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Differential Analysis in Infinite Dimensional Spaces

Proceedings of an AMS Special Session held August 8–10, 1983

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Proceedings of an AMS Special Session held August 8–10, 1983, with partial support from the NSERC (Canada)

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AMERICAN MATHEMATICAL SOCIETY
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1. Complex manifolds—Congresses. 2. Differentiable manifolds—Congresses. I. Sundaresan, Kondagunta, 1931– . II. Swaminathan, Srinivasa. III. Special Session on Differential Analysis in Infinite Dimensional Spaces (1983: Albany, N. Y.) IV. American Mathematical Society. V. Series: Contemporary mathematics (American Mathematical Society); v. 54.
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CONTENTS

PREFACE

During the last two decades there has been a significant development of many topics in differential analysis in infinite dimensional spaces. New techniques, such as ultraproducts and ultrapowers, have thrown light on the relationship between the geometric properties of Banach spaces and the existence of differentiable functions on the spaces.

A special session on Differential Analysis on Infinite Dimensional Spaces was held at the Summer meeting of the American Mathematical Society at SUNY, Albany, N.Y., August 8 - 11, 1983. The session consisted of three meetings of three forty-minute talks each. This volume contains the articles submitted by most of the participants in the special session as well as articles by those who were invited but could not be present at the meeting.

We thank all the participants and the contributors for their cooperation. It is a pleasure to acknowledge our gratitude to the editorial committee of the Contemporary Mathematics Series for including these proceedings in the Series. Finally, we are thankful to the staff of the AMS for their efficient service, help during the Session and cooperation.

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ERRATA

DIFFERENTIAL ANALYSIS IN INFINITE DIMENSIONAL SPACES

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The enclosed material was inadvertently omitted from "Approximation of Differentiable Functions on a Hilbert Space" by M. P. Heble. Please insert it after equation (1), line 15 on page 23. We apologize for any inconvenience this error may have caused.

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In general for any integer $j \in [0,k]$,

$$||D^j\tilde{f}_x(y) - D^jf(y)||_j = ||\Big[D^jf(x) + D^{j+1}f(x)\cdot(y-x) + \cdots + \frac{D^kf(x)}{(k-j)!}(y-x)^{(k-j)}\Big]$$
$$(2) \qquad - \Big[D^jf(x)+D^{j+1}f(x)\cdot(y-x) + \cdots + \int_0^1 \frac{(1-t)^{k-j-1}}{(k-j-1)!} D^kf(x+t(y-x))\cdot \cdot dt(y-x)^{(k)}\Big]||.$$

Now suppose $\eta > 0$; then let $\delta > 0$ such that $\sup_{y\in U}||D^kf(x)-D^kf(y)||_k < \eta$. Then for such y we find that

$$\sup_{y\in U} ||D^j\tilde{f}_x(y) - D^jf(y)||_j < \frac{\eta\cdot\delta^{k-j}}{(k-j)!}, \quad j = 0,1, \ldots, k.$$

This completes the proof of the Corollary.

Next let $X = \{x_1,x_2,\ldots\}$ be a countable dense set in Ω, and write $\varepsilon_n = \varepsilon(x_n)$, for $n = 1,2,\ldots$.

LEMMA 2. (a) ([11], p. 301, p. 308) For each $x \in \Omega$, $\exists$ open ball $B_r(x) \subset \Omega$ satisfying

$$\sup_{y,y'\in B_r(x)} |\varepsilon(y) - \varepsilon(y')| < \inf_{y\in B_r(x)} \frac{\varepsilon(y)}{2}.$$

(b) For each $n = 1,2,\ldots$ and given constant $K_n > 1$ $\exists$ open ball $B_{\rho_n}(x_n) \subset \Omega$ such that (a) holds in $B_{\rho_n}(x_n)$ as also the following: $\exists\ \tilde{f}_n \in C^\infty(\Omega,F)$ satisfying:

$$\forall \text{ integers } j \in [0,k], \quad \sup_{x\in B_{\rho_n}(x_n)} ||D^j\tilde{f}_n(x)-D^jf(x)||_j < \frac{\varepsilon_n\rho_n^{k-j}}{K_n\cdot 2^{n+3}}.$$

PROOF OF LEMMA 2. (a) Let $x \in \Omega$; then $m = \varepsilon(x) > 0$. The continuity of $\varepsilon(\cdot)$ implies that $\exists\, r > 0 \ni ||w-x||<r \Rightarrow |\varepsilon(w) - \varepsilon(x)| < Km$ (where $0 < K < 1$ is a hypothetical constant to be suitably determined presently). Then for $w \in B_r(x)$, $\varepsilon(w) > (1-K)m$, hence

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