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Advanced Analytic Number Theory
Part 1: Ramification Theoretic Methods

Carlos J. Moreno

American Mathematical Society

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PREFACE

These notes are intended as an introduction to those aspects of analytic number theory which depend on and have applications to the theory of algebraic numbers. As is well known the central problem of the theory is the general algebraic formulation of the following distribution results: prime number theorem, Dirichlet's theorem on primes in arithmetic progressions, Cebotarev's density theorem on the distribution of Frobenius conjugacy classes in Galois groups, Hecke's density theorem on the distribution of the arguments of quasi-characters of idele class groups, Sato-Tate conjectural densities for the value distribution of the traces of Frobenius elements, strong multiplicity one theorem for automorphic representations of the general linear group, etc. These are all variants of the following *fundamental distribution problem*. Let G be the Galois group of the separable closure of the rationals; let F_p be the Frobenius conjugacy class associated with the rational prime p; let $r : G \to GL(V)$ be a representation of G on some (l-adic) vector space V. The main problem then is the study of weighted sums of the form

$$\sum_p \chi(F_p)w_p, \qquad (\chi = tr(r)),$$

where p runs over finite sets of primes. The basic analytic tools needed for the study of such sums are the so called explicit formulas of prime number theory which relate sums over primes with sums over zeros of L-functions. The principal aim of these notes is to develop these formulas and to give some of their applications.

Part I: Ramification Theoretic Methods treats all the necessary local aspects of algebraic number theory that appear in the formal description of the structure of the general explicit formulas, notably the integral representation of the Herbrand distribution. We believe that Part I in itself is of independent interest and indeed chapters I, II, IV and V could be used as an introduction to local class field theory.

Part II: L-functions treats certain global aspects of algebraic number theory, e.g. relative Weil groups of number fields and their finite dimensional representations, functional equations of L-functions, distribution of zeros, theorems of Hadamard-De La Vallee Poussin, explicit formulas, generalized prime number theorems, lower bounds for conductors, etc. For a precise description of the new results see Part I, Chapter 0, §2.

We are indebted to Weil's Basic Number Theory for the influence it has had on our presentation. Undoubtedly in our mind this book has paved the way from Hecke's classical approach to algebraic number theory and modular forms to the recent advanced developments in the theory of automorphic forms. Our discussion of the ramification theory of abelian local extensions, Shafarevitch's theorem and the Herbrand distribution follows rather closely that of the third edition.

These notes were originally prepared to serve the author as a dictionary for future reference. It now appears that the elementary and informal nature of our presentation may also serve other number theorists and algebraists as an introduction to an area of mathematics which traditionally has been considered difficult and inaccessible. The author hopes that they are more pleasant to read than they were to write.

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