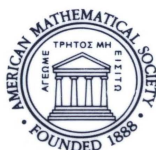


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115

## Statistical Multiple Integration



American Mathematical Society

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# Statistical Multiple Integration

# CONTEMPORARY MATHEMATICS

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115

## Statistical Multiple Integration

Proceedings of a Joint Summer Research Conference  
held at Humboldt University, June 17–23, 1989

Nancy Flourney  
Robert K. Tsutakawa  
Editors



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## Preface

The idea for a conference on Statistical Multiple Integration had its genesis during the years 1986 to 1988 when Nancy Flournoy directed the Program in Statistics and Probability at the National Science Foundation (NSF). Directors of NSF programs are frequently called upon to describe “current thrusts”, “significant breakthroughs”, and “obstacles to progress” on behalf of their field, and thus to keep an ear open for such events while synthesizing grant proposals and reviewers’ comments.

Since multiple integration is not considered to be in the mainstream of the field of statistics, associated problems did not receive prominent or highlighted attention in the grant proposals. However, it became apparent that such problems were repeatedly tucked into the midst of a statistical problem, and often, a single statement unobtrusively positioned in the text would assert that multiple integration was *the* major obstacle in the solution to this or that statistical problem.

Some statistical proposals that included a multiple integration component were submitted to the newly created NSF program in Computational Mathematics, and were reviewed by applied mathematicians and numerical analysts as well as statisticians. Reviewers outside the field of statistics invariably lacked an appreciation for the statisticians’ problems, commonly suggesting that solutions were available if only the statistician would look in the right place or collaborate with the right people.

Bob Tsutakawa and I had collaborated previously on the design of a Bayesian experiment for clinical trials at the Fred Hutchinson Cancer Research Center in Seattle. This work provided the basis for our joint interest in statistical multiple integration, although through different motivations, and led us to the notion of a conference that would bring together mathematicians, statisticians, and computational scientists to make each constituency more aware of the other’s progress.

High dimensional integration arises naturally in two major subfields in statistics: multivariate and Bayesian statistics. Indeed, the most common measures of central tendency, variation, and loss are defined by integrals

over the sample space, the parameter space, or both. Recent advances in computation power have stimulated significant new thrusts in both Bayesian and classical multivariate statistics. Bayesian statisticians are motivated by the need to develop tractable computational methods for the application of Bayesian statistical theory; whereas, multivariate statisticians are motivated by the need to analyze more complex dependencies among variables and objects.

Thus the Joint AMS-IMS-SIAM conference on Statistical Multiple Integration was conceived and this volume speaks to its success. The papers collected together in this volume serve to document the state of the art of multiple integration with respect to problems in statistics, statistical thrusts blocked by problems with multiple integration, and current work directed at expanding our capability to integrate over high dimensional surfaces. Upon reading these works, we trust you will agree that there are many important problems in statistical multiple integration to challenge and stimulate statisticians, numerical analysts, and other computational and algorithmic experts alone and in collaboration. We hope this volume serves to stimulate this effort.

Nancy Flournoy

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