Factorizations of $b^n \pm 1$
$b=2, 3, 5, 6, 7, 10, 11, 12$
Up to High Powers

John Brillhart
D. H. Lehmer
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10 9 8 7 6 5 4 3 2 02 01 00 99 98 97
This work is dedicated to the memory of Lt.-Col. Allan J. C. Cunningham (1842-1928), an industrious computer and maker of mathematical tables.
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3^n - 1 & \quad n \text{ odd} \\
3^n + 1 & \quad n \leq 350 \\
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5^n - 1 & \quad n \text{ odd} \\
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