Index Theory of Elliptic Operators, Foliations, and Operator Algebras

Proceedings of AMS Special Sessions held January 7, 1986 and April 11, 1986
Titles in This Series

Volume

1 Markov random fields and their applications, Ross Kindermann and J. Laurie Snell

2 Proceedings of the conference on integration, topology, and geometry in linear spaces, William H. Graves, Editor

3 The closed graph and P-closed graph properties in general topology, T. R. Hamlett and L. L. Herrington

4 Problems of elastic stability and vibrations, Vadim Komkov, Editor

5 Rational constructions of modules for simple Lie algebras, George B. Seligman

6 Umbral calculus and Hopf algebras, Robert Morris, Editor

7 Complex contour integral representation of cardinal spline functions, Walter Schempp

8 Ordered fields and real algebraic geometry, D. W. Dubois and T. Recio, Editors

9 Papers in algebra, analysis and statistics, R. Lidl, Editor

10 Operator algebras and K-theory, Ronald G. Douglas and Claude Schochet, Editors

11 Plane ellipticity and related problems, Robert P. Gilbert, Editor

12 Symposium on algebraic topology in honor of José Adem, Samuel Gitler, Editor

13 Algebraists' homage: Papers in ring theory and related topics, S. A. Amitsur, D. J. Saltman, and G. B. Seligman, Editors

14 Lectures on Nielsen fixed point theory, Boju Jiang

15 Advanced analytic number theory, Part I: Ramification theoretic methods, Carlos J. Moreno

16 Complex representations of GL(2, K) for finite fields K, Ilya Piatetski-Shapiro

17 Nonlinear partial differential equations, Joel A. Smoller, Editor

18 Fixed points and nonexpansive mappings, Robert C. Sine, Editor

19 Proceedings of the Northwestern homotopy theory conference, Haynes R. Miller and Stewart B. Priddy, Editors

20 Low dimensional topology, Samuel J. Lomonaco, Jr., Editor

21 Topological methods in nonlinear functional analysis, S. P. Singh, S. Thomeier, and B. Watson, Editors

22 Factorizations of $b^n \pm 1$, $b = 2, 3, 5, 6, 7, 10, 11, 12$ up to high powers, John Brillhart, D. H. Lehmer, J. L. Selfridge, Bryant Tuckerman, and S. S. Wagstaff, Jr.

23 Chapter 9 of Ramanujan's second notebook—Infinite series identities, transformations, and evaluations, Bruce C. Berndt and Padmini T. Joshi

24 Central extensions, Galois groups, and ideal class groups of number fields, A. Fröhlich

25 Value distribution theory and its applications, Chung-Chun Yang, Editor

26 Conference in modern analysis and probability, Richard Beals, Anatole Beck, Alexandra Bellow, and Arshag Hajian, Editors

27 Microlocal analysis, M. Salah Baouendi, Richard Beals, and Linda Preiss Rothschild, Editors

28 Fluids and plasmas: geometry and dynamics, Jerrold E. Marsden, Editor

29 Automated theorem proving, W. W. Bledsoe and Donald Loveland, Editors

30 Mathematical applications of category theory, J. W. Gray, Editor

31 Axiomatic set theory, James E. Baumgartner, Donald A. Martin, and Saharon Shelah, Editors

32 Proceedings of the conference on Banach algebras and several complex variables, F. Greenleaf and D. Gulick, Editors

33 Contributions to group theory, Kenneth I. Appel, John G. Ratcliffe, and Paul E. Schupp, Editors

34 Combinatorics and algebra, Curtis Greene, Editor

35 Four-manifold theory, Cameron Gordon and Robion Kirby, Editors

36 Group actions on manifolds, Reinhard Schultz, Editor

37 Conference on algebraic topology in honor of Peter Hilton, Renzo Piccinini and Denis Sjerve, Editors

http://dx.doi.org/10.1090/conm/070
Volume

38 Topics in complex analysis. Dorothy Browne Shaffer, Editor
39 Errett Bishop: Reflections on him and his research. Murray Rosenblatt, Editor
40 Integral bases for affine Lie algebras and their universal enveloping algebras. David Mitzman
41 Particle systems, random media and large deviations. Richard Durrett, Editor
42 Classical real analysis. Daniel Waterman, Editor
43 Group actions on rings. Susan Montgomery, Editor
44 Combinatorial methods in topology and algebraic geometry. John R. Harper and Richard Mandelbaum, Editors
45 Finite groups—coming of age. John McKay, Editor
46 Structure of the standard modules for the affine Lie algebra \( A_1 \). James Lepowsky and Mirko Primc
48 Analytic functions of one complex variable. Chung-chun Yang and Chi-tai Chuang, Editors
49 Complex differential geometry and nonlinear differential equations. Yum-Tong Siu, Editor
51 Nonlinear problems in geometry. Dennis M. DeTurck, Editor
53 The Selberg trace formula and related topics. Dennis A. Hejhal, Peter Sarnak, and Audrey Anne Terras, Editors
54 Differential analysis and infinite dimensional spaces. Kundagunta Sundaresan and Srinivasa Swaminathan, Editors
55 Applications of algebraic K-theory to algebraic geometry and number theory. Spencer J. Bloch, R. Keith Dennis, Eric M. Friedlander, and Michael R. Stein, Editors
56 Multparameter bifurcation theory. Martin Golubitsky and John Guckenheimer, Editors
57 Combinatorics and ordered sets. Ivan Rival, Editor
58.1 The Lefschetz centennial conference. Part I: Proceedings on algebraic geometry. D. Sundararaman, Editor
58.2 The Lefschetz centennial conference. Part II: Proceedings on algebraic topology. S. Gitler, Editor
58.3 The Lefschetz centennial conference. Part III: Proceedings on differential equations. A. Verjovsky, Editor
59 Function estimates. J. S. Marron, Editor
60 Nonstrictly hyperbolic conservation laws. Barbara Lee Keyfitz and Herbert C. Kranzer, Editors
61 Residues and traces of differential forms via Hochschild homology. Joseph Lipman
63 Integral geometry. Robert L. Bryant, Victor Guillemin, Sigurdur Helgason, and R. O. Wells, Jr., Editors
64 The legacy of Sonya Kovalevskaya. Linda Keen, Editor
65 Logic and combinatorics. Stephen G. Simpson, Editor
66 Free group rings. Narian Gupta
67 Current trends in arithmetical algebraic geometry. Kenneth A. Ribet, Editor
68 Differential geometry: The Interface between pure and applied mathematics. Mladen Luksic, Clyde Martin, and William Shadwick, Editors
69 Methods and applications of mathematical logic. Walter A. Carnielli and Luiz Paulo de Alcantara, Editors
70 Index theory of elliptic operators, foliations, and operator algebras. Jerome Kaminker, Kenneth C. Millett, and Claude Schochet, Editors
Index Theory of Elliptic Operators, Foliations, and Operator Algebras
CONTEMPORARY MATHEMATICS
Volume 70

Index Theory of Elliptic Operators, Foliations, and Operator Algebras

Proceedings of AMS Special Sessions held January 7, 1986 and April 11, 1986

Jerome Kaminker, Kenneth C. Millett, and Claude Schochet, Editors

AMERICAN MATHEMATICAL SOCIETY
Providence - Rhode Island
The Proceedings of the Special Session on Operator Algebras and Foliations was held at the 92nd Annual Meeting of the American Mathematical Society in New Orleans, Louisiana, January 7, 1986 and the Proceedings of the Special Session on Index Theory and its Applications was held at the 826th Meeting of the American Mathematical Society in Indianapolis, Indiana, April 11, 1986.

1980 Mathematics Subject Classification. (1985 Revision). Primary 46L80, 53C12, 58G12.

Library of Congress Cataloging-in-Publication Data

Index theory of elliptic operators, foliations, and operator algebras : proceedings of the AMS special sessions held January 7, 1986 and April 11, 1986 / Jerome Kaminker, Kenneth C. Millett, and Claude Schochet, editors.

p. cm. -- (Contemporary mathematics, ISSN 0271-4132 ; v. 70)
Bibliography: p.
ISBN 0-8218-5077-6 (alk. paper)
QA281.152 1986
515.7'242--dc19 87-33653
CIP

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

The appearance of the code on the first page of an article in this book indicates the copyright owner’s consent for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law, provided that the fee of $1.00 plus $.25 per page for each copy be paid directly to the Copyright Clearance Center, Inc., 21 Congress Street, Salem, Massachusetts 01970. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

Copyright ©1988 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights except those granted to the United States Government.
Printed in the United States of America.
This volume was printed directly from author-prepared copy.
The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability. ☺
Table of Contents

Preface ix

John Cantwell and Lawrence Conlon
The theory of levels 1

Ronald G. Douglas, Steven Hurder, and Jerome Kaminker
Toeplitz operators and the eta invariant: the case of $S^1$ 11

Thierry Fack
Sur la Conjecture de Novikov 43

Jeff Fox and Peter Haskell
A new proof of the K-amenability of SU(1,1) 103

James L. Heitsch
Some interesting group actions 113

Conner Lazarov
A relation between index and exotic classes 125

Ib Madsen and Jonathan Rosenberg
The Universal Coefficient Theorem for equivariant K-theory of real and complex C*-algebras 145

N. Christopher Phillips
Equivariant K-theory for proper actions and C*-algebras 175

N. Christopher Phillips
Equivariant K-theory for proper actions II: some cases in which finite dimensional bundles suffice 205

John Roe
Operator algebras and index theory on non-compact manifolds 229

Jonathan Rosenberg
K-theory of group C*-algebras, foliation algebras and crossed products 251

Xiaolu Wang
Non-commutative CW-complexes 303
There were two special sessions held during 1986 on closely related subjects. In January, 1986 at the annual meeting in New Orleans, a Special Session on Operator Algebras and Foliations was organized by Claude Schochet and Ken Millett, while at the Central Sectional meeting in Indianapolis in April, 1986 a Special Session on Index Theory of Elliptic Operators and its Applications was organized by Jerry Kaminker. The topics covered at these meetings were very closely linked, so it seemed appropriate to publish a volume made up of contributions by the participants of both.

Here is a brief description of the subject matter in this volume. The C*-algebra of a foliation, \( C^*(M,F) \), introduced by Alain Connes, plays two distinct roles. First, it is a non-commutative analog of the continuous functions on a locally compact Hausdorff space. From this point of view it provides a substitute for the continuous functions on the space of leaves. This is necessary since the quotient topology on the set of leaves will often be degenerate. This is an important example of the way in which one associates a non-commutative algebra of operators to a singular geometric object so as to capture essential geometric properties.

Second, it plays a quite distinct role as a replacement for the compact operators in the index theory of elliptic operators. On a closed manifold an elliptic operator is invertible modulo the compacts. This no longer holds for a leafwise elliptic operator. However, in that case, the operator is invertible modulo \( C^*(M,F) \). This allows one to carry through much of index theory in this setting.
The articles presented here address these themes by describing the use of operator algebras to reflect properties of geometric objects and by applying index theory in settings where the relevant elliptic operators are invertible modulo a C*-algebra other than the compact operators.

The papers in this volume are in final form and no versions of them will appear for publication elsewhere.

Jerome Kaminker
Kenneth C. Millett
Claude Schochet