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**From Stein to
Weinstein and Back**
**Symplectic Geometry of
Affine Complex Manifolds**

Kai Cieliebak
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10 9 8 7 6 5 4 3 2 1 17 16 15 14 13 12

To my parents, Snut and Hinrich. Kai
To Ada. Yasha

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Preface

In Spring 1996 Yasha Eliashberg gave a Nachdiplomvorlesung (a one semester graduate course) “Symplectic geometry of Stein manifolds” at ETH Zürich. Kai Cieliebak, at the time a graduate student at ETH, was assigned the task to take notes for this course, with the goal of having lecture notes ready for publication by the end of the course. At the end of the semester we had some 70 pages of typed notes, but they were nowhere close to being publishable. So we buried the idea of ever turning these notes into a book.

Seven years later Kai spent his first sabbatical at the Mathematical Sciences Research Institute (MSRI) in Berkeley. By that time, through work of Donaldson and others on approximately holomorphic sections on the one hand, and gluing formulas for holomorphic curves on the other hand, Weinstein manifolds had been recognized as fundamental objects in symplectic topology. Encouraged by the increasing interest in the subject, we dug out the old lecture notes and began turning them into a monograph on Stein and Weinstein manifolds.

Work on the book has continued on and off since then, with most progress happening during Kai’s numerous visits to Stanford University and another sabbatical 2009 that we both spent at MSRI. Over this period of almost 10 years, the content of the book has been repeatedly changed and its scope significantly extended. Some of these changes and extensions were due to our improved understanding of the subject (e.g., a quantitative version of J -convexity which is preserved under approximately holomorphic diffeomorphisms), others due to new developments such as the construction of exotic Stein structures by Seidel and Smith, McLean, and others since 2005, and Murphy’s h -principle for loose Legendrian knots in 2011. In fact, the present formulation of the main theorems in the book only became clear about a year ago. As a result of this process, only a few lines of the original lecture notes have survived in the final text (in Chapters 2–4).

The purpose of the book has also evolved over the past decade. Our original goal was a complete and detailed exposition of the existence theorem for Stein structures in [42]. While this remains an important goal, which we try to achieve in Chapters 2–8, the book has evolved around the following two broader themes: The first one, as indicated by the title, is the correspondence between the complex analytic notion of a Stein manifold and the symplectic notion of a Weinstein manifold. The second one is the extent to which these structures are flexible, i.e., satisfy an h -principle. In fact, until recently we believed the border between flexibility and rigidity to run between subcritical and critical structures, but Murphy’s h -principle extends flexibility well into the critical range.

The book is roughly divided into “complex” and “symplectic” chapters. Thus Chapters 2–5 and 8–10 can be read as an exposition of the theory of J -convex

functions on Stein manifolds, while Chapters 6–7, 9 and 11–14 provide an introduction to Weinstein manifolds and their deformations. However, our selection of material on both the complex and symplectic side is by no means representative for the respective fields. Thus on the complex side we focus only on topological aspects of Stein manifolds, ignoring most of the beautiful subject of several complex variables. On the symplectic side, the most notable omission is the relationship between Weinstein domains and Lefschetz fibrations over the disc.

Over the past 16 years we both gave many lecture courses, seminars, and talks on the subject of this book not only at our home institutions, Ludwig-Maximilians-Universität München and Stanford University, but also at various other places such as the Forschungsinstitut für Mathematik at ETH Zürich, University of Pennsylvania in Philadelphia, Columbia University in New York, the Courant Institute of Mathematical Sciences in New York, University of California in Berkeley, Washington University in St. Louis, the Mathematical Sciences Research Institute in Berkeley, the Institute for Advanced Study in Princeton, and the Alfréd Rényi Institute of Mathematics in Budapest. We thank all these institutions for their support and hospitality.

Many mathematicians and students who attended our lectures and seminars or read parts of preliminary versions of the book provided us with valuable comments and critical remarks. We are very grateful to all of them, and in particular to M. Abouzaid, S. Akbulut, J. Bowden, V. Braungardt, J. Daniel, T. Ekholm, C. Epstein, J. Etnyre, C. Fefferman, F. Forstnerič, U. Frauenfelder, A. Gerstenberger, R. Gompf, A. Huckleberry, P. Landweber, J. Latschev, L. Lempert, R. Lipshitz, C. Llosa Isenrich, D. McDuff, M. McLean, K. Mohnke, J. Morgan, E. Murphy, S. Nemirovski, L. Nirenberg, K. Nguyen, A. Oancea, N. Øvrelid, P. Ozsváth, L. Polterovich, P. Seidel, A. Stadelmaier, A. Stipsicz, D. Thurston, T. Vogel, E. Volkov, J. Wehrheim, and C. Wendl.

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Bibliography

- [1] A. Abbondandolo and M. Schwarz, *On the Floer homology of cotangent bundles*, Comm. Pure Appl. Math. **59**, no. 2, 254–316 (2006).
- [2] A. Abbondandolo and M. Schwarz, *Floer homology of cotangent bundles and the loop product*, Geom. Topol. **14**, no. 3, 1569–1722 (2010).
- [3] M. Abouzaid and P. Seidel, *Altering symplectic manifolds by homologous recombination*, arXiv:1007.3281.
- [4] R. Abraham and J. Robbin, *Transversal Mappings and Flows*, with an appendix by A. Kelley, Benjamin, New York-Amsterdam (1967).
- [5] T. Akahori, *A new approach to the local embedding theorem of CR-structures for $n \geq 4$ (the local solvability for the operator $\bar{\partial}_b$ in the abstract sense)*, Mem. Amer. Math. Soc. **67**, no. 366 (1987).
- [6] A. Akhmedov, J. Etnyre, T. Mark and I. Smith, *A note on Stein fillings of contact manifolds*, Math. Res. Lett. **15**, 1127–1132 (2008).
- [7] A. Andreotti and T. Frankel, *The Lefschetz theorem on hyperplane sections*, Ann. of Math. **69**, 717–717 (1959).
- [8] A. Andreotti and R. Narasimhan, *A topological property of Runge pairs*, Ann. of Math. **76**, 499–509 (1962).
- [9] V. I. Arnold, *Ordinary Differential Equations*, MIT Press, Cambridge, Massachusetts (1973).
- [10] V.I. Arnold, *Mathematical Methods of Classical Mechanics*, Springer (1978).
- [11] V.I. Arnold, *Geometrical Methods in the Theory of Ordinary Differential Equations*, Springer (1983).
- [12] W. Ballmann, *Lectures on Kähler manifolds*, ESI Lectures in Mathematics and Physics, European Mathematical Society, Zürich (2006).
- [13] A. Banyaga, *Sur la structure du groupe des difféomorphismes qui préservent une forme symplectique*, Comment. Math. Helv. **53**, no. 2, 174–227 (1978).
- [14] S. Batterson, *Stephen Smale: The Mathematician Who Broke the Dimension Barrier*, Amer. Math. Soc. (2000).
- [15] E. Bedford and B. Gaveau, *Envelopes of holomorphy of certain 2-spheres in \mathbb{C}^2* , Amer. J. Math. **105**, no. 4, 975–1009 (1983).
- [16] D. Bennequin, *Entrelacements et équations de Pfaff*, Third Schnepfenried geometry conference, Vol. 1 (Schnepfenried 1982), 87–161, Astérisque **107-108**, Soc. Math. France, Paris (1983).
- [17] P. Biran, *Lagrangian barriers and symplectic embeddings*, Geom. Funct. Anal. **11**, no. 3, 407–464 (2001).
- [18] E. Bishop, *Mappings of partially analytic spaces*, Amer. J. Math. **83**, 209–242 (1961).
- [19] E. Bishop, *Differentiable manifolds in complex Euclidean space*, Duke Math. J. **32**, 1–21 (1965).
- [20] F. Bogomolov and B. de Oliveira, *Stein small deformations of strictly pseudoconvex surfaces*, Birational algebraic geometry (Baltimore, 1996), 25–41, Contemp. Math. **207**, Amer. Math. Soc. (1997).
- [21] R. Bott, *The stable homotopy of the classical groups*, Ann. of Math. (2) **70**, 313–337 (1959).
- [22] R. Bott, *Marston Morse and his mathematical works*, Bull. Amer. Math. Soc. **3**, no. 3, 907–950 (1980).
- [23] R. Bott and J. Milnor, *On the parallelizability of the spheres*, Bull. Amer. Math. Soc. **64**, 87–91 (1958).
- [24] F. Bourgeois, T. Ekholm and Y. Eliashberg, *Effect of Legendrian Surgery*, arXiv: 0911.0026.

- [25] F. Bruhat and H. Whitney, *Quelques propriétés fondamentales des ensembles analytiques-réels*, Comment. Math. Helv. **33**, 132–160 (1959).
- [26] A. Cannas da Silva, *Lectures on Symplectic Geometry*, Springer (2001).
- [27] H. Cartan, *Variétés analytiques complexes et cohomologie*, Colloque sur les fonctions de plusieurs variables (tenu à Bruxelles 1953), 41–55, Georges Thone, Liège; Masson & Cie, Paris (1953).
- [28] H. Cartan, *Variétés analytiques réelles et variétés analytiques complexes*, Bull. Soc. Math. France **85**, 77–99 (1957).
- [29] D. Catlin, *A Newlander–Nirenberg theorem for manifolds with boundary*, Michigan Math. J. **35**, no. 2, 233–240 (1988).
- [30] J. Cerf, *La stratification naturelle des espaces de fonctions différentiables réelles et le théorème de la pseudo-isotopie*, Inst. Hautes Études Sci. Publ. Math. **39**, 5–173 (1970).
- [31] Y. Chekanov, *Differential algebra of Legendrian links*, Invent. Math. **150**, no. 3, 441–483 (2002).
- [32] K. Cieliebak, *Handle attaching in symplectic homology and the chord conjecture*, J. Eur. Math. Soc. (JEMS) **4**, no. 2, 115–142 (2002).
- [33] K. Cieliebak, *Subcritical Stein manifolds are split*, preprint 2002.
- [34] K. Cieliebak, A. Floer and H. Hofer, *Symplectic homology II: A general construction*, Math. Z. **218**, no. 1, 103–122 (1995).
- [35] K. Cieliebak, U. Frauenfelder and A. Oancea, *Rabinowitz Floer homology and symplectic homology*, Ann. Sci. Éc. Norm. Supér. (4) **43**, no. 6, 957–1015 (2010).
- [36] J.-P. Demailly, *Complex analytic and differential geometry*, preliminary version available on the author’s homepage, <http://www-fourier.ujf-grenoble.fr/~demailly/books.html>.
- [37] P. De Paepe, *Eva Kallin’s lemma on polynomial convexity*, Bull. London Math. Soc. **33**, no. 1, 1–10 (2001).
- [38] F. Docquier and H. Grauert, *Levisches Problem und Rungescher Satz für Teilgebiete Steinischer Mannigfaltigkeiten*, Math. Ann. **140**, 94–123 (1960).
- [39] K. Dymara, *Legendrian knots in overtwisted contact structures on S^3* , Ann. Global Anal. Geom. **19**, no. 3, 293–305 (2001).
- [40] T. Ekholm, J. Etnyre and M. Sullivan, *Non-isotopic Legendrian submanifolds in \mathbb{R}^{2n+1}* , J. Diff. Geom. **71**, no. 1, 85–128 (2005).
- [41] Y. Eliashberg, *Classification of overtwisted contact structures on 3-manifolds*, Invent. Math. **98**, no. 3, 623–637 (1989).
- [42] Y. Eliashberg, *Topological characterization of Stein manifolds of dimension > 2* , Internat. J. Math. **1**, no. 1, 29–46 (1990).
- [43] Y. Eliashberg, *Filling by holomorphic discs and its applications*, London Math. Soc. Lect. Notes 151, 45–68 (1991).
- [44] Y. Eliashberg, *Contact 3-manifolds 20 years since J. Martinet’s work*, Ann. Inst. Fourier **42**, 165–192 (1992).
- [45] Y. Eliashberg, *A few remarks about symplectic filling*, Geom. Topol. **8**, 277–293 (2004).
- [46] Y. Eliashberg, *Unique holomorphically fillable contact structure on the 3-torus*, Internat. Math. Res. Notices 1996, no. 2, 77–82.
- [47] Y. Eliashberg, *Symplectic geometry of plurisubharmonic functions*, Notes by M. Abreu, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci. 488, Gauge theory and symplectic geometry (Montreal, 1995), 49–67, Kluwer Acad. Publ. (1997).
- [48] Y. Eliashberg and M. Fraser, *Topologically trivial Legendrian knots*, J. Symp. Geom. **7**, no. 2, 77–127 (2009).
- [49] Y. Eliashberg and M. Gromov, *Convex Symplectic Manifolds*, Proceedings of Symposia in Pure Mathematics, vol. 52, Part 2, 135–162 (1991).
- [50] Y. Eliashberg and M. Gromov, *Embeddings of Stein manifolds of dimension n into the affine space of dimension $3n/2 + 1$* , Ann. of Math. **136**, 123–135 (1992).
- [51] Y. Eliashberg and V. Kharlamov, *On the number of complex points of a real surface in a complex surface*, Proc. Leningrad International Topology Conference, Leningrad, 1982, 143–148 (1984).
- [52] Y. Eliashberg and N. Mishachev, *Introduction to the h -Principle*, Graduate Studies in Mathematics 48, Amer. Math. Soc. (2002).
- [53] J. Etnyre, *Introductory lectures on contact geometry*, Topology and geometry of manifolds (Athens, GA, 2001), 81–107, Proc. Sympos. Pure Math. 71, Amer. Math. Soc. (2003).

- [54] J. Etnyre, *On Symplectic Fillings*, *Algebr. Geom. Topol.* **4**, 73–80 (2004).
- [55] J. Etnyre, *Legendrian and Transversal Knots*, *Handbook of Knot Theory*, 105–185, Elsevier (2005).
- [56] J. Etnyre and K. Honda, *On the nonexistence of tight contact structures*, *Ann. of Math. (2)* **153**, no. 3, 749–766 (2001).
- [57] A. Floer and H. Hofer, *Symplectic homology I: Open sets in C^n* , *Math. Z.* **215**, no. 1, 37–88 (1994).
- [58] G. Folland, *Introduction to Partial Differential Equations*, Princeton Univ. Press (1976).
- [59] J. E. Fornaess and B. Stensønes, *Lectures on Counterexamples in Several Complex Variables*, Princeton Univ. Press (1987), reprinted by AMS Chelsea (2007).
- [60] F. Forstnerič, *Stein Manifolds and Holomorphic Mappings*, Springer (2011).
- [61] F. Forstnerič and F. Lárusson, *Survey of Oka theory*, *New York J. Math.* **17a**, 1–28 (2011).
- [62] F. Forstnerič, E. Löw and N. Øvrelid, *Solving the d - and $\bar{\partial}$ -equations in thin tubes and applications to mappings*, *Michigan Math. J.* **49**, 369–416 (2001).
- [63] F. Forstnerič and M. Slapar, *Stein structures and holomorphic mappings*, *Math. Z.* **256**, no. 3, 615–646 (2007).
- [64] H. Geiges, *Symplectic manifolds with disconnected boundary of contact type*, *Int. Math. Res. Not.* **1994**, no. 1, 23–30.
- [65] H. Geiges, *An Introduction to Contact Topology*, Cambridge Univ. Press (2008).
- [66] E. Giroux, *Convexité en topologie de contact*, *Comment. Math. Helv.* **66**, no. 4, 637–677 (1991).
- [67] E. Giroux, *Une infinité de structures de contact tendues sur une infinité de variétés*, *Invent. Math.* **135**, 789–802 (1999).
- [68] E. Giroux, *Structures de contact en dimension trois et bifurcations des feuilletages de surfaces*, *Invent. Math.* **141**, no. 3, 615–689 (2000).
- [69] R. Gompf, *A new construction of symplectic manifolds*, *Ann. of Math.* **142**, 527–595 (1995).
- [70] R. Gompf, *Handlebody construction of Stein surfaces*, *Ann. of Math.* **148**, no. 2, 619–693 (1998).
- [71] R. Gompf, *Stein surfaces as open subsets of C^2* , *Conference on Symplectic Topology*, *J. Symp. Geom.* **3**, no. 4, 565–587 (2005).
- [72] R. Gompf, *Constructing Stein manifolds after Eliashberg*, *New perspectives and challenges in symplectic field theory*, 229–249, *CRM Proc. Lecture Notes* 49, Amer. Math. Soc. (2009).
- [73] R. Gompf, *Smooth embeddings with Stein surface images*, arXiv:1110.1865.
- [74] R. Gompf and A. Stipsicz, *4-Manifolds and Kirby Calculus*, Amer. Math. Soc. (1999).
- [75] J. Gray, *Some global properties of contact structures*, *Ann. of Math. (2)* **69**, 421–450 (1959).
- [76] H. Grauert, *Holomorphe Funktionen mit Werten in komplexen Lieschen Gruppen*, *Math. Ann.* **133**, 450–472 (1957).
- [77] H. Grauert, *On Levi's problem and the imbedding of real-analytic manifolds*, *Ann. of Math. (2)* **68**, 460–472 (1958).
- [78] H. Grauert and R. Remmert, *Theory of Stein Spaces*, Springer (1979).
- [79] H. Grauert and R. Remmert, *Coherent Analytic Sheaves*, Springer (1984).
- [80] P. Griffiths and J. Harris, *Principles of Algebraic Geometry*, John Wiley & Sons, New York (1978).
- [81] M. Gromov, *A topological technique for the construction of solutions of differential equations and inequalities*, *ICM 1970, Nice*, vol. 2, 221–225 (1971).
- [82] M. Gromov, *Convex integration of differential relations I*, *Izv. Akad. Nauk SSSR Ser. Mat.* **37**, 329–343 (1973).
- [83] M. Gromov, *Pseudoholomorphic curves in symplectic manifolds*, *Invent. Math.* **82**, no. 2, 307–347 (1985).
- [84] M. Gromov, *Partial Differential Relations*, *Ergebnisse der Mathematik und ihrer Grenzgebiete (3)* 9, Springer (1986).
- [85] M. Gromov, *Oka's principle for holomorphic sections of elliptic bundles*, *J. Amer. Math. Soc.* **2**, 851–897 (1989).
- [86] M. Gromov and Y. Eliashberg, *Removal of singularities of smooth maps*, *Izv. Akad. Nauk SSSR Ser. Mat.* **35**, 600–626 (1971).
- [87] V. Guillemin and A. Pollack, *Differential Topology*, Prentice-Hall, Englewood Cliffs, New Jersey (1974).

- [88] R. Gunning, *Introduction to Holomorphic Functions of Several Variables*, Vol. III: Homological Theory, Wadsworth & Brooks/Cole, Belmont (1990).
- [89] R. Gunning and H. Rossi, *Analytic Functions of Several Complex Variables*, Prentice-Hall (1965), reprinted by AMS Chelsea (2009).
- [90] A. Haefliger, *Plongements différentiables de variétés dans variétés*, Comment. Math. Helv. **36**, 47–82 (1961).
- [91] A. Hatcher, *Algebraic Topology*, Cambridge Univ. Press (2002).
- [92] A. Hatcher and J. Wagoner, *Pseudo-isotopies of compact manifolds*, Astérisque **6**, Soc. Math. de France (1973).
- [93] G. Henkin and J. Leiterer, *Theory of functions on complex manifolds*, Monographs in Mathematics 79, Birkhäuser (1984).
- [94] C. D. Hill and M. Nacinovich, *Stein fillability and the realization of contact manifolds*, Proc. Amer. Math. Soc. **133**, no. 6, 1843–1850 (2005).
- [95] R. Hind, *Stein fillings of lens spaces*, Commun. Contemp. Math. **5**, no. 6, 967–982 (2003).
- [96] H. Hironaka, *Resolution of singularities of an algebraic variety over a field of characteristic zero I, II*, Ann. of Math. **79**, 109–326 (1964).
- [97] M. Hirsch, *Immersions of manifolds*, Trans. Amer. Math. Soc. **93**, 242–276 (1959).
- [98] M. Hirsch, *Differential Topology*, Springer (1976).
- [99] H. Hofer, *Pseudoholomorphic curves in symplectizations with applications to the Weinstein conjecture in dimension three*, Invent. Math. **114**, no. 3, 515–563 (1993).
- [100] H. Hofer and E. Zehnder, *Symplectic Invariants and Hamiltonian Dynamics*, Birkhäuser (1994).
- [101] K. Honda, *On the classification of tight contact structures I*, Geom. Topol. **4**, 309–368 (2000).
- [102] L. Hörmander, *L^2 estimates and existence theorems for the $\bar{\partial}$ operator*, Acta Math. **113**, 89–152 (1965).
- [103] L. Hörmander, *An Introduction to Complex Analysis in Several Variables*, D. Van Nostrand Co., Princeton (1966), 3rd edition North-Holland (1990).
- [104] L. Hörmander and J. Wermer, *Uniform approximation on compact sets in C^n* , Math. Scand. **23**, 5–23 (1968).
- [105] A. Huckleberry, *Hans Grauert: mathematician pur*, Mitt. Deutsche Math.-Verein. **16**, no. 2, 75–77 (2008).
- [106] A. Huckleberry, *Karl Stein (1913–2000)*, Jahresber. Deutsch. Math.-Verein. **110**, no. 4, 195–206 (2008).
- [107] K. Igusa, *The stability theorem for smooth pseudoisotopies*, K-Theory **2**, no. 1-2 (1988).
- [108] H. Jacobowitz, *An Introduction to CR Structures*, Mathematical Surveys and Monographs 32, Amer. Math. Soc. (1990).
- [109] E. Kallin, *Fat polynomially convex sets*, Function Algebras (Proc. Internat. Sympos. on Function Algebras, Tulane Univ., 1965), Scott-Foresman, 149–152 (1966).
- [110] Y. Kanda, *The classification of tight contact structures on the 3-torus*, Comm. Anal. Geom. **5**, no. 3, 413–438 (1997).
- [111] M. Kervaire, *Non-parallelizability of the n -sphere for $n > 7$* , Proc. Nat. Acad. of Sci. USA **44**, 280–283 (1958).
- [112] M. Kervaire, *Le théorème de Barden-Mazur-Stallings*, Comment. Math. Helv. **40**, 31–42 (1965).
- [113] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry*, Vol. II, Interscience Tracts in Pure and Applied Mathematics No. 15 Vol. II, John Wiley & Sons, New York (1969).
- [114] J. Kohn and H. Rossi, *On the extension of holomorphic functions from the boundary of a complex manifold*, Ann. of Math. **81**, 451–472 (1965).
- [115] A. Kosinski, *Differential Manifolds*, Pure and Applied Mathematics 138, Academic Press, Boston (1993).
- [116] S. Krantz, *Function Theory of Several Complex Variables*, John Wiley & Sons, New York (1982), 2nd edition reprinted by AMS Chelsea (2001).
- [117] P. Kronheimer and T. Mrowka, *The genus of embedded surfaces in the projective plane*, Math. Res. Lett. **1**, no. 6, 797–808 (1994).
- [118] M. Kuranishi, *Strongly pseudo-convex CR structures over small balls, Part III*, Ann. of Math. **116**, 249–330 (1982).

- [119] H.-F. Lai, *Characteristic classes of real manifolds immersed in complex manifolds*, Trans. Amer. Math. Soc. **172**, 1–33 (1972).
- [120] P. Landweber, *Complex structures on open manifolds*, Topology **13**, 69–75 (1974).
- [121] S. Lefschetz, *L'Analysis situs et la géométrie algébrique*, Collection de Monographies publiée sous la direction de M. Emile Borel, Gauthier-Villars, Paris (1924).
- [122] L. Lempert, *Algebraic approximations in analytic geometry*, Invent. Math. **121**, no. 2, 335–353 (1995).
- [123] P. Lisca, *Symplectic fillings and positive scalar curvature*, Geom. Topol. **2**, 103–116 (1998).
- [124] P. Lisca, *On symplectic fillings of lens spaces*, Trans. Amer. Math. Soc. **360**, 765–799 (2008).
- [125] P. Lisca and G. Matić, *Tight contact structures and Seiberg-Witten invariants*, Invent. Math. **129**, 509–525 (1997).
- [126] R. Lutz, *Structures de contact sur les fibrés principaux en cercles de dimension trois*, Ann. Inst. Fourier (Grenoble) **27**, 1–15 (1977).
- [127] J. Marsden and T. Ratiu (eds.), *The breadth of symplectic and Poisson geometry*, Birkhäuser (2005).
- [128] J. Martinet, *Formes de contact sur les variétés de dimension 3*, Proceedings of Liverpool Singularities Symposium II (1969/1970), 142–163, Lecture Notes in Math. 209, Springer (1971).
- [129] J. Martinet, *Singularities of Smooth Functions and Maps*, Cambridge Univ. Press (1982).
- [130] P. Massot, K. Niederkrüger and C. Wendl, *Weak and strong fillability of higher dimensional contact manifolds*, arXiv:1111.6008.
- [131] M. Maydanskiy, *Exotic symplectic manifolds from Lefschetz fibrations*, arXiv:0906.2224.
- [132] M. Maydanskiy and P. Seidel, *Lefschetz fibrations and exotic symplectic structures on cotangent bundles of spheres*, J. Topol. **3**, no. 1, 157–180 (2010).
- [133] D. McDuff, *Symplectic manifolds with contact type boundaries*, Invent. Math. **103**, no. 3, 651–671 (1991).
- [134] D. McDuff, *Blow ups and symplectic embeddings in dimension 4*, Topology **30**, 409–421 (1991).
- [135] D. McDuff, *The local behavior of holomorphic curves in almost complex manifolds*, J. Diff. Geom. **34**, 143–164 (1991).
- [136] D. McDuff and D. Salamon, *Introduction to Symplectic Topology*, 2nd edition, Oxford Univ. Press (1998).
- [137] M. McLean, *Lefschetz fibrations and symplectic homology*, Geom. Topol. **13**, no. 4, 1877–1944 (2009).
- [138] M. Micallef and B. White, *The structure of branch points in minimal surfaces and in pseudoholomorphic curves*, Ann. of Math. **141**, 35–85 (1995).
- [139] J. Milnor, *Morse Theory*, Based on lecture notes by M. Spivak and R. Wells, Annals of Mathematics Studies 51, Princeton University Press, Princeton (1963).
- [140] J. Milnor, *Lectures on the h-Cobordism Theorem*, Notes by L. Siebenmann and J. Sondow, Princeton Univ. Press, Princeton (1965).
- [141] J. Morgan and G. Tian, *Ricci flow and the Poincaré conjecture*, Clay Mathematics Monographs 3, Amer. Math. Soc. (2007).
- [142] J. Munkres, *Obstructions to the smoothing of piecewise-differentiable homeomorphisms*, Ann. of Math. **72**, 521–554 (1960).
- [143] E. Murphy, *Loose Legendrian embeddings in high dimensional contact manifolds*, arXiv:1201.2245.
- [144] R. Narasimhan, *Imbedding of holomorphically complete complex spaces*, Amer. J. Math. **82**, 917–934 (1960).
- [145] R. Narasimhan, *A note on Stein spaces and their normalisations*, Ann. Scuola Norm. Sup. Pisa (3) **16**, no. 4, 327–333 (1962).
- [146] A. Némethi and P. Popescu-Pampu, *Milnor fibers of cyclic quotient singularities*, arXiv:0805.3449v2.
- [147] S. Nemirovski, *Complex analysis and differential topology on complex surfaces*, Russian Math. Surveys **54**, no. 4, 729–752 (1999).
- [148] S. Nemirovski, *Adjunction inequality and coverings of Stein surfaces*, Turkish J. Math. **27**, no. 1, 161–172 (2003).
- [149] A. Newlander and L. Nirenberg, *Complex analytic coordinates in almost complex manifolds*, Ann. of Math. (2) **65**, 391–404 (1957).

- [150] L. Ng, *A Legendrian Thurston–Bennequin bound from Khovanov homology*, *Algebr. Geom. Topol.* **5**, 1637–1653 (2005).
- [151] K. Niederkrüger and O. van Koert, *Every contact manifold can be given a nonfillable contact structure*, *Int. Math. Res. Not. IMRN* 2007, no. 23.
- [152] A. Nijenhuis and W. Wolf, *Some integration problems in almost complex manifolds*, *Ann. of Math. (2)* **77**, 424–489, (1963).
- [153] L. Nirenberg, *Lectures on linear partial differential equations*, Amer. Math. Soc. (1973).
- [154] K. Oka, *Sur les fonctions analytiques de plusieurs variables VII: Sur quelques notions arithmétiques*, *Bull. Soc. Math. France* **78**, 1–27 (1950).
- [155] *Kiyoshi Oka: Collected papers*, translated from the French by R. Narasimhan, with commentaries by H. Cartan, edited by R. Remmert, Springer (1984).
- [156] B. Ozbagci and A. Stipsicz, *Contact 3-manifolds with infinitely many Stein fillings*, *Proc. Amer. Math. Soc.* **132**, 1549–1558 (2004).
- [157] J. Palis and W. de Melo, *Geometric theory of dynamical systems: An introduction*, Springer (1982).
- [158] G. Perelman, *Finite extinction time for the solutions to the Ricci flow on certain three-manifolds*, arXiv:math/0307245.
- [159] O. Plamenevskaya and J. Van Horn-Morris, *Planar open books, monodromy factorizations and symplectic fillings*, *Geom. Topol.* **14**, no. 4, 2077–2101 (2010).
- [160] R. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables*, Springer (1986).
- [161] R. Richberg, *Stetige streng pseudokonvexe Funktionen*, *Math. Annalen* **175**, 251–286 (1968).
- [162] A. Ritter, *Topological quantum field theory structure on symplectic cohomology*, arXiv:1003.1781.
- [163] H. Rossi, *Attaching analytic spaces to an analytic space along a pseudoconcave boundary*, *Proc. Conf. Complex Analysis (Minneapolis, 1964)*, 242–256, Springer (1965).
- [164] L. Rudolph, *Quasipositivity as an obstruction to sliceness*, *Bull. Amer. Math. Soc.* **29**, 51–59 (1993).
- [165] D. Salamon and J. Weber, *Floer homology and the heat flow*, *Geom. Funct. Anal.* **16**, no. 5, 1050–1138 (2006).
- [166] J. Schürmann, *Embeddings of Stein spaces into affine spaces of minimal dimension*, *Math. Ann.* **307**, no. 3, 381–399 (1997).
- [167] P. Seidel and I. Smith, *The symplectic topology of Ramanujam’s surface*, *Comment. Math. Helv.* **80**, no. 4, 859–881 (2005).
- [168] P. Seidel, *Fukaya categories and Picard–Lefschetz theory*, *Zurich Lectures in Advanced Mathematics*, European Math. Soc. (2008).
- [169] P. Seidel, *A biased view of symplectic cohomology*, *Current Developments in Mathematics, 2006*, 211–253, Int. Press (2008).
- [170] J-P. Serre, *Quelques problèmes globaux relatifs aux variétés de Stein*, *Colloque sur les fonctions de plusieurs variables (Bruxelles, 1953)*, 57–68, Georges Thone, Liège; Masson & Cie, Paris (1953).
- [171] S. Smale, *A classification of immersions of the two-sphere*, *Trans. Amer. Math. Soc.* **90**, 281–290 (1958).
- [172] S. Smale, *The classification of immersions of spheres in Euclidean spaces*, *Ann. of Math.* **69**, 327–344 (1959).
- [173] S. Smale, *On the structure of manifolds*, *Amer. J. Math.* **84**, 387–399 (1962).
- [174] I. Smith, *Torus fibrations on symplectic four-manifolds*, *Turkish J. Math.* **25**, 69–95 (2001).
- [175] J. Sotomayor, *Generic bifurcations of dynamical systems*, *Dynamical Systems (Proc. Sympos. Univ. Bahia, Salvador, 1971)*, Academic Press, 561–582 (1973).
- [176] J. Stallings, *The piecewise-linear structure of Euclidean space*, *Proc. Cambridge Philos. Soc.* **58**, 481–488 (1962).
- [177] N. Steenrod, *The topology of fibre bundles*, Princeton Univ. Press, Princeton (1951).
- [178] K. Stein, *Analytische Funktionen mehrerer komplexer Veränderlichen zu vorgegebenen Periodizitätsmoduln und das zweite Cousinsche Problem*, *Math. Ann.* **123**, 201–222 (1951).
- [179] E. Stout, *Polynomial Convexity*, Birkhäuser (2007).
- [180] D. Struppa, *The first eighty years of Hartogs’ theorem*, *Geometry Seminars 1987–1988*, Univ. Stud. Bologna, 127–209 (1988).

- [181] D. Sullivan, *Cycles for the dynamical study of foliated manifolds and complex manifolds*, Invent. Math. **36**, 225–255 (1976).
- [182] S. Tabachnikov, *An invariant of a submanifold that is transversal to a distribution* (Russian), Uspekhi Mat. Nauk **43** (1988), no. 3 (261), 193–194; translation in Russian Math. Surveys **43**, no. 3, 225–226 (1988).
- [183] R. Thompson, *Singular values and diagonal elements of complex symmetric matrices*, Linear Algebra Appl. **26**, 65–106 (1979).
- [184] W. Thurston, *Some simple examples of symplectic manifolds*, Proc. Amer. Math. Soc. **55**, no. 2, 467–468 (1976).
- [185] C. Viterbo, *Functors and computations in Floer homology with applications I*, Geom. Funct. Anal. **9**, no. 5, 985–1033 (1999).
- [186] A. Weinstein, *Symplectic manifolds and their Lagrangian submanifolds*, Advances in Math. **6**, 329–346 (1971).
- [187] A. Weinstein, *Contact surgery and symplectic handlebodies*, Hokkaido Math. J. **20**, 241–251 (1991).
- [188] C. Wendl, *Strongly fillable contact manifolds and J -holomorphic foliations*, Duke Math. J. **151**, no. 3, 337–384 (2010).
- [189] H. Whitney, *Analytic extensions of differentiable functions defined in closed sets*, Trans. Amer. Math. Soc. **36**, no. 1, 63–89 (1934).
- [190] H. Whitney, *Differentiable manifolds*, Ann. of Math. (2) **37**, no. 3, 645–680 (1936).
- [191] H. Whitney, *The self-intersections of a smooth n -manifold in $2n$ -space*, Ann. of Math. (2) **45**, 220–246 (1944).
- [192] H. Whitney, *On singularities of mappings of Euclidean spaces I. Mappings of the plane into the plane*, Ann. of Math. (2) **62**, no. 3, 374–410 (1955).
- [193] H. Whitney, Interview with A. Tucker and W. Aspray, 10 April 1984, The Princeton Mathematics Community in the 1930s, Transcript Number 43 (PMC43), The Trustees of Princeton University (1985).
- [194] W.-T. Wu, *On the isotopy of C^r -manifolds of dimension n in euclidean $(2n + 1)$ -space*, Sci. Record (N.S.) **2** 271–275 (1958).
- [195] R. Ye, *Filling by holomorphic curves in symplectic 4-manifolds*, Trans. Amer. Math. Soc. **350**, no. 1, 213–250 (1998).

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A beautiful and comprehensive introduction to this important field.

—*Dusa McDuff, Barnard College, Columbia University*

This excellent book gives a detailed, clear, and wonderfully written treatment of the interplay between the world of Stein manifolds and the more topological and flexible world of Weinstein manifolds. Devoted to this subject with a long history, the book serves as a superb introduction to this area and also contains the authors' new results.

—*Tomasz Mrowka, MIT*

This book is devoted to the interplay between complex and symplectic geometry in affine complex manifolds. Affine complex (a.k.a. Stein) manifolds have canonically built into them symplectic geometry which is responsible for many phenomena in complex geometry and analysis. The goal of the book is the exploration of this symplectic geometry (the road from “Stein to Weinstein”) and its applications in the complex geometric world of Stein manifolds (the road “back”). This is the first book which systematically explores this connection, thus providing a new approach to the classical subject of Stein manifolds. It also contains the first detailed investigation of Weinstein manifolds, the symplectic counterparts of Stein manifolds, which play an important role in symplectic and contact topology.

Assuming only a general background from differential topology, the book provides introductions to the various techniques from the theory of functions of several complex variables, symplectic geometry, h -principles, and Morse theory that enter the proofs of the main results. The main results of the book are original results of the authors, and several of these results appear here for the first time. The book will be beneficial for all students and mathematicians interested in geometric aspects of complex analysis, symplectic and contact topology, and the interconnections between these subjects.

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