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INTRODUCTION

Dorothy B. Shaffer

Most of the mathematical ideas presented in this volume are based on papers given at the Meeting of the American Mathematical Society, held at Fairfield University in October 1983. The talks were presented in a Special Session on "Topics in Complex Analysis"; the unifying theme of the session was Geometric Function Theory. The twenty-minute talks described the latest results in univalent function theory and related topics.

The papers in this volume generally represent extended versions of the talk presented by one of the authors. In addition, the Proceedings contain several papers that could not be given in person. A few of the papers have been expanded to include further research results obtained in the time between the conference and submission of manuscripts. In most cases, an expository section or history of recent research has been added. The authors' new research results are incorporated into this more general framework. The collection of the papers represents a survey of research carried out in recent years in a variety of topics.

The paper by Professor Y. J. Leung deals with the Loewner equation, classical results on coefficient bodies and modern optimal control theory. Professor Glenn Schober writes about the class $\Sigma$, its support points and extremal configurations. Professor Peter Duren deals with support points for the class $S$, Loewner chains and the process of truncation.

A very complete survey about the role of polynomials and their limits in class $S$ is contributed by Professor T. J. Suffridge.
A generalization of the univalence criterion due to Nehari and its
relation to the hyperbolic metric is contained in the paper by Professor
David Minda. The omitted area problem for functions in class $S$ is solved
in the paper by Roger Barnard. New results on angular derivatives and
domains are represented in the paper by Professors Rodin and Warschawski,
while estimates on the radial growth of the derivative of univalent
functions are given by Professor Thomas McGregor.

In the paper by Bshouty and Hengartner a conjecture of Bombieri
is proved for some cases. Other interesting problems for special
subclasses are solved by Case and Quine; Reade, Silverman and Todorov;
Silverman and Silvia.

New univalence criteria for integral transforms are given by
Professor Edward Merkes. Potential theoretic results are represented in
the paper by Jack Quine with new results on the Star Function and by
Professor David Tepper with free boundary problems in the flow around an
obstacle. Approximation by functions which are the solutions of more
general elliptic equations are treated by Dufresnoy, Gauthier and Ow.

At the time of preparation of these manuscripts, nothing was known
about the proof of the Bieberbach conjecture. Many of the authors of this
volume and other experts in the field were recently interviewed by the
editor regarding the effect of the proof of the conjecture. Their ideas
regarding future trends in research in Complex Analysis are presented in
the epilogue to these papers.
LIST OF SPEAKERS
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