

# CONTEMPORARY MATHEMATICS

PROCEEDINGS OF THE CONFERENCE  
ON

**Integration,  
Topology, and  
Geometry in  
Linear Spaces**

AMERICAN MATHEMATICAL SOCIETY

**VOLUME 2**

# CONTEMPORARY MATHEMATICS

---

## **Titles in this Series**

- VOLUME 1**    **Markov random fields and their applications**  
Ross Kindermann and J. Laurie Snell
- VOLUME 2**    **Proceedings of the conference on integration,  
topology, and geometry in linear spaces**  
William H. Graves, Editor

# **CONTEMPORARY MATHEMATICS**

**Volume 2**

**PROCEEDINGS OF THE CONFERENCE  
ON**

## **Integration, Topology, and Geometry in Linear Spaces**

**AMERICAN MATHEMATICAL SOCIETY**

**Providence • Rhode Island**

PROCEEDINGS OF THE CONFERENCE ON  
INTEGRATION, TOPOLOGY, AND GEOMETRY IN LINEAR SPACES

HELD AT THE UNIVERSITY OF NORTH CAROLINA  
CHAPEL HILL, NORTH CAROLINA

MAY 17–19, 1979

EDITED BY  
WILLIAM H. GRAVES

1980 Mathematics Subject Classifications. 28–02, 28A15, 28A20, 28A25, 28A33,  
28B05, 28B10, 28C05, 46B22, 46G05, 46G10, 46E27, 46E40.

**Library of Congress Cataloging in Publication Data**

Conference on Integration, Topology, and Geometry in Linear Spaces, University of North  
Carolina, 1979.

Proceedings of the Conference on Integration, Topology, and Geometry in Linear Spaces,  
held at the University of North Carolina, Chapel Hill, May 17–19, 1979.

(Contemporary mathematics; v. 2)

Includes bibliographies.

1. Integrals, Generalized—Congresses. 2. Measure theory—Congresses. 3. Linear topo-  
logical spaces—Congresses. 4. Pettis, Billy James. I. Graves, William Howard, 1940— II. Title.  
III. Series: Contemporary mathematics (Providence, R. I.); v. 2.

QA312.C575 1979

515.4'3

80-25417

ISBN 0-8218-5002-4

Copyright © 1980 by the American Mathematical Society

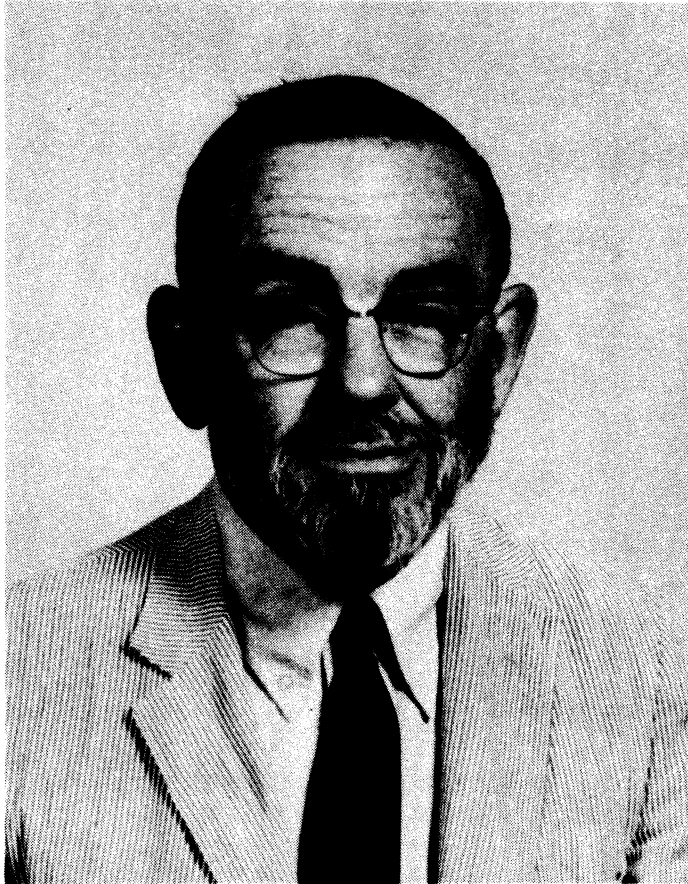
Printed in the United States of America

All rights reserved except those granted to the United States Government

This book may not be reproduced in any form without the permission of the publishers

## CONTENTS

Introduction.....	vii
Strict topologies in measure theory..... HERON S. COLLINS	1
A survey of results related to the Dunford-Pettis property..... JOE DIESTEL	15
An expansion theorem..... NELSON DUNFORD	61
The Radon-Nikodym property..... R. E. HUFF	75
The Orlicz-Pettis theorem..... N. J. KALTON	91
Applications of vector measures..... IGOR KLUVANEK	101
Pettis's measurability theorem..... J. J. UHL, JR.	135
Closed measures..... CECILIA H. BROOK and WILLIAM H. GRAVES	145
Weak and strong compactness in the space of Pettis integrable functions..... JAMES K. BROOKS and NICOLAE DINCULEANU	161
Compactness in spaces of vector-valued measures and a natural Mackey topology for spaces of bounded measurable functions..... WILLIAM H. GRAVES and WOLFGANG RUESS	189
Axiomatic infinite sums—an algebraic approach to integration theory..... DENIS HIGGS	205
On some classes of Banach spaces and generalized harmonic analysis..... KA-SING LAU	213
Applied functorial semantics, III: Characterizing Banach conjugate spaces..... F. E. J. LINTON	227
Stonian differentiation and representation of vector functions and measures..... DENNIS SENTILLES	241



These Proceedings are dedicated to

**B. J. PETTIS**

**1913–1979**

## INTRODUCTION

The papers in this collection are dedicated to the memory of Billy James Pettis who died of cancer at age 65 on April 14, 1979, shortly before his planned retirement as Professor of Mathematics, a position which he had held with distinction for 22 years at the University of North Carolina. His death barely preceded the Conference on Integration, Geometry, and Topology in Linear Spaces, which had been organized to coincide with his retirement. Professors Collins, Diestel, Huff, Kalton, Kluvanek, and Uhl spoke at the conference, and their contributions to this volume reflect their talks of May 17 and 18, 1979. Professor Dunford's paper represents the talk which he would have delivered had he not had to cancel his attendance at the last moment. The remaining papers herein were contributed by conference participants and, in the case of Professors Brooks and Dinculeanu, others who were forced to cancel their planned participation.

B. J. Pettis was raised in Spartanburg, South Carolina, where his father served Wofford College for many years as a teacher of mathematics and physics and his mother taught in the public schools.

He earned his B. A. from Wofford College in 1932, M. A. from the University of North Carolina in 1933, and Ph. D. from the University of Virginia in 1937. He then was Dupont Research Fellow at the University of Virginia in 1937–1938, Sterling Research Fellow at Yale in 1938–1939, and B. O. Peirce Instructor at Harvard from 1939 through early 1941 when he volunteered as a Private in the Army of the United States. After serving five years in the Army in Australia and Luzon and in combat in New Guinea and rising to the rank of Captain, he returned to mathematics. He served at Yale University (1945–1947), Tulane University (1947–1957, including one year as Chairman of the Department of Mathematics), Princeton University (1949–1950 as Visiting Lecturer), the National Science Foundation (1964–1965 as Science Faculty Fellow), Norfolk State College (1966–1977 as part time Visiting Professor), and, of course, the University of North Carolina (1957–1979).

Among his pre-war mathematical achievements were results which are now known to the mathematical world as the Orlicz-Pettis theorem, the Dunford-Pettis theorem, the Pettis theorem on measurability, and the theory of the Pettis integral. These alone would assure his place in mathematical history, but despite the professionally fracturing effect of serving in the Army, he returned from the war to write many widely read research articles and contribute to several books, including a now standard source: *Linear Topological Spaces* by Kelley, Namioka, and others. *Vector Measures* by Diestel and Uhl is dedicated to Bill Pettis. The authors of the latter note that "as we progressed in the study of the history of the basic theorems of the theory of vector measures, we were not surprised by learning that most of them,

in one way or another, have their origins in the fertile mind of one man, B. J. Pettis." This dedication, a conference in 1975 on Pettis integration, and the conference represented by this volume attest to the value of his scientific life.

With his international reputation as a mathematical researcher and scholar came numerous invitations to serve his profession in broad and interesting ways. He was sought as a colloquium speaker and lecturer at universities, conferences, and institutes around the world. Leading mathematical journals such as the *Bulletin of the American Mathematical Society* and the *Duke Journal* sought and received his editorial services. He was a consultant in many capacities to many institutions, agencies, foundations, and societies including the National Science Foundation, the Woodrow Wilson Foundation, the Office of Naval Research, the Educational Testing Service, the American Mathematical Society, the Mathematical Association of America, the National Council of Teachers of Mathematics, the School Mathematics Study group, the Institute of International Education, numerous school systems, and several corporations, including the Education Development Corporation. It was in the service of the Education Development Corporation that over almost a decade he significantly contributed to the development of teacher training and mathematical curricula in eleven developing African countries as a participant and leader in several on-site institutes and writing projects. He held membership in many professional and honorary societies: the American Mathematical Society, the Mathematical Association of America, the Société Mathématique de France, Sigma Xi, the American Association for the Advancement of Science, the North Carolina Academy of Science, the National Council of Teachers of Mathematics, the American Association of University Professors, and Phi Beta Kappa.

No such formal listing of accomplishments can describe this remarkable man. He possessed a powerful yet graceful clarity of mind and expression. He greatly valued humbleness and good manners. That he could do so while holding himself, his colleagues, his friends, and his students to the highest standards is a tribute to his complete lack of pretension and his constant yet unobtrusive concern for the comfort and feelings of those around him. Some students will remember him as an inspirational teacher who taught them to discover their very best efforts within themselves. To many of his students he provided a first, and sometimes an only, encounter with depth and intellectual passion, and he will be remembered by them with a mysterious sense of respect. His ability and penchant for engaging others in gently probing, self-revealing conversation will be fondly remembered by those who knew him.

Bill Pettis loved his wife, Mary, his children, and all of his family. He immensely enjoyed the active appreciation of books, music, and wine which he shared with Mary, his family, and his friends. No theme more fascinated him than that of development. The development into graceful maturity of a young wine or a budding Ph. D. student afforded him a pleasure akin to that he derived and cherished from the development of his children. There was in him a mischievous, boyish quality which relished long evenings of wine and spirited conversation and hastily planned, devil-may-care visits to wine shops, book stores, and museums in distant cities. This impish aspect of his personality delighted his family and friends and complemented his graceful gentlemanly demeanor.



One of the finest assessments of the value of Bill Pettis to his colleagues was written to him by a friend when he learned of Bill's illness. "It wasn't too long after I'd come to know you that I felt 'There is a man to measure yourself by'. Not just things mathematical, for these the measure was obvious and woefully short on my end. But in all things. I felt in meeting you that I had finally come in contact with one of those special individuals, a kind I had only known before through time and distance on the printed page. You seemed a living link with those ideas and men who had fired me from as far back as I could remember. The clarity and depth of your expression remains my envy. I know you're no Newton or Jefferson or Gauss, but in my still boyish imagination I felt that in knowing you I had experienced some touch, however slight, with such as these."

He will long be remembered for his very positive influence on mathematics, on friends and colleagues, and on the shape of the Department of Mathematics at the University of North Carolina in Chapel Hill. He was a gentleman. He was a scholar. He was a man of worth.

William H. Graves  
Robert L. Davis  
Fred B. Wright

