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Proceedings of a Summer Research Conference held July 30–August 5, 1988

AMERICAN MATHEMATICAL SOCIETY

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This volume is dedicated to Roger Brockett
on the occasion of his 50th birthday.
Happy Birthday, Roger!!

Contents

Foreword, by Roger Brockett xiii

Introduction xv

An enumerative theory of equilibrium rotations for planar kinematic chains
J. BAILLIEUL 1

Stability and stiffening of driven and free planar rotating beams
A. M. BLOCH and R. R. RYAN 11

Characterization of Hamiltonian input-output systems
P. E. CROUCH and A. J. VAN DER SCHAFT 27

Robustness of distributed parameter systems
R. F. CURTAIN 47

On the relationship between discrete-time optimal control and recursive dynamics for elastic multibody chains
C. J. DAMAREN and M. T. D'ELEUTERIO 61

Some solvable stochastic control problems in compact symmetric spaces of rank one
T. E. DUNCAN 79

Slew-induced deformation shaping on slow integral manifolds
T. A. W. DWYER III 97

Feedback equivalence and symmetries of Brunowski normal forms
R. B. GARDNER, W. F. SHADWICK, and G. R. WILKENS 115

The application of total energy as a Lyapunov function for mechanical control systems
D. E. KODITSCHEK 131

Classical adiabatic angles for slowly moving mechanical systems
J. KOILLER 159

Eulerian many-body problems
P. S. KRISHNAPRASAD 187

Morse theory for a model space structure
M. LEVI 209

A unified approach for the control of multifingered robot hands
Z. LI and S. SASTRY 217

Tethered satellite system stability
D.-C. LIAW and E. H. ABED 241

Quantum control theory I
E. B. LIN 269

Cartan-Hannay-Berry phases and symmetry
J. E. MARSDEN, R. MONTGOMERY, and T. RATIU 279

Block diagonalization and the energy-momentum method
J. E. MARSDEN, J. C. SIMO, D. LEWIS, and T. A. POSBERGH 297

The dynamics of two coupled rigid bodies in three space
G. W. PATRICK 315

Nonsmooth optimization algorithms for the design of controlled flexible structures
E. POLAK 337

Stability analysis of a rigid body with attached geometrically nonlinear rod by the energy-momentum method
T. A. POSBERGH, J. C. SIMO, and J. E. MARSDEN 371

Controllability of Poisson control systems with symmetries
G. SÁNCHEZ DE ALVAREZ 399

Chaos in a rapidly forced pendulum equation
J. SCHEURLE 411

Accurate time critical control of many body systems
N. SREENATH 421

Hamiltonian control systems: decomposition and clamped dynamics
A. J. VAN DER SCHAFT 441

Graph-theoretical methods in multibody dynamics
J. WITTENBURG 459

Foreword

by Roger Brockett

The conference on control theory and multibody dynamics as conceived and developed by the organizers was a happy event, covering a range of topics all of which could be viewed as applications of geometrical methods to problems arising in dynamics and control. I am pleased to have been honored by its organizers and participants. At several points during the conference I was reminded of a meeting on dynamics and control held at NASA/Ames in 1976. As compared with that earlier meeting, new areas of applications discussed at Bowdoin included work on the mathematics behind numerical methods in mechanics, the discussion of the dynamics of complex kinematic chains such as arise in automotive engineering and robotics, and various applications of strongly nonholonomic systems as in Berry's phase. By the time we shared the Down East conference banquet, those who listened to the lectures closely were able to say something mathematical about the deformation of lobster shells and the kinematic chain model of a lobster claw. For the most part papers in this volume are faithful to the idea that by using mathematics one can say a lot with a few words. This leaves room in the foreword for a few anecdotes and impressions written in a different style.

At breakfast one morning at Bowdoin several of us were sharing a table with a distinguished conferee from another AMS conference. In the same spirit as I might ask "What is elliptic cohomology?", he asked "What is control theory?" Of course television newscasters are able to dispense with much more difficult questions in an arbitrarily small amount of time, but we were reduced to giving examples. We could have done better. There have been researchers such as Rayleigh and Nyquist whose work has had enormous influence in part because they adopted a control systems point of view. The 1943 Gibbs lecture of H. Bateman, "On the Control of an Elastic Fluid," has admirable scope and impressive depth. It is exemplary in that it discusses important control-theoretic matters, feedback stability, the effect of delays, etc., and is able to capture the flavor of the systems point of view. That is, Bateman discusses dynamical systems as filters, passes freely between problems involving the transmission of sound, the transmission of electrical signals, the control of fluids, etc. He is a candidate for the pantheon of control theory because he uses input/output thinking in a way which is mathematically natural and succeeds in bringing out previously unobserved mathematical structures in real problems.

Suppose our breakfast companion has asked, "Why control theory and dynamics?" This is a more precise question and it requires a more detailed answer. I was fortunate to be starting my graduate studies in control in 1960. It was an exciting time; a flurry of new results were completely changing the nature of the subject. Bellman's work on dynamic programming, Pontryagin's maximum principle, recursive estimation, the blending of Fourier methods with Liapunov stability, and the use of Ito calculus were all introduced within a few years. In this period there was a great interest in dynamics because of Sputnik; aerospace problems associated with trajectory optimization and orbit transfer were common fare. (In contrast to the indifference one sees today, I recall that on the day of the first manned Mercury flight my probability class was cut short because the professor said "I can't think with that thing flying around up there.") Moreover, because of what seemed at the time to ban an all-consuming interest in optimal control, the calculus of variations in its various forms, including the variational formulations of dynamics, was never far from the center of the stage. Insofar as mathematics was concerned, the principal mode of thought was analysis. Since the early 1950's Solomon Lefschetz, first at Princeton and later at RIAS, had been attempting to resuscitate the subject of differential equations. Partly because of ongoing work in the Soviet Union, his interests included control theory viewed as an offshoot of differential equations. The style in vogue with respect to dynamics was that of Whittaker and Goldstein, and control theory and dynamics were closely linked.

This history has had a significant effect on the situation we find today even though, to a large extent, geometry has replaced analysis as the *lingua franca* in the common ground between control and dynamics. Although, as chronicled in the pioneering book of Abraham and Marsden, once started the geometrization of dynamics proceeded quite rapidly, control theory changed rather more slowly after the halcyon days of the early 60's. However, in time it became clear that differential geometry had the potential to express key control theoretic ideas in a natural way. A number of people took part in these developments. In my case, I had been fascinated for some time with the idea of nonholonomic constraints, partly because of the role they play in the planar integration scheme found in Vannevar Bush's differential analyzer and partly because they seemed mysterious. As I attempted to find a comfortable way of thinking about them, and the related matter of the Caratheodory statement of the second law of thermodynamics, I eventually realized that there was a wide variety of ways in which geometrical ideas could be helpful in forming a unified view of control.

What does the future hold? Developments in computer engineering have given us microprocessors and digital processing chips which make possible the implementation of a new level of mathematical sophistication in control systems. Developments in material science have put at our disposal magnetostrictive, piezoelectric, electro-rheological, etc., materials whose use in control mechanisms calls for a fresh look at modeling issues. There is significant progress in synthesizing controllable mechanisms on the scale of tens of microns. Since neither science nor technology stands still we can be sure that in the future, as in the past, there will be interesting challenges and a continuing need to meld what we know with ongoing developments.

Introduction

The study of complex, interconnected mechanical systems with rigid and flexible articulated components is of growing interest to engineers and mathematicians. The rich history of the subject derives primarily from the work of mechanism designers and the work of aerospace engineers interested in the modeling and control of complex multibody spacecraft. A more recent source of inspiration is in the area of robotics (eg. the control of multifingered hands, contact problems, etc.).

Recent work in this area reveals a rich geometry underlying the mathematical models that appear in this context. In particular, Lie groups of symmetries, reduction, and Poisson structures play a significant role in explicating the qualitative properties of multibody systems. A fresh look at covariant formulations of elasticity has also proved to be very useful. Geometric ideas should play an even more crucial role in the design of reliable numerical schemes for computer simulation of models of multibody systems, and the underlying control theory of Hamiltonian systems with symmetry is beginning to be worked out in a systematic manner.

In engineering applications, the question of exploiting the special structures of mechanical systems is important. For mechanical systems with symmetry (such as rigid bodies with internal rotors) one wants to take into account conservation of angular momentum and the special Poisson bracket structures on the associated reduced phase space. Certain mechanical problems involving control of interconnected rigid bodies can be formulated as Lie-Poisson systems, possibly with forcing and damping. It is likely that dynamic models of robotic manipulators are also amenable to such a formulation. The utility of this formalism lies in the natural way it describes certain controllability and stability questions.

The dynamics and control of interconnected rigid and flexible structures such as robotic, aeronautic, and space structures, involve difficulties in modeling, in mathematical analysis, and in numerical implementation. First of all, the formulation of the basic dynamical models is a nontrivial step. In modeling rotating systems with continuum-mechanical components such as plates, shells, or beams, nonlinear models (e.g., the so-called geometrically exact models) display behavior which is in certain instances qualitatively quite different from what is observed in linear and semi-linearized models. For example, if one views a plate or beam equation of say Euler-Bernoulli type as an approximation of a geometrically exact model, then the processes of *attachment* to a rapidly rotating rigid body (think of helicopter blades) and *approximation* do not commute. Done in the wrong order, the procedure can lead to spurious softening and hence to completely erroneous results even for small deflections. Thus, proper attention to dynamic modeling is a first crucial step. This is probably important even for "robust" techniques—all too often practical problems are "fixed up" in unsatisfactory ad hoc ways to balance bad modeling.

Some painful modeling lessons of this sort were learned in the early days of the U.S. space exploration program. In particular, they brought home the point that dynamic interactions between rigid and flexible components of a spacecraft are important. A famous example in this regard was the Explorer I mission in 1958. Energy dissipation in whip antennas attached to a passive spinning body caused instability and an end-over-end tumble. The key lesson here was that passive spin stabilization about the minor axis, while feasible in rigid bodies, is not possible in the presence of dissipative flexible components.

Experience with Explorer I and later with missions such as the Orbiting Geophysical Observatory III in 1966 (in this case excessive oscillations were induced by control system interactions with flexible beams) led to a vigorous program of research in multibody systems with flexible components. The approximate analytic and numerical techniques developed in the course of this research were quite successful in suggesting good designs for spacecraft of modest size and flexibility.

A new generation of spacecraft with large flexible components (radar arrays, solar collectors, truss structures) are presenting new challenges to our abilities to model and accurately predict the dynamic behavior of such structures in space. It is necessary to have the proper tools to do this since the requirements on the performance of these new spacecraft are quite unprecedented. The Hubble Space Telescope is expected to have a pointing accuracy of 0.01 arc second on rms jitter of less than 0.007 arc seconds.

Refined mathematical models and analyses will be necessary to attack such problems with a degree of confidence. New control methodologies will be necessary to maintain the effects of dynamics interactions in such large space structures within prescribed limits.

Recent developments in Hamiltonian dynamics and coupling of systems with symmetries (such as invariance under Euclidean motions) has shed new light on some of these issues. Likewise, engineering questions have suggested new mathematical structures. This commonality leads to new and interesting applications of Hamiltonian methods, such as the energy-momentum-Casimir method (for determining nonlinear stability) and bifurcation of Hamiltonian systems with symmetry (for uncovering nontrivial branches of new solutions when system parameters are varied). Other tools borrowed from Hamiltonian and Lagrangian systems theory have led to new results on periodic solutions of systems near equilibria, including information on their spatial symmetry, and to the proof of nonintegrability in certain regimes of phase space. These results also suggest how to construct numerical integration schemes that preserve energy and angular momentum, avoiding systematic biases or oscillations. When dissipation is added, all of these techniques appear to be naturally compatible with Lyapunov and invariance principle methods.

Further information on the above ideas and the larger context of control theory, dynamics, and its applications can be found in the panel report *Future Directions in Control Theory,* chaired by Wendell Fleming, and available through SIAM.

It was these sorts of developments that motivated us to organize this conference and to assemble this volume. We hope it will be a useful addition to the literature and to the cooperation between engineering and mathematics. It is a pleasure to thank all the participants for their enthusiasm and their contribution to the conference and its proceedings. Finally, it is our sincere pleasure to dedicate this volume to Roger Brockett on the occasion of his 50th birthday. He has been a great source of inspiration to many of us.

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