

CONTEMPORARY MATHEMATICS

99

The Connection between Infinite Dimensional and Finite Dimensional Dynamical Systems

Proceedings of a Summer Research Conference
held July 19–25, 1987



Titles in This Series

Volume

- 1 **Markov random fields and their applications**, Ross Kindermann and J. Laurie Snell
- 2 **Proceedings of the conference on integration, topology, and geometry in linear spaces**, William H. Graves, Editor
- 3 **The closed graph and P-closed graph properties in general topology**, T. R. Hamlett and L. L. Herrington
- 4 **Problems of elastic stability and vibrations**, Vadim Komkov, Editor
- 5 **Rational constructions of modules for simple Lie algebras**, George B. Seligman
- 6 **Umbral calculus and Hopf algebras**, Robert Morris, Editor
- 7 **Complex contour integral representation of cardinal spline functions**, Walter Schempp
- 8 **Ordered fields and real algebraic geometry**, D. W. Dubois and T. Recio, Editors
- 9 **Papers in algebra, analysis and statistics**, R. Lidl, Editor
- 10 **Operator algebras and K-theory**, Ronald G. Douglas and Claude Schochet, Editors
- 11 **Plane ellipticity and related problems**, Robert P. Gilbert, Editor
- 12 **Symposium on algebraic topology in honor of José Adem**, Samuel Gitler, Editor
- 13 **Algebraists' homage: Papers in ring theory and related topics**, S. A. Amitsur, D. J. Saltman, and G. B. Seligman, Editors
- 14 **Lectures on Nielsen fixed point theory**, Boju Jiang
- 15 **Advanced analytic number theory. Part I: Ramification theoretic methods**, Carlos J. Moreno
- 16 **Complex representations of $GL(2, K)$ for finite fields K** , Ilya Piatetski-Shapiro
- 17 **Nonlinear partial differential equations**, Joel A. Smoller, Editor
- 18 **Fixed points and nonexpansive mappings**, Robert C. Sine, Editor
- 19 **Proceedings of the Northwestern homotopy theory conference**, Haynes R. Miller and Stewart B. Priddy, Editors
- 20 **Low dimensional topology**, Samuel J. Lomonaco, Jr., Editor
- 21 **Topological methods in nonlinear functional analysis**, S. P. Singh, S. Thomeier, and B. Watson, Editors
- 22 **Factorizations of $b^n \pm 1$, $b = 2$, 3, 5, 6, 7, 10, 11, 12 up to high powers**, John Brillhart, D. H. Lehmer, J. L. Selfridge, Bryant Tuckerman, and S. S. Wagstaff, Jr.
- 23 **Chapter 9 of Ramanujan's second notebook—Infinite series identities, transformations, and evaluations**, Bruce C. Berndt and Padmini T. Joshi
- 24 **Central extensions, Galois groups, and ideal class groups of number fields**, A. Fröhlich
- 25 **Value distribution theory and its applications**, Chung-Chun Yang, Editor
- 26 **Conference in modern analysis and probability**, Richard Beals, Anatole Beck, Alexandra Bellow, and Arshag Hajian, Editors
- 27 **Microlocal analysis**, M. Salah Baouendi, Richard Beals, and Linda Preiss Rothschild, Editors
- 28 **Fluids and plasmas: geometry and dynamics**, Jerrold E. Marsden, Editor
- 29 **Automated theorem proving**, W. W. Bledsoe and Donald Loveland, Editors
- 30 **Mathematical applications of category theory**, J. W. Gray, Editor
- 31 **Axiomatic set theory**, James E. Baumgartner, Donald A. Martin, and Saharon Shelah, Editors
- 32 **Proceedings of the conference on Banach algebras and several complex variables**, F. Greenleaf and D. Gulick, Editors
- 33 **Contributions to group theory**, Kenneth I. Appel, John G. Ratcliffe, and Paul E. Schupp, Editors
- 34 **Combinatorics and algebra**, Curtis Greene, Editor

Titles in This Series

Volume

- 35 **Four-manifold theory**, Cameron Gordon and Robion Kirby, Editors
- 36 **Group actions on manifolds**, Reinhard Schultz, Editor
- 37 **Conference on algebraic topology in honor of Peter Hilton**, Renzo Piccinini and Denis Sjerve, Editors
- 38 **Topics in complex analysis**, Dorothy Browne Shaffer, Editor
- 39 **Errett Bishop: Reflections on him and his research**, Murray Rosenblatt, Editor
- 40 **Integral bases for affine Lie algebras and their universal enveloping algebras**, David Mitzman
- 41 **Particle systems, random media and large deviations**, Richard Durrett, Editor
- 42 **Classical real analysis**, Daniel Waterman, Editor
- 43 **Group actions on rings**, Susan Montgomery, Editor
- 44 **Combinatorial methods in topology and algebraic geometry**, John R. Harper and Richard Mandelbaum, Editors
- 45 **Finite groups—coming of age**, John McKay, Editor
- 46 **Structure of the standard modules for the affine Lie algebra $A_1^{(1)}$** , James Lepowsky and Mirko Primc
- 47 **Linear algebra and its role in systems theory**, Richard A. Brualdi, David H. Carlson, Biswa Nath Datta, Charles R. Johnson, and Robert J. Plemmons, Editors
- 48 **Analytic functions of one complex variable**, Chung-chun Yang and Chi-tai Chuang, Editors
- 49 **Complex differential geometry and nonlinear differential equations**, Yum-Tong Siu, Editor
- 50 **Random matrices and their applications**, Joel E. Cohen, Harry Kesten, and Charles M. Newman, Editors
- 51 **Nonlinear problems in geometry**, Dennis M. DeTurck, Editor
- 52 **Geometry of normed linear spaces**, R. G. Bartle, N. T. Peck, A. L. Peressini, and J. J. Uhl, Editors
- 53 **The Selberg trace formula and related topics**, Dennis A. Hejhal, Peter Sarnak, and Audrey Anne Terras, Editors
- 54 **Differential analysis and infinite dimensional spaces**, Kondagunta Sundaresan and Srinivasa Swaminathan, Editors
- 55 **Applications of algebraic K-theory to algebraic geometry and number theory**, Spencer J. Bloch, R. Keith Dennis, Eric M. Friedlander, and Michael R. Stein, Editors
- 56 **Multiparameter bifurcation theory**, Martin Golubitsky and John Guckenheimer, Editors
- 57 **Combinatorics and ordered sets**, Ivan Rival, Editor
- 58.I **The Lefschetz centennial conference. Part I: Proceedings on algebraic geometry**, D. Sundararaman, Editor
- 58.II **The Lefschetz centennial conference. Part II: Proceedings on algebraic topology**, S. Gitler, Editor
- 58.III **The Lefschetz centennial conference. Part III: Proceedings on differential equations**, A. Verjovsky, Editor
- 59 **Function estimates**, J. S. Marron, Editor
- 60 **Nonstrictly hyperbolic conservation laws**, Barbara Lee Keyfitz and Herbert C. Kranzer, Editors
- 61 **Residues and traces of differential forms via Hochschild homology**, Joseph Lipman
- 62 **Operator algebras and mathematical physics**, Palle E. T. Jorgensen and Paul S. Muhly, Editors
- 63 **Integral geometry**, Robert L. Bryant, Victor Guillemin, Sigurdur Helgason, and R. O. Wells, Jr., Editors
- 64 **The legacy of Sonya Kovalevskaya**, Linda Keen, Editor
- 65 **Logic and combinatorics**, Stephen G. Simpson, Editor
- 66 **Free group rings**, Narian Gupta
- 67 **Current trends in arithmetical algebraic geometry**, Kenneth A. Ribet, Editor
- 68 **Differential geometry: The interface between pure and applied mathematics**,

Titles in This Series

Volume

- Mladen Lukšić, Clyde Martin, and William Shadwick, Editors
- 69 Methods and applications of mathematical logic**, Walter A. Carnielli and Luiz Paulo de Alcantara, Editors
- 70 Index theory of elliptic operators, foliations, and operator algebras**, Jerome Kaminker, Kenneth C. Millett, and Claude Schochet, Editors
- 71 Mathematics and general relativity**, James A. Isenberg, Editor
- 72 Fixed point theory and its applications**, R. F. Brown, Editor
- 73 Geometry of random motion**, Rick Durrett and Mark A. Pinsky, Editors
- 74 Geometry of group representations**, William M. Goldman and Andy R. Magid, Editors
- 75 The finite calculus associated with Bessel functions**, Frank M. Cholewinski
- 76 The structure of finite algebras**, David C. Hobby and Ralph McKenzie
- 77 Number theory and its applications in China**, Wang Yuan, Yang Chung-chun, and Pan Chengbiao, Editors
- 78 Braids**, Joan S. Birman and Anatoly Libgober, Editors
- 79 Regular differential forms**, Ernst Kunz and Rolf Waldi
- 80 Statistical inference from stochastic processes**, N. U. Prabhu, Editor
- 81 Hamiltonian dynamical systems**, Kenneth R. Meyer and Donald G. Saari, Editors
- 82 Classical groups and related topics**, Alexander J. Hahn, Donald G. James, and Zhe-xian Wan, Editors
- 83 Algebraic K-theory and algebraic number theory**, Michael R. Stein and R. Keith Dennis, Editors
- 84 Partition problems in topology**, Stevo Todorčević
- 85 Banach space theory**, Bor-Luh Lin, Editor
- 86 Representation theory and number theory in connection with the local Langlands conjecture**, J. Ritter, Editor
- 87 Abelian group theory**, Laszlo Fuchs, Rüdiger Göbel, and Phillip Schultz, Editors
- 88 Invariant theory**, R. Fossum, W. Haboush, M. Hochster, and V. Lakshmibai, Editors
- 89 Graphs and algorithms**, R. Bruce Richter, Editor
- 90 Singularities**, Richard Randell, Editor
- 91 Commutative harmonic analysis**, David Colella, Editor
- 92 Categories in computer science and logic**, John W. Gray and Andre Scedrov, Editors
- 93 Representation theory, group rings, and coding theory**, M. Isaacs, A. Lichtman, D. Passman, S. Sehgal, N. J. A. Sloane, and H. Zassenhaus, Editors
- 94 Measure and measurable dynamics**, R. Daniel Mauldin, R. M. Shortt, and Cesar E. Silva, Editors
- 95 Infinite algebraic extensions of finite fields**, Joel V. Brawley and George E. Schnibben
- 96 Algebraic topology**, Mark Mahowald and Stewart Priddy, Editors
- 97 Dynamics and control of multibody systems**, J. E. Marsden, P. S. Krishnaprasad, and J. C. Simo, Editors
- 98 Every planar map is four colorable**, Kenneth Appel and Wolfgang Haken
- 99 The connection between infinite dimensional and finite dimensional dynamical systems**, Basil Nicolaenko, Ciprian Foias, and Roger Temam, Editors

The Connection between Infinite Dimensional and Finite Dimensional Dynamical Systems

CONTEMPORARY MATHEMATICS

Volume 99

The Connection between Infinite Dimensional and Finite Dimensional Dynamical Systems

**Proceedings of the AMS-IMS-SIAM
Joint Summer Research Conference
held July 19–25, 1987, with support
from the National Science Foundation
and the Air Force Office of Scientific Research**

**Basil Nicolaenko, Ciprian Foias,
Roger Temam, Editors**

AMERICAN MATHEMATICAL SOCIETY

Providence • Rhode Island

EDITORIAL BOARD

Daniel M. Burns, Jr., managing editor
Richard W. Beals Gerald J. Janusz
Sylvain E. Cappell Jan Mycielski
David Eisenbud Michael E. Taylor
Jonathan Goodman

The AMS-IMS-SIAM Joint Summer Research Conference in the Mathematical Sciences on The Connection between Infinite and Finite Dimensional Dynamical Systems was held at The University of Colorado, Boulder, Colorado, on July 19–25, 1987 with support from the National Science Foundation, Grant DMS-8613199 and the Air Force Office of Scientific Research.

1980 *Mathematics Subject Classification* (1985 Revision). Primary 34Cxx, 35Kxx, 70Kxx, 76-xx.

Library of Congress Cataloging-in-Publication Data

AMS-IMS-SIAM Joint Summer Research Conference in the Mathematical Sciences on the Connection between Infinite and Finite Dimensional Dynamical Systems (1987): University of Colorado)

The connection between infinite dimensional and finite dimensional dynamical systems: proceedings of the AMS-IMS-SIAM joint summer research conference held July 19–25, 1987, with support from the National Science Foundation and the Air Force Office of Scientific Research/Basil Nicolaenko, Ciprian Foias, Roger Temam, editors.

p. cm.—(Contemporary mathematics, ISSN 0271-4132; v. 99)

“AMS-IMS-SIAM Joint Summer Research Conference in the Mathematical Sciences on the Connection Between Infinite and Finite Dimensional Dynamical Systems . . . held at the University of Colorado, Boulder, Colorado”—T. p. verso.

ISBN 0-8218-5105-5 (alk. paper)

1. Differentiable dynamical systems—Congresses. 2. Differential equations, Parabolic—Congresses. 3. Nonlinear theories—Congresses. 4. Fluid mechanics—Congresses. I. Nicolaenko, Basil, 1943–. II. Foias, Ciprian. III. Temam, Roger. IV. American Mathematical Society. V. Institute of Mathematical Statistics. VI. Society for Industrial and Applied Mathematics. VII. Title. VIII. Series: Contemporary mathematics (American Mathematical Society); v. 99.

QA614.8.A47 1987

515'.352—dc20

89-15158

CIP

Copyright ©1989 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights except those granted to the United States Government.

Printed in the United States of America.

Information on copying and reprinting can be found at the back of this volume.

This volume was printed directly from author-prepared copy.

The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability. ♾

Portions of this publication were typeset using $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\text{T}\mathcal{E}\mathcal{X}$, the American Mathematical Society's $\text{T}\mathcal{E}\mathcal{X}$ macro system.

10 9 8 7 6 5 4 3 2 1 94 93 92 91 90 89

Contents

| | |
|--|-----|
| Preface | xi |
| Dynamical systems in infinite dimension R. TEMAM | 1 |
| A construction of inertial manifolds PETER CONSTANTIN | 27 |
| Analytic structure of dynamical systems M. TABOR | 63 |
| Hausdorff and Lyapunov dimensions for gradient systems GEORGE R. SELL | 85 |
| Persistent heteroclinic orbits DIETER ARMBRUSTER | 93 |
| Orientation of saddle connections for a reaction diffusion equation MICHAEL S. JOLLY | 105 |
| Finite dimensionality in the complex Ginzburg-Landau equation C. R. DOERING, J. D. GIBBON, D. D. HOLM and B. NICOLAENKO | 117 |
| Periodic dynamical system with application to Sine-Gordon equations: Estimates on the fractal dimension of the universal attractor JEAN-MICHEL GHIDAGLIA and ROGER TEMAM | 143 |
| Inertial manifolds for models of compressible gas dynamics BASIL NICOLAENKO | 165 |
| Existence and finite-dimensionality of universal attractors for the Landau- Lifschitz equations of ferromagnetism TEPPER L. GILL and W. W. ZACHARY | 181 |
| The nonlinear Schrödinger equation—singularity formation, stability and dispersion MICHAEL I. WEINSTEIN | 213 |

| | |
|--|-----|
| Formal stability of two-dimensional self-gravitating rotating disks | |
| ARTHUR MAZER and TUDOR RATIU | 233 |
| A deterministic approach towards self-organization in continuous media | |
| E. VAN GROESEN | 259 |
| Low dimensional description of complicated phenomena | |
| LAWRENCE SIROVICH | 277 |
| Using dynamic embedding methods to analyze experimental data | |
| ERIC J. KOSTELICH and JAMES A. YORKE | 307 |
| Global bifurcations in maps of the plane and in Rayleigh-Bénard convection | |
| IOANNIS G. KEVREKIDIS and ROBERT E. ECKE | 313 |
| A model of double-diffusive convection with periodic boundary conditions | |
| EDGAR KNOBLOCH, ANIL E. DEANE and JURI TOOMRE | 339 |
| Controversies concerning finite/infinite sequences of fluid corner vortices | |
| K. GUSTAFSON, K. HALASI and R. LEBEN | 351 |

Preface

During the last few years we have seen a number of major developments which show that the longtime behavior of solutions of a very large class of partial differential equations (PDE's) possess a striking resemblance to the behavior of solutions of finite dimensional dynamical systems, or ordinary differential equations (ODE's). The first of these advances was the discovery (by a number of researchers) that a dissipative PDE has a compact, global attractor with finite Hausdorff and fractal dimensions. More recently it was shown that some of these PDE's possess a finite dimensional inertial manifold, i.e., an invariant manifold that contains the attractor and exponentially attractive trajectories. For the latter equations, the connection with ODE's is no longer a mere analogy, instead it has become a striking reality! Indeed, when one restricts the PDE to the inertial manifold one obtains an ODE, which we call an *inertial form* for the given PDE; since an inertial manifold contains the global attractor, it follows that the longtime behavior of solutions of a PDE with an inertial manifold is *completely* determined by the inertial form.

Now that one is obtaining a better understanding of the exact connection between finite dimensional dynamical systems and various classes of dissipative PDE's, it is realistic to hope that the wealth of studies of such topics as bifurcations of finite vector fields and “strange” fractal attractors can be brought to bear on various mathematical models including continuum flows. Surprisingly, a number of distributed systems from continuum mechanics—as well as their infinite-dimensional models—have been found to exhibit the same nontrivial dynamic behavior, including routes to deterministic chaos, as observed in low-dimensional dynamical systems. As a natural consequence of these observations, a new direction of research has arisen: detection and analysis of finite dimensional dynamical characteristics of infinite-dimensional systems.

The Summer Seminar on “The Connection between Infinite and Finite Dimensional Dynamical Systems” was hosted by the University of Colorado at Boulder and brought together both mathematicians and mathematical physicists. It succeeded as an effective catalyst to bring forward the latest developments in the field, and fostered lively interactions on open questions and future directions. Besides aspects of global attractors, inertial manifolds and global bifurcations, problems of non-integrable dynamical systems were also discussed. A major component was the application of these ideas to fluid dynamical systems, where practitioners have sometimes diagnosed

effective low-dimensional behavior in the transition to turbulence. The practical implications for a low-dimensional description of complex global bifurcations in fluid systems are especially promising. More generally the finite-dimensional behavior of turbulent flows and the reduction of the number of determining modes is of great importance.

The organizers wish to thank the National Science Foundation and the Air Force Office for Scientific Research for their generous support. Also the Center for Nonlinear Studies at Los Alamos National Laboratory provided valuable indirect support. Finally, Ms. Betty Verducci and Carole Kohanski from the American Mathematical Society deserve special recognition for their relentless efforts in making this Summer Seminar successful.

Ciprian Foias
Basil Nicolaenko
Roger Temam

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

The appearance of the code on the first page of an article in this book indicates the copyright owner's consent for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law, provided that the fee of \$1.00 plus \$.25 per page for each copy be paid directly to the Copyright Clearance Center, Inc., 21 Congress Street, Salem, Massachusetts 01970. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

ISBN 0-8218-5105-5



9 780821 851050

CONM/99

AMS *on the Web*
www.ams.org