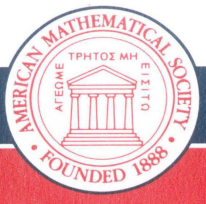

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Primes Associated to an Ideal

Stephen McAdam

http://dx.doi.org/10.1090/conm/102

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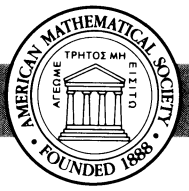

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Stephen McAdam

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1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 13A17.

Library of Congress Cataloging-in-Publication Data

McAdam, Stephen.
Primes associated to an ideal/Stephen McAdam.
p. cm. — (Contemporary mathematics, ISSN 0271-4132; v. 102)
Includes bibliographical references (p.).
ISBN 0-8218-5108-X (alk. paper)
1. Noetherian rings. 2. Ideals (Algebra) I. Title. II. Series: Contemporary mathematics (American Mathematical Society); v. 102.
QA251.3.M3726 1989
512'.4—dc20 89-27624
CIP

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To Martha

(It bears repeating.)

ACKNOWLEDGEMENTS

I deeply thank my esteemed colleagues Jack Ratliff and Dan Katz, who contributed immensely to the material presented herein. After writing an earlier monograph, I swore to never again undertake such a task. A large part of the blame for talking me out of that wise oath (which I again take) falls on the heads of the "Purdue Crew", Bill Heinzer, Craig Huneke, and their spate of recent graduates, in particular, Sam Huckaba and Jugal Verma. Nor does Judy Sally escape blameless. Ray Heitmann was, as always, the person who instantly told me the answers to any questions over which I had struggled for weeks.

Despite the marvels of modern microprocessors, turning a floppy disk into camera ready copy is no mean feat. Dave and Joan Sands of Longhorn Copies more than lived up to their titles of Saint Dave and Joan of Arc with their patient and skilled help.

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PREFACE

This text discusses five closely related sets of prime ideals associated to an ideal I in a Noetherian ring, the persistent, asymptotic, quintasymptotic, essential, and quintessential primes of I. The first two of these were studied in [M2]. Since the appearance of that monograph, the other three sets were developed, and more was learned about the first two sets as well. Matters have reached a state in which the known results are scattered throughout some three dozen papers, making it difficult for an interested person to learn the subject. The aim of this work is to present the most important and interesting of the ideas in an efficient manner, easing the burden of those who wish to learn much of, or simply refer to part of what is known concerning these sets.

The background required is little more than a standard year course in commutative ring theory. Thus, the work should be accessible to many graduate students, and I would be delighted should a dissertation or two be conceived while it is being read. While this work is primarily intended for commutative ring theorists, I would like to think that noncommutative ring theorists and algebraic geometers might also find it of interest.

As the ideas studied in [M2] are closely related to the ideas discussed here, there is some overlap between that work and this one. I adopted the following strategy. If a result from [M2] could be reproved in a way which added insight into the relationship between the older and newer ideas, then I included the new proof here. Otherwise, I simply refer to results in [M2] as needed.

ISBN 978-0-8218-5108-1

CONM/102