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From Stein to Weinstein and Back

Symplectic Geometry of Affine Complex Manifolds

Kai Cieliebak
Yakov Eliashberg

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To my parents, Snut and Hinrich. Kai
To Ada. Yasha

Contents

Preface

In Spring 1996 Yasha Eliashberg gave a Nachdiplomvorlesung (a one semester graduate course) “Symplectic geometry of Stein manifolds” at ETH Zürich. Kai Cieliebak, at the time a graduate student at ETH, was assigned the task to take notes for this course, with the goal of having lecture notes ready for publication by the end of the course. At the end of the semester we had some 70 pages of typed notes, but they were nowhere close to being publishable. So we buried the idea of ever turning these notes into a book.

Seven years later Kai spent his first sabbatical at the Mathematical Sciences Research Institute (MSRI) in Berkeley. By that time, through work of Donaldson and others on approximately holomorphic sections on the one hand, and gluing formulas for holomorphic curves on the other hand, Weinstein manifolds had been recognized as fundamental objects in symplectic topology. Encouraged by the increasing interest in the subject, we dug out the old lecture notes and began turning them into a monograph on Stein and Weinstein manifolds.

Work on the book has continued on and off since then, with most progress happening during Kai’s numerous visits to Stanford University and another sabbatical 2009 that we both spent at MSRI. Over this period of almost 10 years, the content of the book has been repeatedly changed and its scope significantly extended. Some of these changes and extensions were due to our improved understanding of the subject (e.g., a quantitative version of J-convexity which is preserved under approximately holomorphic diffeomorphisms), others due to new developments such as the construction of exotic Stein structures by Seidel and Smith, McLean, and others since 2005, and Murphy’s h-principle for loose Legendrian knots in 2011. In fact, the present formulation of the main theorems in the book only became clear about a year ago. As a result of this process, only a few lines of the original lecture notes have survived in the final text (in Chapters 2–4).

The purpose of the book has also evolved over the past decade. Our original goal was a complete and detailed exposition of the existence theorem for Stein structures in [**42**]. While this remains an important goal, which we try to achieve in Chapters 2–8, the book has evolved around the following two broader themes: The first one, as indicated by the title, is the correspondence between the complex analytic notion of a Stein manifold and the symplectic notion of a Weinstein manifold. The second one is the extent to which these structures are flexible, i.e., satisfy an h-principle. In fact, until recently we believed the border between flexibility and rigidity to run between subcritical and critical structures, but Murphy’s h-principle extends flexibility well into the critical range.

The book is roughly divided into “complex” and “symplectic” chapters. Thus Chapters 2–5 and 8–10 can be read as an exposition of the theory of J-convex functions on Stein manifolds, while Chapters 6–7, 9 and 11–14 provide an introduction to Weinstein manifolds and their deformations. However, our selection of material on both the complex and symplectic side is by no means representative for the respective fields. Thus on the complex side we focus only on topological aspects of Stein manifolds, ignoring most of the beautiful subject of several complex variables. On the symplectic side, the most notable omission is the relationship between Weinstein domains and Lefschetz fibrations over the disc.

Over the past 16 years we both gave many lecture courses, seminars, and talks on the subject of this book not only at our home institutions, Ludwig-Maximilians-Universität München and Stanford University, but also at various other places such as the Forschungsinstitut für Mathematik at ETH Zürich, University of Pennsylvania in Philadelphia, Columbia University in New York, the Courant Institute of Mathematical Sciences in New York, University of California in Berkeley, Washington University in St. Louis, the Mathematical Sciences Research Institute in Berkeley, the Institute for Advanced Study in Princeton, and the Alfréd Rényi Institute of Mathematics in Budapest. We thank all these institutions for their support and hospitality.

Many mathematicians and students who attended our lectures and seminars or read parts of preliminary versions of the book provided us with valuable comments and critical remarks. We are very grateful to all of them, and in particular to M. Abouzaid, S. Akbulut, J. Bowden, V. Braungardt, J. Daniel, T. Ekholm, C. Epstein, J. Etnyre, C. Fefferman, F. Forstnerič, U. Frauenfelder, A. Gerstenberger, R. Gompf, A. Huckleberry, P. Landweber, J. Latschev, L. Lempert, R. Lipshitz, C. Llosa Isenrich, D. McDuff, M. McLean, K. Mohnke, J. Morgan, E. Murphy, S. Nemirovski, L. Nirenberg, K. Nguyen, A. Oancea, N. Øvrelid, P. Ozsváth, L. Polterovich, P. Seidel, A. Stadelmaier, A. Stipsicz, D. Thurston, T. Vogel, E. Volkov, J. Wehrheim, and C. Wendl.

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A beautiful and comprehensive introduction to this important field.

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This excellent book gives a detailed, clear, and wonderfully written treatment of the interplay between the world of Stein manifolds and the more topological and flexible world of Weinstein manifolds. Devoted to this subject with a long history, the book serves as a superb introduction to this area and also contains the authors' new results.

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This book is devoted to the interplay between complex and symplectic geometry in affine complex manifolds. Affine complex (a.k.a. Stein) manifolds have canonically built into them symplectic geometry which is responsible for many phenomena in complex geometry and analysis. The goal of the book is the exploration of this symplectic geometry (the road from "Stein to Weinstein") and its applications in the complex geometric world of Stein manifolds (the road "back"). This is the first book which systematically explores this connection, thus providing a new approach to the classical subject of Stein manifolds. It also contains the first detailed investigation of Weinstein manifolds, the symplectic counterparts of Stein manifolds, which play an important role in symplectic and contact topology.

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