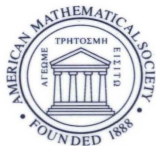


CONTEMPORARY MATHEMATICS

121

Spinor Construction of Vertex Operator Algebras, Triality, and $E_8^{(1)}$

Alex J. Feingold
Igor B. Frenkel
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Contents

Chapter 0	Introduction	1
Chapter 1	Summary	13
Chapter 2	Affine Algebras and Representations	34
Chapter 3	Spinor Construction of Vertex Operator Superalgebras	44
Chapter 4	Spinor Construction of the Chevalley Algebra and Triality for D_4	74
Chapter 5	Spinor Construction of Triality for $D_4^{(1)}$	91
Chapter 6	Spinor Construction of a Vertex Operator Para-algebra for $D_4^{(1)}$	106
Chapter 7	Spinor Construction of E_8	128
Chapter 8	Spinor Construction of Vertex Operator Algebras for $E_8^{(1)}$	132
References	.	144

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