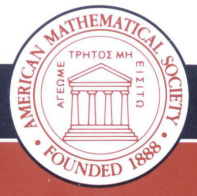


CONTEMPORARY MATHEMATICS

AMERICAN MATHEMATICAL SOCIETY

109

Combinatorial Group Theory

Proceedings of the AMS Special Session in Combinatorial Group Theory—Infinite Groups held April 23–24, 1988

http://dx.doi.org/10.1090/conm/109

Titles in This Series

Volume

1 **Markov random fields and their applications,** Ross Kindermann and J. Laurie Snell

2 **Proceedings of the conference on integration, topology, and geometry in linear spaces,** William H. Graves, Editor

3 **The closed graph and P-closed graph properties in general topology,** T. R. Hamlett and L. L. Herrington

4 **Problems of elastic stability and vibrations,** Vadim Komkov, Editor

5 **Rational constructions of modules for simple Lie algebras,** George B. Seligman

6 **Umbral calculus and Hopf algebras,** Robert Morris, Editor

7 **Complex contour integral representation of cardinal spline functions,** Walter Schempp

8 **Ordered fields and real algebraic geometry,** D. W. Dubois and T. Recio, Editors

9 **Papers in algebra, analysis and statistics,** R. Lidl, Editor

10 **Operator algebras and K-theory,** Ronald G. Douglas and Claude Schochet, Editors

11 **Plane ellipticity and related problems,** Robert P. Gilbert, Editor

12 **Symposium on algebraic topology in honor of José Adem,** Samuel Gitler, Editor

13 **Algebraists' homage: Papers in ring theory and related topics,** S. A. Amitsur, D. J. Saltman, and G. B. Seligman, Editors

14 **Lectures on Nielsen fixed point theory,** Boju Jiang

15 **Advanced analytic number theory. Part I: Ramification theoretic methods,** Carlos J. Moreno

16 **Complex representations of GL(2, K) for finite fields K,** Ilya Piatetski-Shapiro

17 **Nonlinear partial differential equations,** Joel A. Smoller, Editor

18 **Fixed points and nonexpansive mappings,** Robert C. Sine, Editor

19 **Proceedings of the Northwestern homotopy theory conference,** Haynes R. Miller and Stewart B. Priddy, Editors

20 **Low dimensional topology,** Samuel J. Lomonaco, Jr., Editor

21 **Topological methods in nonlinear functional analysis,** S. P. Singh, S. Thomeier, and B. Watson, Editors

22 **Factorizations of $b^n \pm 1$, b = 2, 3, 5, 6, 7, 10, 11, 12 up to high powers,** John Brillhart, D. H. Lehmer, J. L. Selfridge, Bryant Tuckerman, and S. S. Wagstaff, Jr.

23 **Chapter 9 of Ramanujan's second notebook—Infinite series identities, transformations, and evaluations,** Bruce C. Berndt and Padmini T. Joshi

24 **Central extensions, Galois groups, and ideal class groups of number fields,** A. Fröhlich

25 **Value distribution theory and its applications,** Chung-Chun Yang, Editor

26 **Conference in modern analysis and probability,** Richard Beals, Anatole Beck, Alexandra Bellow, and Arshag Hajian, Editors

27 **Microlocal analysis,** M. Salah Baouendi, Richard Beals, and Linda Preiss Rothschild, Editors

28 **Fluids and plasmas: geometry and dynamics,** Jerrold E. Marsden, Editor

29 **Automated theorem proving,** W. W. Bledsoe and Donald Loveland, Editors

30 **Mathematical applications of category theory,** J. W. Gray, Editor

31 **Axiomatic set theory,** James E. Baumgartner, Donald A. Martin, and Saharon Shelah, Editors

32 **Proceedings of the conference on Banach algebras and several complex variables,** F. Greenleaf and D. Gulick, Editors

33 **Contributions to group theory,** Kenneth I. Appel, John G. Ratcliffe, and Paul E. Schupp, Editors

34 **Combinatorics and algebra,** Curtis Greene, Editor

Titles in This Series

Volume

35 **Four-manifold theory,** Cameron Gordon and Robion Kirby, Editors

36 **Group actions on manifolds,** Reinhard Schultz, Editor

37 **Conference on algebraic topology in honor of Peter Hilton,** Renzo Piccinini and Denis Sjerve, Editors

38 **Topics in complex analysis,** Dorothy Browne Shaffer, Editor

39 **Errett Bishop: Reflections on him and his research,** Murray Rosenblatt, Editor

40 **Integral bases for affine Lie algebras and their universal enveloping algebras,** David Mitzman

41 **Particle systems, random media and large deviations,** Richard Durrett, Editor

42 **Classical real analysis,** Daniel Waterman, Editor

43 **Group actions on rings,** Susan Montgomery, Editor

44 **Combinatorial methods in topology and algebraic geometry,** John R. Harper and Richard Mandelbaum, Editors

45 **Finite groups–coming of age,** John McKay, Editor

46 **Structure of the standard modules for the affine Lie algebra $A_1^{(1)}$,** James Lepowsky and Mirko Primc

47 **Linear algebra and its role in systems theory,** Richard A. Brualdi, David H. Carlson, Biswa Nath Datta, Charles R. Johnson, and Robert J. Plemmons, Editors

48 **Analytic functions of one complex variable,** Chung-chun Yang and Chi-tai Chuang, Editors

49 **Complex differential geometry and nonlinear differential equations,** Yum-Tong Siu, Editor

50 **Random matrices and their applications,** Joel E. Cohen, Harry Kesten, and Charles M. Newman, Editors

51 **Nonlinear problems in geometry,** Dennis M. DeTurck, Editor

52 **Geometry of normed linear spaces,** R. G. Bartle, N. T. Peck, A. L. Peressini, and J. J. Uhl, Editors

53 **The Selberg trace formula and related topics,** Dennis A. Hejhal, Peter Sarnak, and Audrey Anne Terras, Editors

54 **Differential analysis and infinite dimensional spaces,** Kondagunta Sundaresan and Srinivasa Swaminathan, Editors

55 **Applications of algebraic K-theory to algebraic geometry and number theory,** Spencer J. Bloch, R. Keith Dennis, Eric M. Friedlander, and Michael R. Stein, Editors

56 **Multiparameter bifurcation theory,** Martin Golubitsky and John Guckenheimer, Editors

57 **Combinatorics and ordered sets,** Ivan Rival, Editor

58.I **The Lefschetz centennial conference. Proceedings on algebraic geometry,** D. Sundararaman, Editor

58.II **The Lefschetz centennial conference. Proceedings on algebraic topology,** S. Gitler, Editor

58.III **The Lefschetz centennial conference. Proceedings on differential equations,** A. Verjovsky, Editor

59 **Function estimates,** J. S. Marron, Editor

60 **Nonstrictly hyperbolic conservation laws,** Barbara Lee Keyfitz and Herbert C. Kranzer, Editors

61 **Residues and traces of differential forms via Hochschild homology,** Joseph Lipman

62 **Operator algebras and mathematical physics,** Palle E. T. Jorgensen and Paul S. Muhly, Editors

63 **Integral geometry,** Robert L. Bryant, Victor Guillemin, Sigurdur Helgason, and R. O. Wells, Jr., Editors

64 **The legacy of Sonya Kovalevskaya,** Linda Keen, Editor

65 **Logic and combinatorics,** Stephen G. Simpson, Editor

66 **Free group rings,** Narian Gupta

67 **Current trends in arithmetical algebraic geometry,** Kenneth A. Ribet, Editor

Titles in This Series

Volume

68 **Differential geometry: The interface between pure and applied mathematics,** Mladen Luksic, Clyde Martin, and William Shadwick, Editors

69 **Methods and applications of mathematical logic,** Walter A. Carnielli and Luiz Paulo de Alcantara, Editors

70 **Index theory of elliptic operators, foliations, and operator algebras,** Jerome Kaminker, Kenneth C. Millett, and Claude Schochet, Editors

71 **Mathematics and general relativity,** James A. Isenberg, Editor

72 **Fixed point theory and its applications,** R. F. Brown, Editor

73 **Geometry of random motion,** Rick Durrett and Mark A. Pinsky, Editors

74 **Geometry of group representations,** William M. Goldman and Andy R. Magid, Editors

75 **The finite calculus associated with Bessel functions,** Frank M. Cholewinski

76 **The structure of finite algebras,** David C. Hobby and Ralph Mckenzie

77 **Number theory and its applications in China,** Wang Yuan, Yang Chung-chun, and Pan Chengbiao, Editors

78 **Braids,** Joan S. Birman and Anatoly Libgober, Editors

79 **Regular differential forms,** Ernst Kunz and Rolf Waldi

80 **Statistical inference from stochastic processes,** N. U. Prabhu, Editor

81 **Hamiltonian dynamical systems,** Kenneth R. Meyer and Donald G. Saari, Editors

82 **Classical groups and related topics,** Alexander J. Hahn, Donald G. James, and Zhe-xian Wan, Editors

83 **Algebraic K-theory and algebraic number theory,** Michael R. Stein and R. Keith Dennis, Editors

84 **Partition problems in topology,** Stevo Todorcevic

85 **Banach space theory,** Bor-Luh Lin, Editor

86 **Representation theory and number theory in connection with the local Langlands conjecture,** J. Ritter, Editor

87 **Abelian group theory,** Laszlo Fuchs, Rüdiger Göbel, and Phillip Schultz, Editors

88 **Invariant theory,** R. Fossum, W. Haboush, M. Hochster, and V. Lakshmibai, Editors

89 **Graphs and algorithms,** R. Bruce Richter, Editor

90 **Singularities,** Richard Randell, Editor

91 **Commutative harmonic analysis,** David Colella, Editor

92 **Categories in computer science and logic,** John W. Gray and Andre Scedrov, Editors

93 **Representation theory, group rings, and coding theory,** M. Isaacs, A. Lichtman, D. Passman, S. Sehgal, N. J. A. Sloane, and H. Zassenhaus, Editors

94 **Measure and measurable dynamics,** R. Daniel Mauldin, R. M. Shortt, and Cesar E. Silva, Editors

95 **Infinite algebraic extensions of finite fields,** Joel V. Brawley and George E. Schnibben

96 **Algebraic topology,** Mark Mahowald and Stewart Priddy, Editors

97 **Dynamics and control of multibody systems,** J. E. Marsden, P. S. Krishnaprasad, and J. C. Simo, Editors

98 **Every planar map is four colorable,** Kenneth Appel and Wolfgang Haken

99 **The connection between infinite dimensional and finite dimensional dynamical systems,** Basil Nicolaenko, Ciprian Foias, and Roger Temam, Editors

100 **Current progress in hyperbolic systems: Riemann problems and computations,** W. Brent Lindquist, Editor

101 **Recent developments in geometry,** S.-Y. Cheng, H. Choi, and Robert E. Greene, Editors

102 **Primes associated to an ideal,** Stephen McAdam

103 **Coloring theories,** Steve Fisk

Titles in This Series

Volume

104 Accessible categories: The foundations of categorical model theory, Michael Makkai and Robert Paré

105 Geometric and topological invariants of elliptic operators, Jerome Kaminker, Editor

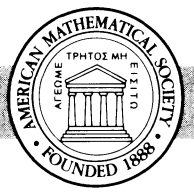
106 Logic and computation, Wilfried Sieg, Editor

107 Harmonic analysis and partial differential equations, Mario Milman and Tomas Schonbek, Editors

108 Mathematics of nonlinear science, Melvyn S. Berger, Editor

109 Combinatorial group theory, Benjamin Fine, Anthony Gaglione, and Francis C. Y. Tang, Editors

Combinatorial Group Theory



CONTEMPORARY MATHEMATICS

109

Combinatorial Group Theory

Proceedings of the AMS Special Session in Combinatorial Group Theory—Infinite Groups held April 23–24, 1988

Benjamin Fine, Anthony Gaglione, and Francis C. Y. Tang, Editors

AMERICAN MATHEMATICAL SOCIETY • PROVIDENCE, RHODE ISLAND

EDITORIAL BOARD

Richard W. Beals, managing editor

Sylvain E. Cappell	Jan Mycielski
Jonathan Goodman	Linda Preiss Rothschild
Craig Huneke	Michael E. Taylor

The AMS Special Session in Combinatorial Group Theory–Infinite Groups was held at the University of Maryland, College Park, Maryland, April 23–24, 1988.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 20EXX, 20FXX.

Library of Congress Cataloging-in-Publication Data

Combinatorial group theory: proceedings of the AMS special session in combinatorial group theory-infinite groups, April 23–24, 1988/Benjamin Fine, Anthony Gaglione, and Francis C. Y. Tang, editors.

p. cm.—(Contemporary mathematics, ISSN 0271-4132; v. 109)

Includes bibliographical references.

ISBN 0-8218-5116-0 (alk. paper)

1. Combinatorial group theory—Congresses. 2. Infinite groups—Congresses. I. Fine, Benjamin, 1948–. II. Gaglione, Anthony M. III. Tang, Francis C. Y., 1935–. IV. American Mathematical Society. V. Series: Contemporary mathematics (American Mathematical Society); v. 109.

QA171.C67843 1990 90-40475

515′.2—dc20 CIP

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940-6248.

The appearance of the code on the first page of an article in this book indicates the copyright owner's consent for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law, provided that the fee of $1.00 plus $.25 per page for each copy be paid directly to the Copyright Clearance Center, Inc., 27 Congress Street, Salem, Massachusetts 01970. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

Copyright ©1990 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights except those granted
to the United States Government.
Printed in the United States of America.
The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability. ♾

This volume was prepared by the authors using $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX,
the American Mathematical Society's TEX macro system.

10 9 8 7 6 5 4 3 2 1 95 94 93 92 91 90

Contents

Preface xi

Some Reflections on Finitely Generated Metabelian Groups
G. BAUMSLAG 1

Conjugacy Separability of Fuchsian Groups and Related Questions
B. FINE and G. ROSENBERGER 11

Two-Generator Subgroups of Certain HNN Groups
B. FINE, F. ROHL, and G. ROSENBERGER 19

A Geometric Approach to Some Group Presentations
T. FOURNELLE and K. WESTON 25

$\gamma_{n+1}(F)$ and $F/\gamma_{n+1}(F)$ Revisited
A. GAGLIONE and D. SPELLMAN 35

The Commutator Collection Process
A. GAGLIONE and H. WALDINGER 43

Levi-Properties in Metabelian Groups
L. KAPPE and R. MORSE 59

Monodromy Groups of Differential Equations on Riemann Surfaces of Genus 1
K. KUIKEN and J. MASTERSON 73

The Lie Algebra Associated to the Lower Central Series of a Free Product of Cyclic Groups of Prime Order p
J. LABUTE 91

Algebraic Closure of Groups
J. LEVINE 99

Automorphisms of Riemann Surfaces
A. MACBEATH 107

On Commutator Laws in Groups, 2
I. MACDONALD and B. H. NEUMANN 113

Two-Dimensional Linear Characters and Automorphisms of Free Groups
J. MCCOOL 131

On the Uniqueness of Amalgamated Product Decompositions of a Group
J. RATCLIFFE 139

The Cartesian Subgroup of a Free Product of Profinite Groups
L. RIBES 147

A Note on Local Coboundaries for Locally Nilpotent Groups
D. ROBINSON 159

Some Results on 1-Relator Quotients of Free Products
C. Y. TANG 165

Covering Spaces, Subgroup Separability, and the Generalized M. Hall Property
M. TRETKOFF 179

Preface

This volume contains papers in final form from the American Mathematical Society special session on combinatorial group theory—infinite groups held at the University of Maryland April 1988. This session was organized by A. Gaglione of the Naval Academy, C. Y. Tang of the University of Waterloo, and B. Fine of Fairfield University. This special session was designed to bring together researchers in various areas of infinite group theory, especially combinatorial group theory, to share methods and results.

We were impressed with the vitality and interest in infinite group theory as reflected in this conference. There were eighteen speakers at the session, and the talks covered a wide range of group theory from purely logical questions to geometric methods. For many reasons—a large number originating in topological and geometric results—there is heightened interest in classical combinatorial group theory and its questions. This was reflected by the sheer volume of work presented at the conference.

The papers in this volume represent a mix of pure research papers and expository papers. In keeping with what we feel is the spirit of the Contemporary Mathematics series we hope that the majority of the papers are clear enough to the nonspecialist to convey a sense of the direction of the field. Some of the papers of course contain entirely new results and the editors wish to thank the participants for using this volume as an outlet for their work.

While no single volume can present the total state of affairs in modern combinatorial group theory we hope that this work will serve as an entry point for people interested in learning where the discipline is.

We wish to thank the referees for their dedicated work in evaluating the papers. We would also like to thank Connie Engle, Pat Struse , and Diane Berezowski for their fine typing job.

The participants at the conference were Seymour Bachmuth, Gilbert Baumslag, Katalin Bencsath, Ben Fine, Thomas Fournelle, Anthony Gaglione, John Labute, Luise Kappe, Kathryn Kuiken, Jerry Levine, Alex Lichtman, Murray

Macbeath, Ian Macdonald, Jim McCool, John Ratcliffe, Luis Ribes, Derek Robinson, Gerhard Rosenberger, Dennis Spellman, Francis Tang, Marvin Tretkoff, Herman Waldinger, and Ken Weston.

Ben Fine
Anthony Gaglione
Francis Tang

ISBN: 0-8218-5116-0