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Fluids and Plasmas: Geometry and Dynamics

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INTRODUCTION

The intention of this conference was to foster interaction among people working on mathematical, numerical and physical aspects of fluid and plasma dynamics. To this end, the organizing committee consisting of Jerry Marsden (Chairman), Philip Holmes and Andy Majda, with Alex Chorin and Alan Weinstein as advisors, chose 27 speakers from the three sub-areas whom we felt would foster good interaction. We worried, though, that the conference would degenerate into three subconferences with specialists talking to only themselves and leaving the audience bewildered. As it turned out, the opposite happened. The conference developed a sense of camaraderie and the speakers made every effort to bridge communication gaps, despite the inevitable differences in taste and background needs that could not all be met. Another worry, prompted by our common experiences at numerous conferences, was that speakers, in their enthusiasm, would run overtime. We offered a non-NSF sponsored prize of $100 for the best lecture with the imposed necessary condition of not running overtime. This worked marvelously -- it was worth every penny. The conference participants voted in the last session to award Harry Swinney the prize for his lecture "Observations of instabilities and chaos in hydrodynamic and chemical systems". Runners-up were Alan Weinstein, Allan Kaufman, Norman Zabusky, and Alan Newell, who all presented exceptional lectures.

It is impossible for me to give a fair and adequate survey of the highlights of the conference, but I shall try to convey the flavor of a few points that I knew about or caught my attention.
The organizing committee envisioned bringing together three groups of people working on the following topics in fluid and plasma dynamics:

1. **Geometric aspects**: Hamiltonian structures, perturbation theory and nonlinear stability by variational methods,

2. **Analytical and numerical methods**: contour dynamics, spectral methods, and functional analytic techniques,

3. **Dynamical systems aspects**: experimental and numerical methods, bifurcation theory, and chaos.

Of course, we could have easily spent our entire budget on any one of these areas. But our purpose was to emphasize interaction rather than comprehensiveness.

Let me comment a little on some of the background for these three items, why they are all exciting developing areas, and how they interrelate.

The geometric methods center on outgrowths of Arnold's article "Sur la géométrie différentielle des groupes de Lie de dimension infinie et ses applications à l'hydrodynamic des fluids parfaits", Ann. Inst. Fourier, Grenoble, 16 (1966) 319-361. Arnold discovered the relationship between the Lagrangian and Eulerian description of an incompressible fluid in group theoretic terms. In the Lagrangian description, the phase space is the tangent or cotangent bundle of the group of volume preserving diffeomorphisms with its usual canonical symplectic structure. Each such diffeomorphism represents a possible fluid configuration relative to a fixed reference configuration. In the Eulerian description, the phase space is its Lie algebra (or its dual), the space of divergence free vector fields (or the space of vorticities). The passage from the canonical Lagrangian description to the noncanonical Eulerian description is an example of what we now call reduction, a general procedure for elimination symmetries in a system (see the books on classical mechanics by Arnold, "Mathematical Methods of Classical Mechanics", Springer (1978),
INTRODUCTION

and Abraham and Marsden, "Foundations of Mechanics", Addison Wesley, (1978), for accounts). Arnold worked with the Lie algebra but it is now generally preferred to use its dual, which carries a natural bracket structure on real valued functions on the dual; this is the Lie-Poisson bracket discovered by Lie in 1890. For incompressible fluids, this dual is identified with the space of vorticities and in two dimensions (for example) the Lie-Poisson bracket on functions of scalar vorticity $\omega$ is given by the vorticity bracket

$$\{F,G\}(\omega) = \int_D \omega \left( \frac{\delta F}{\delta \omega}, \frac{\delta G}{\delta \omega} \right)_{xy} \, dx \, dy$$

where $D \subset \mathbb{R}^2$, is the domain for the fluid, $\delta F/\delta \omega$ is the functional derivative and $\{ , \}_{xy}$ is the standard Poisson bracket in the plane with $x$ and $y$ as conjugate variables. The vorticity equations of motion become $\dot{F} = \{F,H\}$, where $H$ is the kinetic energy, expressed in terms of the vorticity.

Arnold used Hamiltonian methods (constrained second variations and convexity estimates) to study the stability of two dimensional incompressible flows, obtaining a nonlinear version of the classical Rayleigh inflection point criterion for linearized stability. This was a brilliant achievement that received only a fraction of the attention it deserved. Arnold's work is found in several references around 1966 that are cited in Appendix 2 of his mechanics book. In one especially important work, Arnold supplies rigorous convexity estimates; in English translation, it is "On an a priori estimate in the theory of hydrodynamic stability", Trans. Am. Math. Soc. 79 (1969) 267-269.

Formal stability results based on second variation methods occurred in the plasma literature, independently of Arnold's ideas. Results of Newcomb, Rosenbluth, Kruskal, Bernstein, Gardner and others, were published between 1958 and 1965. An account of this development to 1969 can be found in the book of Clemow and Dougherty "Electrodynamics of
INTRODUCTION


Poisson brackets for MHD and the Maxwell-Vlasov equations governing plasma motion were found in 1980 by Morrison and Greene. Allan Kaufman played an important role in bridging the mathematics-physics gap by explaining this work to Alan Weinstein and me. We subsequently showed (Physica 4D (1982) 394-406) how to obtain the Maxwell-Vlasov bracket by Arnold's methods (again a reduction from a Lagrangian to an Eulerian description) and by utilizing this method, corrected one of the terms in Morrison's bracket. The method of Clebsch variables was developed shortly afterwards by Morrison, Holm and Kupershmidt. These various approaches quickly became united and were applied to a variety of systems as the understanding of Hamiltonian structures deepened and the stability results were extended. In fact, they are currently being applied to rather exciting problems such as tokamaks, three dimensional multifluid plasmas, internal waves in the ocean, and to externally stabilized plasmas.

The above setting provides a backdrop for the subjects treated in the lectures or contributions of Darryl Holm, Robert Littlejohn, Richard Montgomery, Phil Morrison, Meinhard Mayer, Allan Kaufman, Alan Weinstein, and Tudor Ratiu. Peter Olver talked about a way one might bridge the gap between these bracket structures and those for water waves, by asymptotic expansion methods. Gerald Goldin explained how one might use these classical structures in quantum field theoretic situations via represenations of semi-direct products involving the diffeomorphism group. Chuck Leith described how enstrophy and its generalizations are used in geostrophic turbulence. Generalized enstrophy is, not coincidentally, a key ingredient in Arnold's stability method. It is a
Casimir in the sense that, using the vorticity bracket, it commutes with every function of vorticity. The papers of Miroslav Grmela, Harvey Segur, and the joint paper of myself, Phil Morrison and Alan Weinstein deal with various aspects of kinetic theory, emphasizing Hamiltonian structures.

The analytic and numerical areas stressed were those that had some relationship with the basic mathematical structures for fluids and plasmas. Techniques available for specific numerical implementation naturally came up, but were not emphasized. The interaction between theory and practice is nicely illustrated by the work of Glimm and Chorin. The random choice methods they use for both compressible and incompressible flow are based on a deep understanding of the basic theory and are very successful numerically. (See the books of Chorin and Marsden, "A Mathematical Introduction to Fluid Mechanics," Springer (1979) and Smoller "Shock Waves and Reaction-Diffusion Equations", Springer (1983) for further details and references). Not only do existence and uniqueness theorems tie into these methods, but so do the geometric aspects of the equations. For example, asking that a code be consistent with the Hamiltonian structure as far as possible could be a useful way to improve or debug it, or even to design new numerical algorithms.

Existence and Uniqueness theorems for the Poisson-Vlasov equations were discussed by Steve Wollman, Robert Glassey and Walter Strauss. Tom Beale and Andy Majda discussed the obstruction to continuability of three dimensional solutions to the Euler equations in terms of sharp bounds on the vorticity. This is related to numerical studies of the problem that were reported by Dan Meiron. Numerical aspects of Chorin's vorticity algorithm were presented by James Sethian. Zabusky described his program for numerical implementation of contour dynamics. His methods have had a very useful influence on the theory, as was demonstrated in Steve Wan's lecture in which he used inspiration from both Zabusky's work and that of Arnold to show the dynamical stability of circular vortex patches. Phillip Spallart and Steve Orszag presented
state of the art methods for numerical fluid problems using spectral
methods, while Jerry Brackbill and John Dawson concentrated on particle
methods in plasma problems and Bob Miller dealt with numerical methods
in oceanography.

Most of the remaining talks fell into the third category of dynamical
systems aspects. Thirteen years ago, when Ruelle and Takens first intro-
duced ideas of chaos into fluid mechanics, many people thought it was a
crazy idea. However, even by then, dynamical systems methods were already
making large strides. Lorenz in his famous 1963 paper (J. Atmos Sci. 20
(1963), 130-141) had already very convincingly shown the presence of chaos
in a deterministic system. In the late 1960's and early 1970's, Judovich,
Sattinger, Joseph, Iooss, and Marsden had shown how the Hopf bifurcation
and ideas of infinite dimensional dynamical systems can be rigorously
applied to yield an understanding of fluid oscillations. Presently "chaos
is in"; skeptics have been largely converted to the usefulness of the
ideas.

In John David Crawford's talk, dynamical systems ideas and the Hopf
bifurcation especially were applied to the beam-plasma instability. In
Harry Swinney's lecture we saw the Hopf bifurcation used as a basic building
block toward understanding the more complex chaotic dynamical behavior he
was observing in his experiments. His data on Couette flow indicates the
presence of a strange attractor of small fractal dimension, this dimension
ranging continuously from about 4 to 7 in the experiments reported. Alan
Newell gave a marvelous illustration of the mixture of ideas from solitons
and chaos in laser optics. Philip Holmes' contribution deals with chaotic
particle paths near a solitary wave that occurs in helical flow in a cylindri-
cal tube. In John Guckenheimer's lecture we saw a program emerging for
how one might distinguish noise from deterministic chaos. In this regard,
one should keep in mind that in many systems (such as the Henon attractor)
it is not a priori clear if the chaos is due to a genuine strange attrac-
tor or to a slightly noisy but complicated tangle of horseshoes and sinks.
It is thus important to develop tests and basic theory which try to make these distinctions. As the lectures and contributions of James Curry, Ed Ott, Jurgen Scheurle, Eric Siggia and Jim Swift demonstrated, while great strides in the basic theory of bifurcations and chaos have been made, the full story is by no means complete. For example, how strange attractors come and go and are related to the more analytically tractible transverse homoclinic bifurcations is still a subject of research interest.

I wish to thank all the conference participants for their energetic and thoughtful lectures, contributions, questions and interaction. Special thanks go to the AMS for administering the conference, especially Carole Kohanski who did most of the detailed work and saw that things ran smoothly. Ruth Edmonds was a great help with organizing the conference and this volume. Connie Calica did a beautiful job typing many of the papers. Finally, the NSF is gratefully acknowledged for their wisely spent financial support.

Jerrold E. Marsden
Berkeley, January 1984
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