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Logic and Computation

Proceedings of a Workshop held at
Carnegie Mellon University, June 30–July 2, 1987

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Titles in This Series

Volume

Titles in This Series

Volume

Titles in This Series

Volume

68 **Differential geometry: The interface between pure and applied mathematics,** Mladen Luksic, Clyde Martin, and William Shadwick, Editors

69 **Methods and applications of mathematical logic,** Walter A. Carnielli and Luiz Paulo de Alcantara, Editors

70 **Index theory of elliptic operators, foliations, and operator algebras,** Jerome Kaminker, Kenneth C. Millett, and Claude Schochet, Editors

71 **Mathematics and general relativity,** James A. Isenberg, Editor

72 **Fixed point theory and its applications,** R. F. Brown, Editor

73 **Geometry of random motion,** Rick Durrett and Mark A. Pinsky, Editors

74 **Geometry of group representations,** William M. Goldman and Andy R. Magid, Editors

75 **The finite calculus associated with Bessel functions,** Frank M. Cholewinski

76 **The structure of finite algebras,** David C. Hobby and Ralph Mckenzie

77 **Number theory and its applications in China,** Wang Yuan, Yang Chung-chun, and Pan Chengbiao, Editors

78 **Braids,** Joan S. Birman and Anatoly Libgober, Editors

79 **Regular differential forms,** Ernst Kunz and Rolf Waldi

80 **Statistical inference from stochastic processes,** N. U. Prabhu, Editor

81 **Hamiltonian dynamical systems,** Kenneth R. Meyer and Donald G. Saari, Editors

82 **Classical groups and related topics,** Alexander J. Hahn, Donald G. James, and Zhe-xian Wan, Editors

83 **Algebraic K-theory and algebraic number theory,** Michael R. Stein and R. Keith Dennis, Editors

84 **Partition problems in topology,** Stevo Todorcevic

85 **Banach space theory,** Bor-Luh Lin, Editor

86 **Representation theory and number theory in connection with the local Langlands conjecture,** J. Ritter, Editor

87 **Abelian group theory,** Laszlo Fuchs, Rüdiger Göbel, and Phillip Schultz, Editors

88 **Invariant theory,** R. Fossum, W. Haboush, M. Hochster, and V. Lakshmibai, Editors

89 **Graphs and algorithms,** R. Bruce Richter, Editor

90 **Singularities,** Richard Randell, Editor

91 **Commutative harmonic analysis,** David Colella, Editor

92 **Categories in computer science and logic,** John W. Gray and Andre Scedrov, Editors

93 **Representation theory, group rings, and coding theory,** M. Isaacs, A. Lichtman, D. Passman, S. Sehgal, N. J. A. Sloane, and H. Zassenhaus, Editors

94 **Measure and measurable dynamics,** R. Daniel Mauldin, R. M. Shortt, and Cesar E. Silva, Editors

95 **Infinite algebraic extensions of finite fields,** Joel V. Brawley and George E. Schnibben

96 **Algebraic topology,** Mark Mahowald and Stewart Priddy, Editors

97 **Dynamics and control of multibody systems,** J. E. Marsden, P. S. Krishnaprasad, and J. C. Simo, Editors

98 **Every planar map is four colorable,** Kenneth Appel and Wolfgang Haken

99 **The connection between infinite dimensional and finite dimensional dynamical systems,** Basil Nicolaenko, Ciprian Foias, and Roger Temam, Editors

100 **Current progress in hyperbolic systems: Riemann problems and computations,** W. Brent Lindquist, Editor

101 **Recent developments in geometry,** S.-Y. Cheng, H. Choi, and Robert E. Greene, Editors

102 **Primes associated to an ideal,** Stephen McAdam

103 **Coloring theories,** Steve Fisk

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Logic and Computation

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Carnegie Mellon University, June 30–July 2, 1987

Wilfried Sieg, Editor

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Contents

Preface

The motivation for organizing the Workshop on Logic and Computation was straightforward. The refined interaction of mathematics and computation theory is one of the most fascinating and potentially most fruitful developments in logic. This interaction has been accelerated by the emergence of computers as a powerful research tool for mathematicians; but, independent of this development, it has been intrinsic to various attempts at carrying out significant parts of mathematics in computationally informative ways. The broad issue is indeed a long-standing one and, historically, connected with particular views on the nature of mathematics and appropriate methodologies. It came to the fore in response to the use of analytic methods in number theory. I allude, of course, to Dirichlet's famous proof of the fact that arithmetical progressions of the form $ax + b$, with a and b relatively prime, contain infinitely many primes;[1] but, I also allude to Dedekind's and Kronecker's responses.

Let me point to one, pertinent feature of Kronecker's views; he considered a proof of an existential statement as completely rigorous only *if it contains a method that allows us to find* [*in finitely many steps*] *the magnitude whose existence has been claimed.*[2] Assuming the statement contains parameters, the proof presumably has to provide a uniform method for finding the magnitude, i.e. an effective function. The Kroneckerian viewpoint is appealed to later in the French predicativists' rejection of abstract, set-theoretic methods. Let me quote from Borel's most interesting contribution to the controversy surrounding Zermelo's proof of the well-ordering principle:

> One may wonder what is the real value of these arguments that I do not regard as absolutely valid but that still lead ultimately to effective results.

[1]G. P. Lejeune Dirichlet, Beweis des Satzes, dass jede arithmetische Progression, deren erstes Glied und Differenz ganze Zahlen ohne gemeinschaftlichen Faktor sind, unendlich viele Primzahlen enthält (1837), reprinted in Dirichlet's *Gesammelte Werke I*, L. Kronecker (editor).

[2]Reported by K. Hensel in his introduction to Kronecker's *Vorlesungen zur Zahlentheorie*, Leipzig, 1901.

> In fact, it seems that if they were completely devoid of value. They could not lead to anything, since they would be meaningless collections of words. This, I believe, would be too harsh. They have a value analogous to certain theories in mathematical physics through which we do not claim to express reality but, rather, to have a guide that aids us, by analogy, in predicting new phenomena which must be verified. It would require considerable research to learn what is the real and precise sense that can be attributed to arguments of this sort.[3]

Borel, however, did not have high hopes for such an enterprise; he continued in his letter:

> Such research would be useless, or at least it would require more effort than it would be worth. How these overly abstract arguments are related to the concrete becomes clear when the need is felt.

Much work in mathematical logic has been concerned with constructive interpretations of classical mathematics and can be understood as doing just the kind of research Borel suggested (to be useless).—Jumping almost a century of detailed mathematical and logical work, in particular, on subsystems of analysis and fragments of arithmetic, I come to recent and related developments. What seems to me to be their most important aspect is this: the emphasis is no longer on computability in principle, but rather feasibility in practice. One way of approaching this goal uses the axiomatic method to analyze parts of mathematics under minimal set-theoretic assumptions; that means, in weak formal theories **T**. It is in terms of the "small" class $\mathscr{R}$ of **T**'s provably recursive functions that systematic algorithmic information can be gained: we have immediately an effective $\mathscr{R}$-bound in case **T** proves a Π^0_2-statement. More specific information concerning bounds may be obtained by a detailed investigation of the given (formal) proof or, as Kreisel puts matters, by unwinding it.[4]

This kind of global and local information is most appropriately obtained by exploiting the mechanical character of formal theories, more particularly, by treating derivations as computations and as data for computations. From this perspective three topics emerge as crucial:

(1) mathematical developments in weak theories,
(2) connections between weak theories and small classes of recursive functions, in particular complexity classes, and
(3) (mechanical) transformations of proofs and programs.

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[3] Baire, Borel, Hadamard, Lebesgue, *Cinq lettres sur la théorie des ensembles*; Bulletin de la Société Mathématique de France 33 (1905), 261–273; translated in Moore's book *Zermelo's Axiom of Choice*, New York, Heidelberg, Berlin, 1982, 311–320.

[4] See as a mathematically significant example H. Luckhardt, *Herbrand-Analysen zweier Beweise des Satzes von Roth: polynomiale Anzahlschranken*; Journal of Symbolic Logic 54(1), 1989, 234–263.

These topics are directly reflected in the organization of the workshop; the resulting proceedings seem to me to be rich in results and replete with mathematical, logical, and computational problems.

It is a pleasure to acknowledge the support I received from my colleagues Peter Andrews, Dana Scott, Rick Statman, and, most of all, Daniel Leivant. I am grateful to the Department of Philosophy and the Department (now: School) of Computer Science at Carnegie Mellon who provided the financial means to make the workshop at all possible. Finally, my warm thanks to the participants, contributors, and last, but not least, the referees of the papers in these proceedings.

Wilfried Sieg

Pittsburgh

August 19, 1989

Schedule of Presentations and Speakers

June 30:

8:30 *Registration*
9:30 *Opening remarks*: D. Scott, W. Sieg

(1) Mathematical developments (in weak theories)

10:00 S. Feferman, *Putting* Explicit Mathematics *to work*
11:00 A. Nerode, *Complexity-theoretic algebra*
2:00 S. Simpson, *The meta-mathematics of Hilbert's basis theorem*

(2) Research Reports

3:15 A. Woods, *A provable pigeon-hole principle*
4:30 X. Yu, *Radon-Nikodym theorem is equivalent to arithmetic comprehension*
5:15 J. Hirst, *Combinatorics in subsystems of second order arithmetic*

July 1:

(3) Weak theories and computational complexity classes

8:30 S. Buss, *Weak axioms for arithmetic and connections to computational complexity*
10:00 G. Takeuti, *Weak consistencies of bounded arithmetic*
11:00 P. Clote, *Recursion theoretic characterization of some complexity classes*

(4) Research Reports

2:00 F. Ferreira, *A conservation result over polynomial time arithmetic*
2:45 R. Mansfield, *Curry—an equational programming language*
4:00 **H & SS—Distinguished Lecture:** S. Feferman, *Turing's oracle*

July 2:

(5) Transformations of proofs and programs

8:30 D. Leivant, *Confined computational models: A bridge between descriptive type theory and computational complexity*
10:00 C. Goad, *Meta-programming in* SIL
11:00 K. McAloon, *Logic programming and linear programming*

(6) Research Reports

2:00 G. Bellin, *Transforming the proof of the Infinite Ramsey Theorem*
2:45 F. Pfenning, *Program development through proof transformations*
4:00 M. Beeson, *Some theories conservative over intuitionistic arithmetic*
4:45 R. Statman, *Some interpretations of Scott's theory* LCF *based on a notion of rate of convergence*

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CONM/106