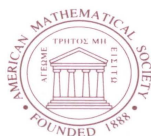


# CONTEMPORARY MATHEMATICS

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## The Structure of Finite Algebras

David Hobby  
Ralph McKenzie



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# The Structure of Finite Algebras

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1991 *Mathematics Subject Classification*. Primary 08A05.

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### Library of Congress Cataloging-in-Publication Data

Hobby, David Charles.

The structure of finite algebras.

(Contemporary mathematics, v. 76)

Bibliography: p.

Includes indexes.

1. Algebra, Universal. I. McKenzie, Ralph. II. Title. III. Series: Contemporary mathematics (American Mathematical Society, v. 76)

QA251.H65 1988

512

88-16712

ISBN 0-8218-5073-3

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Second printing, with an added Appendix and Bibliography, 1996

Printed in the United States of America.

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Dedicated to

PROFESSOR JAMES DONALD MONK

An inspiring teacher of  
congruence lore

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ISBN 0-8218-5073-3



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