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PREFACE

Recently there have appeared numerous articles on the beautiful and classical Nevanlinna value distribution theory and its applications. More specifically, in areas such as defect relations, growth estimation of solutions of differential equations, distribution of zeros and poles of meromorphic functions, entire functions of bounded index, factorization theory, etc. These results have appeared in many different journals in various parts of the world.

In April of 1983 a special session on value distribution theory and its application was held in New York City as part of the 803rd meeting of the American Mathematical Society. The purpose of this session was to bring together some of the mathematicians working in this active field, to present the results of their research, to explore and exchange problems among themselves and to stimulate further research in this area.

Most of the papers in this collection were presented at the special session and have never before been published. For instance, announced here by Charles Osgood is his complete proof of the validity of the analogous Nevanlinna second fundamental theorem for small functions (i.e. defect relations for small functions). This has been a challenge problem for complex analysts for a long time.

I wish to thank all the mathematicians who participated in the special session and for their contributions to this volume. Special thanks are extended to those from abroad: Professors Chi-Tai Chuang and Yong-Xing Gu from China, Professors Kiyoshi Niino and Seiki Mori from Japan, Professor Shlomo Strelitz from Israel, Professor Sakari Toppila from Finland, and Professor L.S.O. Liverpool from Nigeria. Also I wish to thank the American Mathematical Society and the editorial board of the Contemporary Mathematics Series for their approval of this publication and also the editorial department for many helpful suggestions.

In conclusion, it is the unanimous feeling of all the contributors to this volume that the late Professor Rolf Nevanlinna has inspired us in many different ways in developing and searching for further results in this field, both in one variable and in several variables. Therefore it is a privilege and our opportunity to acknowledge our debt to him and to dedicate this collection to his memory.

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