CONTEMPORARY MATHEMATICS

52

Geometry of Normed Linear Spaces

Proceedings of a Conference
held June 9–12, 1983
in honor of Mahlon Marsh Day

American Mathematical Society

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CONTEMPORARY MATHEMATICS

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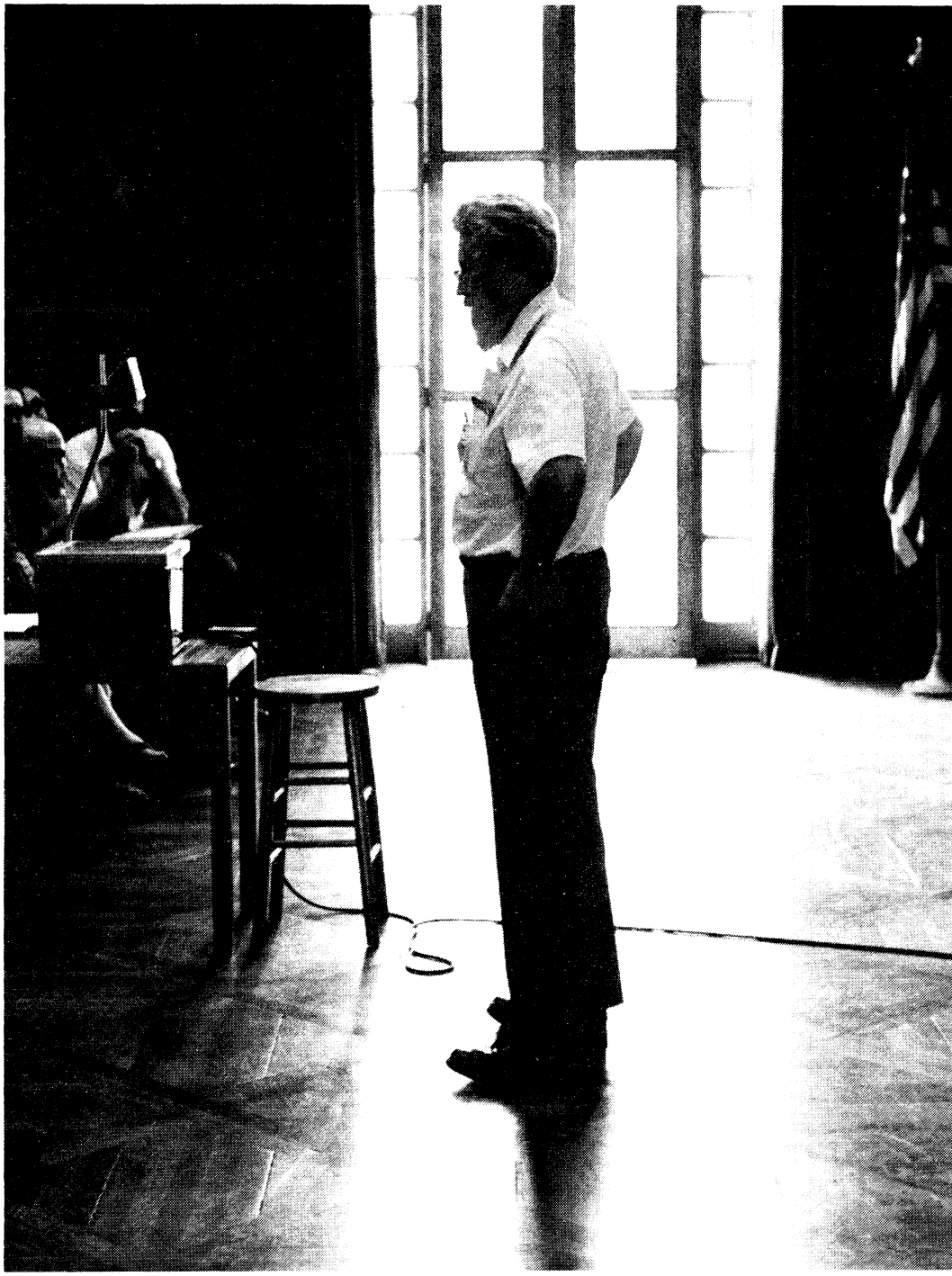
Geometry of Normed Linear Spaces



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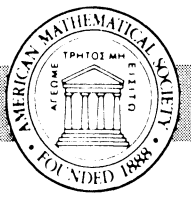
Mahlon Marsh Day



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R. G. Bartle, N. T. Peck,
A. L. Peressini, and J. J. Uhl,
Editors

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The Conference on the Geometry of Normed Linear Spaces was held, in honor of Mahlon Marsh Day, at the University of Illinois at Urbana-Champaign, June 9–12, 1983. The conference was supported by the National Science Foundation, Grant MCS 83-01738, and the Research Board of the University of Illinois.

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Foreword

Mahlon Marsh Day was a faculty member at the University of Illinois from 1940 to 1983, and still is active there in his retirement. His influence has been felt through his pioneering articles and his classic monograph, "Normed Linear Spaces", and through his students - undergraduate and graduate.

Mahlon Day, J. A. Clarkson, and Nelson Dunford are all mathematical brothers; all were thesis students of J. D. Tamarkin at Brown University, where Day received his Ph.D. in 1939. He has long been fascinated by all forms of rotundity and convexity, and has been heard to joke, "If I can draw a picture of it, then I can prove it!" He was the first American mathematician to study normed spaces from a geometric standpoint and, for a number of years, he and R. C. James were the only American mathematicians working on the structure of Banach spaces. In recognition of the variety of Day's contributions to Banach space theory each of the main speakers at the conference was invited to take some aspect of Day's work as a starting point for his lecture. Several of Day's students contributed papers to these proceedings.

From 1958 to 1965, at a critical time for the Illinois mathematics department, he served at its Head. He gave needed leadership and significantly shaped the future course of the department. Today, his is a welcome and familiar face in seminars; he is active as an editor and in matters of the Illinois Journal; and he still teaches an occasional course.

His home and extended family are important to him and to his wife, Frankie. They are active members of the Society of Friends and take their beliefs seriously. Their home is always open to anyone who needs a home, whether direct family, foster children, refugees, or misplaced mathematicians.

Mahlon is a profound source of information of all kinds on many topics - ranging from convexity and rotundity to CCC camps in Oregon (where he grew up), to interurban transit (a good way to get to Danville, in those days), to how the glaciers shaped the terrain of east central Illinois and what rivers they left behind.

From his beard, his very deep voice, his red jacket, his bluff yet merry manner and his wisdom, we do not know whether he is Moses or Santa Claus. But when he snoozes at a lecture, and wakes up to ask a penetrating

question that hadn't occurred to us, we are reminded that he is uniquely Mahlon.

We salute him and wish him and Frankie well in their retirement.

The Organizing Committee

R. G. Bartle

N. T. Peck

A. L. Peressini

J. J. Uhl

Acknowledgments

We wish to thank: Pat Coombs and Betsy Gillies, for serving ably as conference secretaries; Jennifer Eickmann, for seeing to it that Allerton House hospitality never flagged; Hiram Paley, for his excellent conference photographs; Peter Casazza, for making same memorable; and Mabel Jones, for her patient, careful typing of the papers. We also thank the National Science Foundation and the Research Board of the University of Illinois for their support of the conference.

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DAYSONG

by

Bruce Reznick

Once upon a thesis dreary, as I pondered weak and weary,
Looking for a reference to a theorem that had gone astray,
While I nodded, nearly napping, suddenly there came a tapping,
As of something gently snapping, snapping in my matter grey;
Just the notion of a bounded mapping in my matter grey.
How I wished it go away!

Presently do I recall now, my advisor in the hall now,
How the thought makes me feel small now, when Per Enflo told me: "Hey!
Banach space has yoost one standard, you mean you have yoost meandered
On your own and haven't gandered on the book by M. M. Day?
Get yourself a copy of the latest Normed Linear Spa-
ces, the book of M. M. Day!"

Oh it was a revelation! How I stared in fascination,
When I saw the terse notation: PND implies QA.
What a book of condensation! What a job of integration!
What a tool for education! What a man, this M. M. Day!
Yes I went out to the Union, bought the book of M. M. Day;
Eighteen dollars did I pay.

So I bought it and I read it, but I add to my discredit,
Rapidly did I forget it, as I wrote my résumés.
Then I heard from P. T. Bateman: "Do not try to tempt your fate, man!"
Illinois' a groovy state, man!" (Here you note I paraphrase.)
"Take a job here, in the evenings, you may like the staff soirées,
In any case, you'll love the Days."

By the bronze of Alma Mater, by the Boneyard's rippling waters,
Mathematics was in Altgeld, and Illini, also Coble.
There I saw a man with white beard, pens in pocket, smile on visage,
Eyes a-twinkle, voice a-rumble, there I met him: Mahlon Day.
"Have some coffee. That much sugar? We'll get along," said Mahlon Day.
And he was right, I want to say.

Franky's not, in principle, the subject of our session,
But she's so much of Mahlon that I must make this digression.
Teacher, colleague, friend to students, counselor and mother,
Opening their house as refuge, keepers of their brother.
Franky, Mahlon: You're the youngest couple in Urbana,
And you'd keep the title if I threw in Indiana!

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Illinois, in Banach Spaces, has so many stars and aces.
Mahlon's uncondish'nal bass is all agreed to as our voice.
Seminars and learned meetings, welcomes with the fondest greeting,
You will hear his fans repeating: "Mahlon is the an'lyst's choice."
Through the years in West Urbana, Altgeld's halls with pride rejoice:
Mahlon is the an'lyst's choice."

Now, upon this banquet cheery, as we sit here, stuffed and beery,
We should not feel sad and teary, for the man we toast today
At this fine and festive dinner, eager as a fresh beginner,
Going strong and still a winner, we salute you, Mahlon Day!
Going strong and still a winner, we salute you, Mahlon Day!

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