Chapter 9
of Ramanujan's
Second Notebook

Infinite Series Identities,
Transformations, and Evaluations

Bruce C. Berndt
Padmini T. Joshi
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Second Notebook

Infinite Series Identities,
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American Mathematical Society
Providence, Rhode Island
1980 *Mathematics Subject Classification.* Primary 33A15, 33A30, 30A10, 10A40, 40-00.

Library of Congress Cataloging in Publication Data
Berndt, Bruce C., 1939—
Chapter 9 of Ramanujan's second notebook.

(Contemporary mathematics, ISSN 0271-4132; v. 23)
Bibliography: p.
IV. Series: Contemporary mathematics (American Mathematical Society); v. 23.

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The entries in Chapter 9 of Ramanujan's Second Notebook 3
Chapter 9 represents one of the more unified chapters in the second notebook. As in other chapters, it contains a mixture of both new and old results. This chapter is characteristic of Ramanujan's greatest talent and love—dazzlingly beautiful formulas for infinite series. Apéry's proof of the irrationality of \( \zeta(3) \) is but one example of many demonstrating that elegant formulas are often very useful as well. Chapter 9 contains many formulas bearing the same features as Apéry's formula for \( \zeta(3) \). We trust that some of Ramanujan's formulas herein will be valuable, indeed, to present day researchers. But we also hope that these marvelous formulas will foster the same kind of majestic thrill and sublime upliftment that listening, for example, to a Beethoven symphony engenders.

Urbana, Illinois
June, 1983