used in Khrushchev's theory, $d\nu_{n,\ell}(\theta) = \frac{\lvert\varphi_n(e^{i\theta})\rvert^2}{\lvert\varphi_{n+\ell}(e^{i\theta})\rvert^2}\frac{d\theta}{2\pi}$; see (9.3.14)
$\Xi_n(z)$	vector recursion for OPUC; see (2.2.44)
$\pi_g^\times$	extended abelian periods; see (11.8.2)
π_n	decaying CMV function, $\pi_n = \Upsilon_n + F(z)\chi_n$; see (4.4.5)
$\Pi_k(\tilde B_j)$	k-th periodic associated to band, $\tilde B_j$; see (11.7.16)
$\Pi_k(G_j)$	k-th periodic associated to gap, G_j; see (11.7.16)
ρ_j	$\rho_j = (1-\lvert\alpha_j\rvert^2)^{1/2}$; see (1.5.21)
$\rho(B)$	resolvent set of an operator B, $\rho(B) = \{z \mid (B-z)^{-1} \text{ exists}\}$; see Subsection 1.4.2
$\rho(T_n)$	Kotani-Ushiroya ratio; see (10.5.83)
$\sigma_{\rm d}(A)$	discrete spectrum of an operator A; see Subsection 1.4.6
$\sigma_{\rm ess}(A)$	essential spectrum of an operator A, $\sigma_{\rm ess}(A) = \sigma(A)\backslash\sigma_{\rm d}(A)$; see Subsection 1.4.6
σ_n	half of CMV basis, $\sigma_n = \chi_{2n} = z^{-n}\varphi_{2n}^*$; see (4.2.8)
$\sigma(B)$	spectrum of an operator B, $\sigma(B) = \mathbb{C}\backslash\rho(B)$; see (1.4.2)
$d\sigma_{2n}(\theta)$	measure used in studying varying measures; see (9.8.4)
Σ	used for essential spectrum of subshift; see Theorem 12.8.1
τ_n	half of CMV basis, $\tau_n = \chi_{2n-1} = z^{-n+1}\varphi_{2n-1}$; see (4.2.7)
Υ_n	second kind CMV basis; see (4.4.3)
φ_n	normalized OPUC; see (1.5.1)
$\varphi_n^\pm$	Jacobi solutions at ± 2; see (13.1.32)
φ_n^λ	OPUC for Aleksandrov measures; see (3.2.1)
$\tilde\varphi_p^*$	symmetrized OPUC, $z^{-p/2}\varphi_p^*$; see (11.3.3)
Φ_n	monic OPUC; see (1.5.3)
$\Phi_n^{\rm R}$	right polynomials in the theory of matrix-valued measures; see (2.13.16)
$\Phi_n^{\rm L}$	left polynomials in the theory of matrix-valued measures; see (2.13.17)
$\Phi_n^*(z)$	dual of monic OPUC, $z^n\,\overline{\Phi_n(1/\bar z)}$; see (1.5.9)
$\chi_\alpha(g)$	group character; see Section 6.3
χ_n	CMV basis element; see (4.2.5)
$\chi_n^{(0)}$	free CMV basis element; see (4.2.4)

Ψ_n	second kind OPUC; see (3.2.3)
$\Omega^\pm$	wave operators; see (10.7.46) and (10.7.80)

Roman Alphabet

a_n	Jacobi parameter; see (1.2.13)
$a_n(x)$	continued fraction coefficients; see (11.9.1)
$a^\flat(z)$	normalized coefficient in quadratic equation for F; see (11.7.66)
A	support of the a.c. part; see (10.10.4)
A	set where $\gamma(E) = 0$ for subshifts; see Section 12.8
A_n	Wall polynomial; see (1.3.65)
$Av_n(f)$	ergodic average; see Theorem 10.5A.2
$\mathbb{A}$	alphabet for subshift; see Appendix 10.5
$\mathbb{A}_R$	annulus, $\{z \mid R^{-1} < \lvert z\rvert < R\}$; see (7.1.1)
$\mathfrak{A}$	Abel map; see (11.7.26)
$\mathfrak{A}_\times$	extended Abel map; see (11.8.5)
$\mathfrak{A}_\nu$	two-sided Beurling algebra, $\{a \mid \sum_n (1+\lvert n\rvert)^\nu \lvert a_n\rvert < \infty\}$; see (5.1.44)
$\mathfrak{A}_\nu^+$	positive Beurling algebra, $\{a \in \mathfrak{A}_\nu \mid a(n) = 0 \text{ for } n < 0\}$; see (5.1.44)
$\mathfrak{A}_\nu^-$	negative Beurling algebra, $\{a \in \mathfrak{A}_\nu \mid a(n) = 0 \text{ for } n > 0\}$; see (5.1.44)
b_n	Jacobi parameter; see (1.2.13)
$b_{n,\ell}$	OP ratio error; see (9.1.1)
$b_n(z, d\mu)$	inverse Schur iterates; see (9.2.14)
$b^\flat(z)$	normalized coefficient in quadratic equation for F; see (11.7.67)
B	set of z's in $\partial\mathbb{D}$ for which $\lim_{r\uparrow 1} F(rz)$ does not exist; see (10.10.5)
B	set for which $\limsup x_n(z) < \infty$ in theory of subshifts; see (12.8.3)
B_j	bands for periodic Verblunsky coefficients, see (11.1.6)
$\tilde{B}_j$	union of touching bands; see Section 11.7
B_n	Wall polynomials; see (1.3.66)
$B(f)$	ratio $D/\bar{D}$; see (6.2.40)
$B(\mu, \nu)$	bilinear potential; see (8.1.9)
$\mathcal{B}$	factor in CMV proof of Geronimus relations; see (13.2.16)
c_n	moments of a measure, $\int e^{-in\theta}\, d\mu(\theta)$; see (1.1.20)
c_n	OP half of solution of the Geronimo-Case equations; see (13.6.10)
$c_{n,\ell}$	OP ratio; see (9.1.13)
$c^\flat(z)$	normalized coefficient in quadratic equation for F; see (11.7.68)
$\text{cap}(K)$	logarithmic capacity of a compact set K; see Section 8.1
C	union of singular continuous supports over an Aleksandrov family; see (10.10.3)
C_n	normalized solution of Geronimo-Case equations; see (13.6.42)
$C_{i,o}^\pm$	the contours $\lvert z\rvert = 1$ on the Riemann surface $\mathcal{S}$, with $\pm$ indicating which sheet and i, o whether the contour runs inside or outside; see (11.7.12)
C_β	Cantor set with middle β-th removed; see Example 2.12.2
$C^{(\alpha)}$	combined Hausdorff dimension α support; see (10.10.6)
$C(f)$	multiplication operator whose Fourier transform action is convolution; see (6.2.10)
$C(z, w)$	complete Cauchy kernel, $\frac{w+z}{w-z}$; see (1.3.15)
$\mathbb{C}$	the complex numbers
$\mathbb{C}_+$	$\{z \in \mathbb{C} \mid \text{Im}\, z > 0\}$

$\mathcal{C}_{ij}(d\mu)$	CMV matrix; see (4.2.12)
$\tilde{\mathcal{C}}_{ij}(d\mu)$	alternate CMV matrix; see (4.2.13)
$d_H(A)$	Hausdorff dimension of a set A; see (2.12.8)
$d_{j,1}$	Taylor coefficients of $D(z)$; see (7.2.4)
$d_{j,-1}$	Taylor coefficients of $D(z)^{-1}$; see (7.1.5)
d_n	OP square root integral; see (9.1.14)
$\deg(f)$	degree of meromorphic function, i.e., the number of solutions of $\deg(f) = w$ for generic $w \in \mathbb{C}$; see (11.7.11)
$\det$	Fredholm determinant of a trace class operator; see (1.4.64)
$\det_2$	renormalized Fredholm determinant of a Hilbert-Schmidt operator; see (1.4.82)
$D_{\rm ac}^{-1}(e^{i\theta})$	the function in $L^2(\partial\mathbb{D}, d\mu)$, which is the boundary value of $D(z)^{-1}$, set to zero on a support for $d\mu_{\rm s}$; see (2.4.33)
$D_n(d\mu)$	Toeplitz determinant; see (1.3.12)
$D_n(f)$	Dirichlet approximation; see (2.12.32)
D_A	Dirac operator; see the Notes to Section 13.1
$D(z)$	Szegő function, $\exp(\int \frac{e^{i\theta}+z}{e^{i\theta}-z}\log(w(\theta))\frac{d\theta}{4\pi})$; see (2.4.2)
$D_\mu^\alpha(z_0)$	local infinitesimal Hausdorff dimension; see (10.8.28)
$\mathbb{D}$	unit disk, $\{z \mid \lvert z\rvert < 1\}$
$\mathbb{D}_R$	$\{z \mid \lvert z\rvert < R\}$
$\mathbb{D}^{\infty,c}$	set of Schur parameters; see Subsection 1.3.6
$\mathcal{D}(\{G_j\})$	Dirichlet data torus; see Section 11.4
$E_j^\pm$	eigenvalues of Jacobi matrices; see (13.8.21)
E_n	Coulomb energy; see (6.4.2)
$\mathbb{E}$	probability expectation
$\mathcal{E}$	extended CMV matrix; see (10.5.34)
$\mathcal{E}_0(J)$	energy term in C_0 sum rule, $\mathcal{E}_0(J) = \sum_{j,\pm}\log\lvert\beta_j^\pm(J)\rvert$; see (13.8.52)
$\mathcal{E}_q(\beta)$	periodized CMV matrix; see (11.2.4)
$\mathcal{E}(d\mu)$	Coulomb energy for potential theory $\mathcal{E}(d\mu) = \int \log\lvert z-w\rvert^{-1}\ d\mu(z)\,d\mu(w)$; see (8.1.7)
$f_+(z)$	used for Schur function when two-sided sequences are involved; see (10.11.24)
$f_-(z)$	left side Schur function; see (10.11.24) and Theorem 10.11.16
$f_a(z)$	Schur function for Geronimus polynomials, $f_a(z) = \frac{z-1+[(1-z)^2+4a^2z]^{1/2}}{2az}$; see (1.6.82) and (9.5.2)
$f_n(z)$	Schur iterates; see (1.3.37)
$f^{[n]}(z)$	Schur approximants; see (1.3.41)
$f(k)$	Jost function for Schrödinger operators; see (10.7.29)
$f(x,k)$	Jost solution for Schrödinger operators; see (10.7.26)
$f(z)$	Schur function for a measure $\frac{1+zf(z)}{1-zf(z)} = \int \frac{e^{i\theta}+z}{e^{i\theta}-z}\,d\mu(\theta)$; see Section 1.3
F	function in P_2 sum rule; for $E > 2$, $F(E) = \frac{1}{2}\int_2^{\lvert E\rvert}\lvert E^2-4\rvert^{1/2}\,dE$; see (13.8.27) and (13.8.33)
$F_n(f)$	Fejér approximation; see (2.12.33)
$\tilde{F}^{(N)}$	coefficient-stripped Carathéodory function; see (3.4.14) and (3.4.18)
$F(z)$	Carathéodory function of a measure, $F(z) = \int \frac{e^{i\theta}+z}{e^{i\theta}-z}\,d\mu(\theta)$; see Section 1.3

$F(d\mu)$	leading term in limit of Toeplitz matrices, $F(d\mu) = \lim_{n\to\infty} D_n(d\mu)^{1/n} = \prod_{j=0}^{\infty}(1-\lvert\alpha_j\rvert^2)$; see Theorem 2.1.2
$\mathcal{F}$	Fourier-CMV transform, $(\mathcal{F}f)_n = \int \overline{\chi_n(e^{i\theta})}\, f(e^{i\theta})\, d\mu(\theta)$; see (10.7.79)
$\mathcal{F}_0$	free Fourier-CMV transform, $\mathcal{F}$ for $d\mu = \frac{d\theta}{2\pi}$; see (10.7.79)
$\mathcal{F}_{k\ell}(d\mu)$	full GGT matrix; see (4.1.25) and Proposition 4.1.3
g	matrix Schur function; see (4.5.13)
g_0	unperturbed matrix Schur function; see (4.5.14)
g_n	approximate Jost function half of a solution of Geronimo-Case equations; see (13.6.13)
$g_n(\theta, d\mu)$	function needed in Khrushchev theory; see (9.2.32)
G	auxiliary function for P_2 sum rule given by $G(a) = a^2 - 1 - \log(a^2)$; see (13.8.28)
G_∞	limit value for Geronimo-Case equations; see (13.6.43)
$G_0(z)$	unperturbed matrix Carathéodory function; see (4.5.12)
$G_{a,\lambda}(z)$	allowed limit for ratio asymptotics, $G_{a,\lambda}(z) = \frac{1}{2}[(1+\lambda z) + [(1-\lambda z)^2 + 4a^2\lambda z]^{1/2}]$; see (9.5.5)
G_j	gaps in periodic essential spectrum; see Section 11.7
G_n	normalized half solution of Geronimo-Case equation; see (13.6.42)
$G_n(z)$	function studied by Golinskii, $G_n(z) = F(z)\Phi_n(z) + \Psi_n(z)$; see (3.2.40). Up to a constant, this is the u_k of (9.2.28).
$G_n^*(z)$	formal dual function to Golinskii's G_n, $G_n^*(z) = F(z)\Phi_n^* - \Psi_n^*(z)$; see (3.2.41). Up to a constant, this is the u_k^* of (9.2.28).
$G(d\mu)$	second term in Toeplitz determinant asymptotics, $G(d\mu) = \prod_{j=0}^{\infty}(1-\lvert\alpha_j\rvert^2)^{-j-1}$; see (2.1.3)
$G(z)$	matrix Carathéodory function, $G(z) = P[\frac{V+z}{V-z}]P$; see (4.5.11)
$G(z)$	$\int \frac{d\mu(\theta)}{\lvert e^{i\theta}-z\rvert^2}$; see (10.1.5)
$G(z)$	Green's function for $\mathcal{E}$; see (10.11.25)
$\mathcal{G}_\infty$	noneigenvalue set; see Theorem 10.1.5
$\mathcal{G}_{k\ell}(\{\alpha_n\}_{n=0}^{\infty})$	GGT matrix; see (4.1.4) and (4.1.5)
$\mathcal{G}_n$	MNT integral operator; see (9.4.28)
$h_\alpha(A)$	Hausdorff α-dimensional measure; see (2.12.8)
h_N	Hankel determinant; see (1.2.29)
$h(a)$	Hankel operator; see (6.2.2)
$h(\tilde{a})$	Hankel alternate operator; see (6.2.2)
$H^{1/2}$	Sobolev space, $H^{1/2} = \{f \in L^2(\partial\mathbb{D}, \frac{d\theta}{2\pi}) \mid \sum \lvert k\rvert \lvert\hat{f}_k\rvert^2 < \infty\}$; see (6.2.36)
H^2, H^∞	Hardy spaces
$H(f)$	Hankel operator with symbol f; see (6.2.6)
$\mathcal{H}_{\rm ac}$	$L^2(\partial\mathbb{D}, d\mu_{\rm ac})$; see Subsection 1.4.7
$\mathcal{H}_{\rm pp}$	space of eigenvectors; see Subsection 1.4.7
$\mathcal{H}_{\rm sc}$	$L^2(\partial\mathbb{D}, d\mu_{\rm sc})$; see Subsection 1.4.7
$I(f)$	$\log(D/\bar{D})$; see (6.2.39)
$\mathcal{I}_1$	trace class, $\{A \mid \mathrm{Tr}(\lvert A\rvert) < \infty\}$; see Subsection 1.4.12
$\mathcal{I}_2$	Hilbert-Schmidt ideal, $\{A \mid \mathrm{Tr}(A^*A) < \infty\}$; see Subsection 1.4.13
$\mathcal{I}_\mu$	invariant measures on $\partial\mathbb{D}$ for a measure μ on $\mathbb{U}(1,1)$; see Proposition 10.6.2
$\mathcal{I}_p$	trace ideal, $\{A \mid \mathrm{Tr}(\lvert A\rvert^p) < \infty\}$; see Subsection 1.4.13

$\mathcal{I}(A)$ set of measures on $\partial\mathbb{D}$ left invariant by the projection action of A; see Theorem 10.4.12

J $J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, relevant to the definition of $\mathbb{U}(1,1)$; see (10.4.1)
$J^{(1)}$ once stripped Jacobi matrix; see Subsection 1.2.12
J_μ Jacobi matrix; see (1.2.17)
J_n normalization factor of $\Phi_n^{\rm L}$ in the theory of matrix-valued measures; see (2.13.22)
$J_{n;F}$ $n \times n$ matrix obtained from upper right corner of a Jacobi matrix; see (1.2.61)
$J(\mathcal{S})$ Jacobi variety of the Riemann surface $\mathcal{S}$; see (11.7.25)
$\mathcal{J}_e$ restriction of $\mathcal{S}(\mathcal{C}+\mathcal{C}^{-1})\mathcal{S}$ to the even subspace; see Theorem 13.2.1
$\mathcal{J}_o$ restriction of $\mathcal{S}(\mathcal{C}+\mathcal{C}^{-1})\mathcal{S}$ to the odd subspace; see Theorem 13.2.1
$\mathcal{J}^+$ restriction of $\mathcal{J}(\mathcal{C}+\mathcal{C}^{-1})\mathcal{J}^{-1}$ to the even subspace; see Theorem 13.2.2
$\mathcal{J}^-$ restriction of $\mathcal{J}(\mathcal{C}+\mathcal{C}^{-1})\mathcal{J}^{-1}$ to the odd subspace; see Theorem 13.2.2
$\mathcal{J}^\times(\mathcal{S})$ extended Jacobi variety; see (11.8.5)

K_n normalization factor of $\Phi_n^{\rm R}$ in the theory of matrix-valued measures; see (2.13.22)
$K_n(z,\zeta)$ Christoffel-Darboux kernel; see (1.2.36) and (2.2.17)

L_A induced projective action of A on $\partial\mathbb{D}$; see (10.4.26)
$\hat{L}_m$ Fourier coefficients of $\log(w)$, i.e., $\hat{L}_m = \int e^{-im\theta}\log(w(\theta))\frac{d\theta}{2\pi}$; see (1.1.23)
$\mathcal{L}$ period lattice of the Riemann surface $\mathcal{S}$; see (11.7.25)
$\mathcal{L}_{ij}(d\mu)$ $\mathcal{L}$ half of CMV $\mathcal{LM}$ factorization, $\mathcal{L}_{ij} = \langle \chi_i, zx_j\rangle$; see (4.2.17)
$\mathcal{L}_q$ periodized $\mathcal{L}$ matrix; see (11.2.7)
$\mathcal{L}(\omega_0)$ left limit points of $\{T_n w_0\}$; see (10.5A.1)
$\langle\,\cdot\,,\,\cdot\,\rangle_L$ scalar inner product on matrix-valued functions; see Section 1.1
$\langle\langle\,\cdot\,,\,\cdot\,\rangle\rangle_{\rm L}$ matrix inner product on matrix-valued functions; see (2.13.3)

m^+ m-function used in Kotani theory, $m^+ = \rho_0^{-1}z(1-\bar\alpha_0 f)$; see (10.11.5) and Theorem 10.11.6
$m^*(x_1,\dots,x_j)$ function in DHK formula, $m^*(x_1,\dots x_j) = \max(0, x_1, x_1+x_2,\dots, x_1+\cdots+x_j)$; see (6.5.15)
$m_\mu(E)$ Weyl m-function for Jacobi matrices, $m_\mu(E) = \int \frac{d\mu(x)}{x-E}$; see (1.2.6)
$M_{1/2}(d\mu)$ GI size on measures, $M_{1/2}(d\mu) = \sum_{n=1}^\infty n|\hat{L}_n(d\mu)|^2$; see (6.1.16)
$M_{ij}^{(n)}$ matrix inverse to matrix-valued Toeplitz matrix; see (2.13.14)
$M(\alpha_0,\dots,\alpha_{p-1})$ modulus of a point in $\mathbb{D}^p$; see (11.4.29)
Mf reflection map associated to CMV, $\mathcal{M}$, $(Mf)(z) = f(\bar z)$; see (13.2.1)
$M(z)$ M-function for analyzing effect of stripping on meromorphic Carathéodory functions, $M(z) = z(1+\alpha_0)(1+F(z)) + (1+\bar\alpha_0)(1-F(z))$; see (11.7.76)
$\mathcal{M}_{+,1}(X)$ probability measures on a compact Hausdorff space, X
$\mathcal{M}_{ij}(d\mu)$ $\mathcal{M}$ half of CMV $\mathcal{LM}$ factorization, $\mathcal{M}_{ij} = \langle x_i, \chi_j\rangle$; see (4.2.16)
$\mathcal{M}_q(\beta)$ periodized $\mathcal{M}$ matrix; see (11.2.7)

$N_A(e^{i\theta})$ norm associated to a projective action of A, if $u_\theta = 2^{-1/2}(e^{i\theta})^t$, $N_a(e^{i\theta}) = \|Au_\theta\|$; see (10.4.30)

$O(f;z_0)$ degree of a meromorphic function; see (11.7.11)
$\mathcal{O}(\omega_0)$ orbit of a dynamical system, $\mathcal{O}(\omega_0) = \{T^n\omega\}_{n=-\infty}^\infty$; see (10.5A.1)

p_n normalized OPRL; see (1.2.5)

p_n	decaying CMV function, $p_n = y_n + F(z)x_n$; see (4.4.4)
P	pure point support, $\cup_x\{z \mid \mu_\lambda(\{z\}) \neq 0\}$ for an Aleksandrov family; see (10.10.1)
P_+	fundamental projection in Wiener-Hopf theory; see Section 5.1
P_-	$1 - P_+$ in Wiener-Hopf theory; see (5.1.8)
P_n	monic OPRL; see (1.2.5)
$P_r(\theta, \varphi)$	Poisson kernel, $P_r(\theta, \varphi) = \frac{1-r^2}{1+r^2-2\cos(\theta-\varphi)}$; see (1.3.14)
$P(x, \varphi)$	positive elements of $\mathbb{SU}(1,1)$; see (10.4.17)
$\mathcal{P}$	Poisson map in MNT theory, $\mathcal{P}(F, z) = \int_0^{2\pi} \frac{1-\lvert z\rvert^2}{\lvert z-e^{i\theta}\rvert^2} F(e^{i\theta}) \frac{d\theta}{2\pi}$; see (9.4.29)
$\mathcal{P}(d\nu)$	dual of Poisson map, $\mathcal{M}_{+,1}(\bar{\mathbb{D}}) \to \mathcal{M}_{+,1}(\partial\mathbb{D})$ by $\int f(e^{i\theta})\mathcal{P}(d\nu)(\theta) = \int P_r(e^{i\theta}, e^{i\varphi}) f(\theta)\, d\nu(re^{i\varphi})$; see Proposition 8.2.2
$q_n(x)$	second kind OPRL; see Subsection 1.2.10
$(n)_q$	q-factorial, $(n)_q = (1-q)(1-q^2)\dots(1-q^n)$; see (1.6.42)
$\left[\begin{smallmatrix} n \\ j \end{smallmatrix}\right]_q$	q-binomial coefficient, $\left[\begin{smallmatrix} n \\ j \end{smallmatrix}\right]_q = \frac{(n)_q}{(j)_q (n-j)_q}$; see (1.6.43)
$Q_{n,m}(x, y)$	polynomial used in López theory; see (9.9.10)
$Q_n(x)$	second kind OPRL; see Subsection 1.2.10
$Q(J)$	entropy in P_2 sum rule; see (13.8.34)
$\mathbb{Q}$	rational numbers
$r_n(x)$	remainder in continued fraction expansion of a real number x; see (11.9.1)
$r_n(z)$	OPUC normalized Prüfer radius; see (10.12.3)
$R_n(z)$	OPUC Prüfer radius; see (10.12.2)
R_D	R_D = radius of convergence of D if $d\mu_s = 0$; see (7.1.1)
$R_{D^{-1}}$	$R_{D^{-1}}$ = radius of convergence of D^{-1} if $d\mu_s = 0$; see (7.1.2)
R_F	R_F = radius of convergence of $F(z)$ about $z = 0$; see (7.1.1)
R_{NT}	Nevai-Totik radius; see (7.1.3)
R_{Φ^*}	$R_{\Phi^*} = \sup\{r \mid \sup_{n,\, \lvert z\rvert \leq r} \lvert\Phi_n^*(z)\rvert < \infty\}$; see (7.1.2)
R_W	R_W = annular radius for $w(\theta)$; see (7.1.1)
$R(x_1, \dots, x_j)$	function used in Kac proof of the strong Szegő theorem; see (6.5.16)
$R(z)$	function whose square root defines the Riemann surface $\mathcal{S}$; see (11.7.1)
$\mathbb{R}$	real line
$\mathcal{R}$	the matrix indexed by $0, 1, 2, \dots$ with $\mathcal{R}_{ij} = (-1)^j \delta_{ij}$; see (13.2.15)
$\mathcal{R}(\omega_0)$	right limit points of $\{T^n \omega_0\}$; see (10.5A.1)
$\langle \cdot, \cdot \rangle_{\mathrm{R}}$	scalar inner product on matrix-valued functions; see Section 1.1
$\langle\langle \cdot, \cdot \rangle\rangle_{\mathrm{R}}$	matrix inner product on matrix-valued functions; see (2.13.2)
s_n	half of alternate CMV basis, $s_n = x_{2n} = z^{-n}\varphi_{2n}$; see (4.2.10)
$s_n(f)$	n-th Taylor coefficient of f; see (1.3.42)
$s(\mu)$	entropy of μ in Furstenberg's theorem; see (10.6.10)
S	combined singular supports of an Aleksandrov family; see (10.10.2)
$S_{\alpha,\delta}(A)$	Hausdorff measure constructor; see (2.12.7)
S_λ	support of singular part of $d\mu_\lambda$; see (10.1.2)
$S(\alpha)$	matrix used in the Killip-Nenciu proof of the Geronimus relations; see (13.2.13)
$S(\mu \mid \nu)$	relative entropy; see (2.3.2)
$S(z)$	$S(z) = \sum_{n=0}^\infty \alpha_n z^n$; see (7.2.3)
$\mathrm{Sz}(\xi)$	Szegő transform of measures; see (13.1.4)
$\mathbb{SL}(2, \mathbb{C})$	2×2 complex matrices of determinant 1

$\mathbb{SL}(2,\mathbb{R})$	2×2 real matrices of determinant 1
$\mathbb{SU}(1,1)$	$A \in \mathbb{SL}(2,\mathbb{C})$ so that $A^*JA = J$; see (10.4.1)
$\mathbb{SU}(1,1;J_r)$	$A \in \mathbb{SL}(2,\mathbb{C})$ so that $A^*J_rA = J_r$; in fact, it equals $\mathbb{SL}(2,\mathbb{R})$; see Proposition 10.4.1
$\mathcal{S}$	Schur map, from Schur functions to Schur parameters; see Subsection 1.3.6
$\mathcal{S}$	Riemann surface; see Section 11.7
$\mathcal{S}$	map used in Killip-Nenciu proof of the Geronimus connection; see (13.2.14)
$\mathcal{S}_+$	principal sheet of the Riemann surface $\mathcal{S}$; see Section 11.7
$\mathcal{S}_-$	second sheet of the Riemann surface $\mathcal{S}$; see Section 11.7
$\mathcal{S}(f;\mu,\nu)$	Gibbs variational energy for the entropy; see (2.3.7)
$\mathcal{S}(\mu,\nu)$	entropy used in proof of Furstenberg's theorem; see (10.6.9)
t_n	half of alternate CMV basis, $t_n = x_{2n-1} = z^{-n}\varphi^*_{2n-1}$; see (4.2.9)
$t(a)$	Toeplitz operator; see (6.2.2)
$\widetilde{T}_\infty(z)$	limit of modified transfer matrices with $\alpha_n \in \ell^1$; see (10.7.7)
$T^{(n)}_{ij}$	Toeplitz matrix; see (1.3.11)
$T_n(z)$	transfer matrix; see (3.2.27)
$\widetilde{T}_n(z)$	modified transfer matrix; see (10.7.3)
$T_{t_1,\dots,t_k}(g)$	multicharacter for $SU(n)$; see (6.3.6)
$T(f)$	Toeplitz operator with symbol f, see (6.2.6)
$\mathrm{Tr}(A)$	trace on trace class; see Subsection 1.4.12
$\mathcal{T}$	map used in Killip-Nenciu proof of the connection formulae of Berriochoa, Cachafeiro, and García-Amor; see (13.2.36)
u_k	top half of decaying solution of transfer matrix, $u_k = \psi_k + F(z)\varphi_k$; see (9.2.28)
u^*_k	bottom half of decaying solution of transfer matrix, $u^*_k = -\psi^*_k + F(z)\varphi^*_k$; see (9.2.28)
u_n	ratio used in inverse Geronimus relations, $u_n = \varphi^*_{n+2}/\varphi^*_{n+1}$; see (13.1.34) and (13.1.37)
$u_n(z,J)$	Jost function for OPRL; see Section 13.6
$U^\mu(x)$	potential of a measure μ on $\mathbb{C}$; see (8.1.8)
$U(P,Q)$	map in Kato similarity transforms, $U(P,Q) = \sum_{j=1}^{\ell} Q_jP_j$; see (12.1.10)
$\mathbb{U}(1,1)$	$A \in \mathbb{GL}(2, bbC)$ so $A^* = JA = J$; see Section 10.4
$\mathbb{U}(n)$	$n \times n$ unitary matrices
v_n	ratio used in inverse Geronimus relations, $v_n = -\varphi^-_{n+2}/\varphi^-_{n+1}$; see (13.1.34) and (13.1.37)
V_k	canonical basis of first kind holomorphic differentials on the Riemann surface $\mathcal{S}$; see Proposition 11.7.4
$V^{(n)}$	Verblunsky remainder term; see (1.5.53)
$V(P,Q)$	map in Kato similarity transform, $V(P,Q) = \sum_{j=1}^{\ell} P_jQ_j$; see (12.1.11)
$V(\theta)$	density of zeros for periodic problems, $V(\theta) = \frac{1}{p}\frac{\lvert\Delta'(e^{i\theta})\rvert}{(4-\Delta^2(e^{i\theta}))^{1/2}}$; see (11.1.19)
w_n	Weyl solution; see (13.9.5)
$w(\theta)$	$\frac{2\pi d\mu}{d\theta}$, i.e., $d\mu = w(\theta)\frac{d\theta}{2\pi} + d\mu_s$; see (1.1.5)
W_ℓ	vector component of CMV solution; see (4.4.21)
$W(\varphi)$	Coulomb interaction on $\partial\mathbb{D}$; see (6.4.4)
$W(\theta)$	OPUC boost group; see (10.14.5)

x_n	alternate CMV basis; see (4.2.6)
$x_n^{(0)}$	free alternate CMV basis; see (4.2.6)
X_β	Floquet space; see (11.2.2)
y_n	second kind alternate CMV basis; see (4.4.2)
z_α	$z_\alpha = \frac{1+\bar\alpha}{1+\alpha}$; see (11.1.30)
z_{jn}	j-th zero of $\Phi_n(z)$; see Example 1.7.17
$Z_\infty(d\mu)$	limit points of zeros of Φ_n and zeros of some Φ_n; see the remark following Theorem 1.7.12
$Z_n(d\mu)$	zeros of Φ_n; see the remark following Theorem 1.7.12
$Z_{\rm L}(d\mu)$	limit points of zeros of Φ_n; see the remark following Theorem 1.7.12
$Z_{\rm SL}(d\mu)$	strictly limits of zeros of Φ_n; see the remark following Theorem 1.7.12
Z_ω	zero set for γ; see (10.11.1)
$Z(J)$	entropy in C_0 sum rule; see (13.8.22)
$Z(J \mid J^{(1)})$	step-by-step entropy in C_0 sum rule; see (13.8.24)
$\mathbb{Z}$	integers $\{0, \pm1, \pm2, \dots\}$
$\mathbb{Z}_+$	nonnegative integers $\{0, 1, 2, \dots\}$

Nonalphabetic

$\lvert A\rvert$	absolute value of an operator; see (1.4.28)
$(\,\cdot\,; q)_\infty = \prod_{j=0}^\infty (1 - wq^j)$	used for theta function; see (1.6.54)
Ⓢ	group semidirect product
$\{\,\cdot\,,\,\cdot\,\}$	Poisson bracket on $\mathbb{D}^p$; see (11.11.9)
$\{\,\cdot\,,\,\cdot\,\}_0$	free Poisson bracket on $\mathbb{D}$; see (11.11.1)
$Q_n^*(z)$	reversed polynomials, $Q_n^*(z) = z^n\,\overline{Q_n(1/\bar z)}$; see (1.1.6)
$\restriction$	"restriction" $f \restriction A$ is the function (or operator), f, restricted to a set A
$\triangle$	set difference $A \triangle B = (A\backslash B) \cup (B\backslash A)$

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Bibliography

Note on Russian Names: Anyone writing about a subject like OPUC on which there have been substantial Russian (and Ukrainian) contributions has to deal with the issue illustrated by the fact that in their English translations, the name in the book for the author of [**17**] is Akhiezer, while the name of the author in [**13**] is Achieser! I have decided to use in both references and objects (e.g., Chebyshev polynomials) a consistent spelling. But the reader is warned that this may produce a difficulty if you are trying to order a book on Amazon using an author spelling from this bibliography.

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