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84

Partition Problems in Topology

Stevo Todorcevic

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PREFACE

This book deals with a topological version of the Ramsey problem for the uncountable. Simple cases of this problem include, for example, the famous Souslin problem and the well-known problem from topological measure theory asking if all regular Radon measures are σ-finite. The essence of the problem is combinatorial and set-theoretical, but unlike most of the problems of this type (such as the Whitehead problem from group theory or the problem of automatic continuity from the theory of Banach algebras) the set-theoretical methods needed for the solution of the problem have not yet been developed. One of the reasons for this is that the basic notions of set theory such as the notion of cardinality or the notion of a stationary set seem to be quite irrelevant to the problem.

The book is the result of rewriting a set of handwritten notes which were produced during the last few years and distributed informally to a large number of experts in the area. My special thanks are due to Liz Stimmel of the University of Colorado for the excellent typing.

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