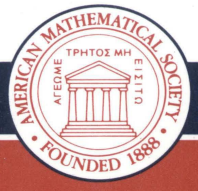

CONTEMPORARY MATHEMATICS

AMERICAN MATHEMATICAL SOCIETY

101

Recent Developments in Geometry

Proceedings of the AMS Special Session in Geometry
November 14–15, 1987

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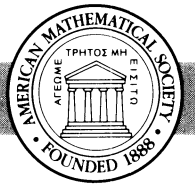
Recent Developments in Geometry



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10 9 8 7 6 5 4 3 2 1 94 93 92 91 90 89

Contents

Preface xi

Some recent developments in Riemannian geometry by ROBERT E. GREENE 1

Designer metrics on Riemannian manifolds by ROBERT BROOKS . . . 31

Open manifolds of nonnegative curvature by GERARD WALSCHAP . . . 47

An extrinsic average variational method by S. WALTER WEI 55

The Dirichlet problem and Fatou's theorem for harmonic mappings on regular domains by PATRICIO AVILES 79

New examples of singly-periodic minimal surfaces and their qualitative behavior by DAVID A. HOFFMAN 97

The Gauss curvature and Minkowski problems in space forms by VLADIMIR I. OLIKER 107

Self-duality, twistor theory, its generalization and application by Y. SUN POON 125

Spectral geometry of Riemannian manifolds by PETER B. GILKEY . . . 147

Eigenvalue asymptotics and their geometric applications by MARK A. PINSKY 155

A lower bound for the number of isospectral surfaces by RICHARD M. TSE 161

The conformal deformation equation and isospectral set of conformal metrics by SUN-YUNG A. CHANG and PAUL C. YANG . . . 165

Analytic torsion for group actions by JOHN LOTT 179

Regularity of entropy for geodesic flows by G. KNIEPER and H. WEISS 191

Twistor and Gauss lifts of surfaces in four-manifolds by GARY R. JENSEN and MARCO RIGOLI 197

Affine differential geometry of complex hypersurfaces by WEIQI GAO 233

Domains with noncompact automorphism groups by KANG-TAE KIM 249

Affine approach to complex geometry by SIDNEY FRANKEL 263

Compactification of complete Kähler–Einstein manifolds of finite volume by NGAIMING MOK 287

Topological types of isolated hypersurface singularities by STEPHEN S.-T. YAU 303

Complex foliations by T. DUCHAMP and M. KALKA 323

Preface

This volume of papers is an outgrowth of a special session on geometry at the November 1987 meeting at UCLA of the American Mathematical Society (meeting No. 838). The organizers of the special session were S. Y. Cheng, H. Choi and R. E. Greene. This special session was unusually well attended. More than forty addresses were given, and the audience (including the lecturers themselves) numbered over sixty. During the meeting, Alan Weinstein, who was serving as an editor of *Contemporary Mathematics*, suggested to the organizers that, in view of the breadth of the coverage of geometry in session, an interesting volume might be prepared by soliciting manuscripts from the participants; and in particular he expressed interest in such a volume for publication in the *Contemporary Mathematics* series. In the event, the participants greeted this idea with enthusiasm; by common consent, it was decided that the papers should be surveys of relatively broad areas of geometry, rather than detailed presentations of new research results as such.

Given the startling diversity and extent of contemporary geometry, it is perhaps not to be expected that a volume such as this, nor indeed a weekend special session, should cover anything like the field in toto. On the other hand, it is certainly our hope that this volume will serve the purpose of providing geometers, whatever their own specialties, with some insight into recent developments in a variety of divergent specialities in geometry. Weinstein had

originally suggested that the volume might appropriately begin with a general survey of the field as a whole and of the organizers' views of future prospects for geometry. In practice, contemplating the enormousness of geometry's recent progress we decided that such a survey to be complete and uniform would have to be so extended in scope as to be disproportionate to the size of the volume itself. And at the same time it seemed to us that such a survey might be more appropriate to the truly systematic coverage of the field that will occur in the Proceedings of the American Mathematical Society Summer Institute on Geometry, scheduled to occur in 1990.

For all our hesitancy to undertake a general survey, it did occur to us that this might be an appropriate moment to survey some recent developments in a most classical and fundamental general problem of Riemannian geometry in its purest form: what complete Riemannian manifolds exist that satisfy a given set of geometric conditions, i.e. conditions on curvature and other geometric invariants? The slogan is curvature controls topology. The problem is to make the slogan into specific mathematical results. This problem is so general as to encompass a great deal of Riemannian geometry from its inception to the present. But as it happens, the last year or two have seen some exciting new developments along these lines. The first article to follow summarizes some of these; while it is without pretence of completeness, we hope that it will draw the reader's attention to the continuing vitality of the purely geodesic-geometric aspect of the subject while at the same time noting some of the remarkable interactions between these classical topics and recent developments in partial differential equations, interactions which are transforming the face of geometry.

As always, the editors are indebted to their contributors, without whom there would have been nothing to edit. We are also indebted to Alan Weinstein, who as noted was the originator of the concept of this volume. With-

out his encouragement, the November 1987 geometry session would have been memorialized at most in the memories of the participants. All the papers that follow are in final form in the sense specified by *Mathematical Reviews* and will not appear elsewhere.

S. Y. Cheng
H. Choi
R. E. Greene

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