

# CONTEMPORARY MATHEMATICS

## Representation Theory, Group Rings, and Coding Theory

Papers in Honor of  
S. D. Berman  
(1922–1987)

AMERICAN MATHEMATICAL SOCIETY

**VOLUME 93**

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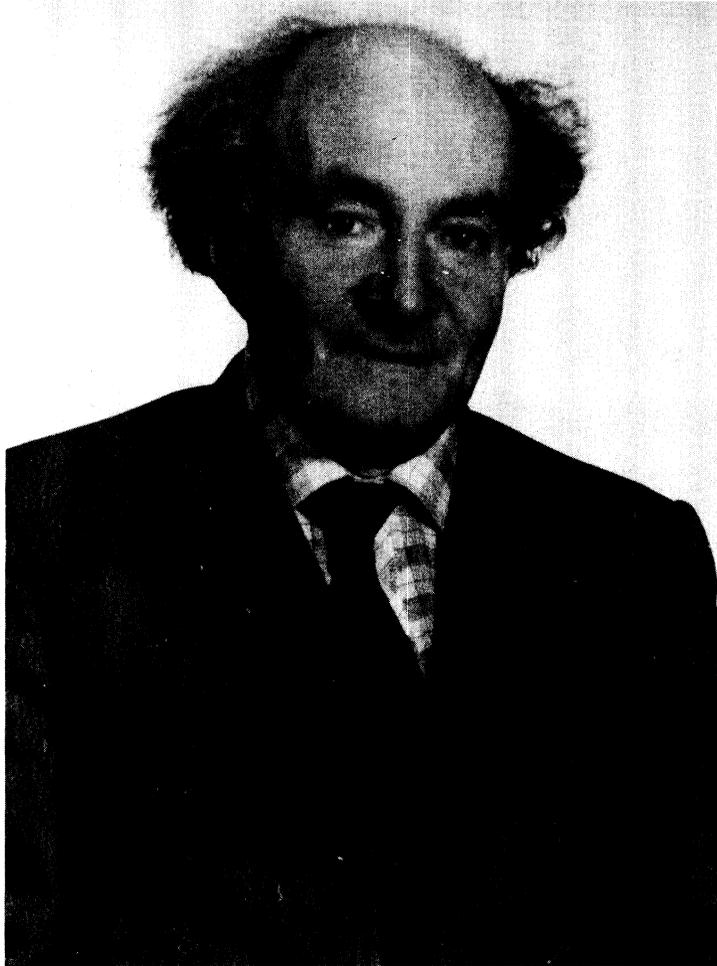
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**Representation Theory,  
Group Rings,  
and Coding Theory**



**S. D. Berman**  
**1922-1987**

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H. Zassenhaus, Editors**

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**Dedicated to the memory of**

**S. D. Berman**

**(1922–1987)**

## Contents

Preface	xv
S. D. Berman (1922–1987): A Biographical Note A. I. Lichtman	xvii
Publications of S. D. Berman	xxv
Ph.D. Students of S. D. Berman	xxxi
The S. D. Berman work on coding theory and on theory of threshold functions B. N. Gartshtein and I. I. Grushko	1
Berman's contributions to group representation over fields G. Karpilovsky	17
S. D. Berman's contribution to the theory of integral representations of finite groups A. I. Lichtman	27
The work of S. D. Berman on infinite abelian group rings Warren May	39
Berman's work on units in group rings Robert Sandling	47
On $p$ -subgroups of finite symmetric and alternating groups Ja. G. Berkovic	67
Parameters of abelian codes in the group algebra $KG$ of $G = (a) \times (b)$ , $a^p = b^p = 1$ , $p$ prime, over a finite field $K$ with primitive $p$ th root of unity and related MDS-codes S. D. Berman and I. I. Grushko	77
Locally cyclic modules over a valuation ring S. D. Berman and N. I. Vishnyakova	85

On the modules over a real crossed group algebra K. Buzási	105
The minimal number of generators of wreath products of nilpotent groups K. Buzási and L. G. Kovács	115
Torsion free space groups and permutation lattices for finite groups Gerald Cliff and Alfred Weiss	123
Clifford theory and induction from subgroups Everett C. Dade	133
The isomorphism problem for modular group algebras of groups with large centres Vesselin Drensky	145
Zero divisors in group rings: Something old, something new Daniel R. Farkas and Peter A. Linnell	155
The augmentation quotients of finite abelian $p$ -groups A. W. Hales and I. B. S. Passi	167
Free groups in normal subgroups of unit groups and arithmetic groups B. Hartley	173
On the Bass-Milnor index of abelian $p$ -groups Klaus Hoechsmann	179
A remark on algebraic geometric codes G. L. Katsman and M. A. Tsfasman	197
On decomposition of a binomial in a skew polynomial ring E. Sh. Kerer	201
The associated graded algebra of a group algebra L. Krop	209
On nilpotent and soluble subgroups of linear groups over fields of fractions of enveloping algebras and of group rings, I A. I. Lichtman	247
The structure of $G_0$ for certain polycyclic group algebras and related algebras Martin Lorenz and D. S. Passman	283
The direct factor problem for modular abelian group algebras Warren May	303

On the isomorphism of the semi-simple group algebras of the primary abelian groups Todor Zh. Mollov	309
On the canonical irreducible representations of countable locally finite groups and infinite Frobenius group S. I. Ostrovskaya	321
Generators of subgroups of $U(ZG)$ Jürgen Ritter and Sudarshan K. Sehgal	331
Connectivity of the finite group and generalized characters E. M. Žmud'	349

## Preface

This volume is dedicated to the memory of the outstanding Soviet mathematician S. D. Berman (1922–1987).

S. D. Berman worked, for the most part, in representation theory, group rings and coding theory. His important mathematical contributions are discussed here in a number of review articles. First B. N. Gartshtein and I. I. Grushko describe Berman's achievements in coding theory, including his pioneering work on abelian codes and his results on the theory of threshold functions, produced during the last ten years of his life. G. Karpilovskii considers Berman's contributions to the representation theory of groups over fields. The article by A. I. Lichtman discusses Berman's work on integral representations of groups, especially his fundamental theorem on groups of finite representation type, proved in the period 1959–1962. Next W. May describes Berman's accomplishments in infinite abelian group rings, in particular the results of his famous Debrecen paper (1967). The article by R. Sandling is devoted to Berman's fundamental results on units in integral group rings. Finally the biographical sketch of S. D. Berman is written by A. I. Lichtman.

This volume also contains twenty-two research articles in the areas of S. D. Berman's major interests. These papers are presented by mathematicians from many countries: United States, Soviet Union, England, Canada, Australia, Hungary, Bulgaria, South Africa—a sign of honor from the international mathematical community to a mathematician who contributed so much.

We would like to thank all the authors, the editors of the American Mathematical Society and all the people who made this collection possible.

M. Isaacs, University of Wisconsin-Madison  
A. Lichtman, University of Wisconsin-Parkside  
D. Passman, University of Wisconsin-Madison  
S. Sehgal, University of Alberta, Edmonton, Canada  
N. J. A. Sloane, AT & T Bell Labs  
H. Zassenhaus, Ohio State University

## **S. D. Berman (1922–1987): A Biographical Note**

A. I. LICHTMAN

The contributions of S. D. Berman to mathematics are widely recognized. His theorems in the theory of representations of finite groups over a field, integral representations and integral group rings are among the most important and deep ones. His work on the group rings of infinite abelian groups is pioneering and the results are fundamental. He was also a pioneer in the study of abelian codes. The review articles in this volume will discuss all these results in detail, but almost no one among the mathematicians in the West met S. D. Berman personally. I would like to tell here about the life of this remarkable man, to describe him as he was seen in the eyes of his friends and students.

Samuil Davidovich Berman was born on January 3, 1922 in a small town in the Ukraine. In 1939 he became a student at Moscow University, but his studies did not continue for long: the greatest and the most fierce war in Russian history broke out and the nineteen-year-old boy became for five years (1941–1946) a soldier and later a sergeant in the Red Army, serving in a combat unit. He took part in the great battle on the Volga River (Stalingrad) and was wounded there, recovered and returned to the army. Many years later, when he talked about the war he always recalled not only the tragedy and horror of the war but spoke with great warmth and sympathy about the people who fought together with him and became his friends in those difficult years. But few of them came back from the war!

He continued serving in a combat unit until the end of the war, fought in Hungary, the Ukraine, Czechoslovakia, and Bulgaria, took part in the battles for Budapest and Vienna, and was awarded a few medals. It was not until 1946 that he was demobilized and could renew his studies, first in the University of Moscow and then in Lvov University.

His Ph.D. thesis advisor was Ya. B. Lopatinsky, an outstanding Soviet mathematician, whose main interests at that time were in differential equations. Berman, however, was interested in differential equations only for a

short time; he published one article and then became involved in the theory of representations of finite groups and in the properties of integral group rings. His Ph.D. thesis and his first publications [1]–[4] (1952–1953) include many of the fundamental results and ideas about the group of units in integral group rings: the group of torsion units of the integral group ring  $ZG$  is  $\pm G$  if and only if the group  $G$  is abelian or Hamiltonian of order  $2^m$ ; the result that the order of a normalized torsion unit is a divisor of the order of the group; the important fact that for every automorphism  $\phi$  of  $ZG$  and a class sum  $k$  the element  $\phi(k)$  is, up to sign, a class sum; the first studies (after Higman's unpublished thesis, 1940) on the isomorphism problem; the properties and the applications of  $K$ -conjugacy classes.

Here, in Lvov, he met Bella Naumovna Gartshstein; she was a graduate student in probability theory and she was also a veteran of the war. She became his wife and later mother of their three children. It was a family full of love and devotion.

In 1952 the couple moved to Uzgorod, a small town near the Carpathian mountains, where they spent fifteen years. The university had no graduate school; the two young candidates of science (the equivalent of the Ph.D. degree in the Soviet Union) taught a lot of courses; in total, they taught more than 1000 hours every year. Samuil Davidovich spent an enormous amount of time and energy working with many of us, the undergraduate students, who were interested in algebra. I recall how in 1956–1960 a few of us (P. Gudivok, B. Bovdi, E. Drobotenko, V. Drobotenko and I) used to wonder how he could find time and energy more than twenty hours of classes per week, service on many committees (he became chairman of the department of algebra) and long discussions with us—sometimes at the department of algebra, sometimes in his apartment, sometimes at the river bank where he often sat, holding in his hands a sheet of paper with formulas—to work on his theorems. He was a man of unlimited generosity and patience, always ready to offer us mathematical problems and ideas and to listen carefully, not interrupting, when we came to him with our theorems, whose proofs almost always contained gaps or mistakes!

His work during the fifties was mostly on representations of finite groups. The theorem on the number of irreducible representations over an arbitrary field and the theorem about the character ring of a finite group over an arbitrary field (the Berman-Witt Theorem) were proven at this time (see Berman [1], [5]). These deep and important theorems, which generalize Brauer's results, became classical theorems in representation theory. During these years he also developed methods, based on the theory of induced representations, for effective constructions of representations of different classes of groups: representations of  $p$ -groups over an arbitrary field of characteristic zero, abelian-by-supersoluble groups over an algebraically closed field of arbitrary characteristic. Later he continued this work and obtained a number of important results on the Schur index (see [6]–[10]).

He began working on integral representations of finite groups in 1959. He believed that the theory of integral representations of groups would have important applications in algebra, especially in the theory of groups. The theory of relation modules (see Gruenberg [11]) can now be considered as an example of such an application.

During 1959–1960 most of the discussions and the talks in the algebra seminar were on integral representations. As usual, Samuil Davidovich was interested in many other things—modular representations of  $p$ -groups, group algebras of infinite abelian groups, but he became fascinated by the depth and difficulty of the problems related to integral representations of groups. I will describe his work in detail in the review paper devoted to his contribution to the theory of integral representations. Here I will mention only some of his results.

He worked in 1959–1960 with P. Gudivok, his graduate student, and during 1960 they showed that a cyclic group of order  $p^2$  ( $p > 2$ ) has a finite number of indecomposable representations over the ring of  $p$ -adic integers, and they gave a description of all of these representations (see [12]–[14]). (For the cyclic group of order 4 this was done earlier by A. Roiter [15]; for cyclic groups of prime order by Diederichsen [16] and Reiner [17].) Berman and Gudivok also showed that if a  $p$ -group is not cyclic or is cyclic of order greater than  $p^2$ , then it has an infinite number of indecomposable representations (see [12]–[14]).

But an important problem was open: what is the class of groups which have only a finite number of indecomposable integral representations? It was in the Fall of 1961 when S. D. Berman proved one of the most remarkable theorems in the theory of integral representations: A finite group  $G$  has a finite number of indecomposable integral representations if and only if for each prime  $p$ , dividing the order of  $G$ , the Sylow  $p$ -subgroups of  $G$  are cyclic of order  $p$  or  $p^2$  (see [13], [18]).

He was on leave then in Moscow and Leningrad, completing his doctoral dissertation; it included mainly his recent results on integral representations—a breakthrough in this area. The impression created by his dissertation was expressed by the opinion of I. R. Shafarevich, one of the judges; I. R. Shafarevich believed that S. D. Berman moved the theory of integral representations far ahead and he called Berman's result on finiteness of the number of integral indecomposable representations "brilliant." S. D. Berman became a Doctor of Science in 1963. This is the highest degree in science in the Soviet Union; there is no equivalent for it in the United States.

The same breakthrough in the theory of integral representations was made independently, at the same time, by a group of mathematicians in the USA (see Heller and Reiner [19], [20], Jones [21], and Dade [22]). Although Berman and Reiner were always very interested in each other's work, these two outstanding mathematicians met only once—in 1966, at the Moscow Mathematical Congress. They spent a lot of time together; it appeared that

at this time French was the most convenient language for their conversations. Later Samuil Davidovich improved his English to such an extent that he could enjoy English poetry.

After 1967 S. D. Berman worked at the Kharkov Institute of Radioelectronics. He continued working in different areas in algebra and was at the center of algebraic life in Kharkov. But in addition to this he became involved in another area of mathematics—coding theory. An account of S. D. Berman's results on abelian codes, published in [23], [24] (1967) is given in the book, *The Mathematical Theory of Coding* by I. Blake and R. Mullin. They consider his work on abelian codes “very significant” and then discuss his results on the minimum distance of any cyclic code of length  $p^n$ , rating them very high. His work in coding theory is described in detail in the review article by B. N. Gartshtein and I. I. Grushko.

S. D. Berman was a pioneer in the study of group rings of infinite abelian groups. His work on this topic began in 1960([25]). In 1967 he published his two famous papers which included a number of fundamental results (see [26], [27]). I will mention two of them here.

If  $K$  is a field of characteristic  $p$  and  $G_1$  and  $G_2$  are countable abelian  $p$ -groups such that  $KG_1$  is isomorphic to  $KG_2$ , then  $G_1$  is isomorphic to  $G_2$ .

If  $K$  is an algebraically closed field and  $G_1$  and  $G_2$  are torsion abelian groups of the same cardinality which have no elements of order equal to  $\text{char } K$ , then  $KG_1$  is isomorphic to  $KG_2$ .

S. D. Berman had many students; twenty of them became Ph.D. (A list of these students is included in this volume.) But the list of mathematicians who worked on problems posed by S. D. Berman or were influenced by his ideas is much longer; it includes many mathematicians from different cities of the Soviet Union: Moscow, Leningrad, Kiev, Kharkov, Erevan, Lvov and Uzgorod. He worked also with mathematicians from Hungary and Bulgaria. In many cases his ideas were picked up quickly by mathematicians from the Soviet Union but sometimes it took many years before they reached mathematicians in the West. This happened with his results on integral group rings and with some of his results on the Schur index. One of the main reasons for this is that few mathematicians in the West know the Russian language, and the systematic translation of Russian mathematical journals began only in the middle of the 1960's.

I believe that some exceedingly interesting ideas of S. D. Berman still have not been followed up as much as they deserve. Most important of them are his results on representations of discrete infinite groups. One of his articles on this topic (see Berman and Saraya [28]) gives a method for construction of series of irreducible representations (over an algebraically closed field  $K$ ) for abelian-by-supersoluble groups; these series contain all the finite dimensional irreducible representations. In particular, when  $G$  is a finitely generated supersoluble group and  $\text{char } K = p > 0$ , this result implies, via Roseblade's

solution of Hall's problem, that all the irreducible representations of  $G$  over  $K$  are obtained by this method.

I have already mentioned that S. D. Berman never traveled to the West and communicated only occasionally with Western mathematicians. He maintained, however, scientific contacts with many Soviet mathematicians and especially with I. R. Shafarevich, who was always very interested in Berman's work in representation theory. S. D. Berman's interest in integral representations was inspired by D. K. Faddeev, who was a pioneer in this field in the Soviet Union.

Reading S. D. Berman's papers gives a strong impression of his remarkable intuition which made it possible for him to begin working in areas completely new to him several times in his life. He also had an excellent memory, not only for mathematical theorems but for poetry as well, mostly Russian classical poetry and above all the great Pushkin. He loved the novels of Tolstoy and Dostoyevsky very much, a counterexample to the infamous opinion that one person cannot love both of them at the same time (see Veresaev [29]). One of his favorite Western authors was Thomas Mann. During discussions on literature and history, which he enjoyed very much, he was willing to listen to opinions which were very different from his point of view, but it was felt that he was disgusted by any kind of extremism or dishonesty. Another great love of his, or perhaps his first love, was classical music.

During the last ten years of his life he was deeply interested in philosophy especially in its connections with physics and cybernetics. Even his love of art became inseparable from his fascination with philosophy. In both cases it was probably his infatuation with the search for truth that nurtured such deep interest.

S. D. Berman was an exceptionally generous and kind man, and at the same time he had a strong will and character. He was not broken after he was fired in 1976 from his position as professor in the Kharkov Institute of Radioelectronics, but continued his research as energetically as before. It would be impossible to describe in a few pages all the harassment this outstanding scientist endured in his struggle against lawlessness and despotism—a struggle that lasted over ten years. During this difficult time his principles and high morale remained the same as throughout his whole life, so did his courage, and love and respect for his friends and students.

His main interests in mathematics during the last ten years were in threshold partitions; he worked towards creating a general theory of them. Most of the results which he obtained in this area have not been published. Article [30] and the review paper by B. N. Gartshtein and I. I. Grushko list only a small part of these results.

Samuil Davidovich Berman died on February 18, 1987. We will never see his kind eyes and will never hear his voice again. But his theorems and ideas are alive; the mark which he left on the lives of many of us, his friends and students, will not fade, and we will never forget him.

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DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF WISCONSIN-PARKSIDE  
KENOSHA, WISCONSIN 53141

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## **Ph.D. Students of S. D. Berman**

1. Yu. L. Barannik
2. B. Bovdi (partly)
3. K. Buzasi
4. N. N. Eisenberg
5. P. M. Gudivok
6. E. S. Drobotenko
7. V. S. Drobotenko
8. G. B. Karpilovsky
9. G. K. Kladov
10. V. F. Kniazeva
11. E. Sh. Kerer
12. A. I. Lichtman (partly)
13. T. Z. Mollov
14. S. I. Ostrovskaya
15. S. S. Poliak
16. V. S. Rudko
17. A. R. Rossa
18. V. S. Šaraya
19. Z. P. Zhilinskaya
20. N. I. Vishniakova

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