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The Connection between Infinite Dimensional and Finite Dimensional Dynamical Systems

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Basil Nicolaenko, Ciprian Foias,
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Preface

During the last few years we have seen a number of major developments which show that the longtime behavior of solutions of a very large class of partial differential equations (PDE's) possess a striking resemblance to the behavior of solutions of finite dimensional dynamical systems, or ordinary differential equations (ODE's). The first of these advances was the discovery (by a number of researchers) that a dissipative PDE has a compact, global attractor with finite Hausdorff and fractal dimensions. More recently it was shown that some of these PDE's possess a finite dimensional inertial manifold, i.e., an invariant manifold that contains the attractor and exponentially attractive trajectories. For the latter equations, the connection with ODE's is no longer a mere analogy, instead it has become a striking reality! Indeed, when one restricts the PDE to the inertial manifold one obtains an ODE, which we call an *inertial form* for the given PDE; since an inertial manifold contains the global attractor, it follows that the longtime behavior of solutions of a PDE with an inertial manifold is *completely* determined by the inertial form.

Now that one is obtaining a better understanding of the exact connection between finite dimensional dynamical systems and various classes of dissipative PDE's, it is realistic to hope that the wealth of studies of such topics as bifurcations of finite vector fields and "strange" fractal attractors can be brought to bear on various mathematical models including continuum flows. Surprisingly, a number of distributed systems from continuum mechanics—as well as their infinite-dimensional models—have been found to exhibit the same nontrivial dynamic behavior, including routes to deterministic chaos, as observed in low-dimensional dynamical systems. As a natural consequence of these observations, a new direction of research has arisen: detection and analysis of finite dimensional dynamical characteristics of infinite-dimensional systems.

The Summer Seminar on "The Connection between Infinite and Finite Dimensional Dynamical Systems" was hosted by the University of Colorado at Boulder and brought together both mathematicians and mathematical physicists. It succeeded as an effective catalyst to bring forward the latest developments in the field, and fostered lively interactions on open questions and future directions. Besides aspects of global attractors, inertial manifolds and global bifurcations, problems of non-integrable dynamical systems were also discussed. A major component was the application of these ideas to fluid dynamical systems, where practitioners have sometimes diagnosed effective low-dimensional behavior in the transition to turbulence. The practical implications for a low-dimensional description of complex global bifurcations in fluid systems are especially promising. More generally the finite-dimensional behavior of turbulent flows and the reduction of the number of determining modes is of great importance.

The organizers wish to thank the National Science Foundation and the Air Force Office for Scientific Research for their generous support. Also the Center for Nonlinear Studies at Los Alamos National Laboratory provided valuable indirect support. Finally, Ms. Betty Verducci and Carole Kohanski from the American Mathematical Society deserve special recognition for their relentless efforts in making this Summer Seminar successful.

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Basil Nicolaenko
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