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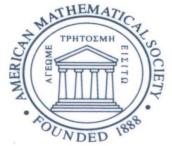
New Approaches in Spectral Decomposition

Ridgley Lange
Shengwang Wang



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PREFACE

In this expository volume on some recent developments in the various theories of spectral decomposition, we attempt to bring to a general mathematics audience a readable, reasonably advanced account of the techniques used in the theory and applications of decomposable operators and related classes of operators.

The first chapter on the fundamental properties of decomposable operators begins with a characterization theorem for these operators; this material is drawn mostly from several recent papers by the authors. The remainder of Chapter I is devoted to the study of various subclassses of decomposable operators. For example, in Section 3 we give criteria for strongly decomposable and quasi-strongly decomposable operators as well as those which are decomposable relative to the identity. In the last section of the chapter we study the relationships among those classes already mentioned and many others (ten altogether) which have been introduced in the literature. We show that these ten classes are linearly ordered by inclusion and that most of them are distinct.

In Chapter II, Section 1, we present several types of results on perturbation of a decomposable operator by a commuting operator. In the next section we specialize from Banach to Hilbert space and we prove that certain compact perturbations of a normal operator having spectrum in a Jordan curve are decomposable. This result hinges on the estimate of the growth of the local resolvent of the perturbed operator. Many results in the last section on quasisimilarity follow from the earlier perturbation theorems.

Chapter III introduces a class of operators which we call "weakly decomposable relative to the identity." We begin with the basic properties of these operators; for example, they have the important single-valued extension property. The next section discusses the multiplication operator associated with one weakly decomposable relative to the identity. By example, we show that, although weakly decomposable operators in the present sense are quasidecomposable, they need not be decomposable. In fact, unlike

the situation in Chapter I, this new class is not comparable to other classes. We also generalize to our new class some old results of Colojoara and Foias for maximal hyperinvariant subspace chains of a given A-spectral operator.

Our principal aim in Chapter III comes in Section 5, where we prove that any linear transformation intertwining a decomposable operator and one weakly decomposable relative to the identity is automatically bounded (given supplementary conditions). (Operator A intertwines T and S if AT = SA). The following section of the chapter gives several applications of the automatic continuity theorem, and in the final section of the chapter we give further perspectives on this notion.

Chapter IV deals with some recent applications of subdecomposable operators, i.e. those which are restrictions of decomposable operators. The principal general result here is that if the restriction of an "unconditional- ly" decomposable operator has spectrum with nonempty interior, then such a restriction has nontrivial invariant subspaces.

Next we prove that hyponormal operators are subdecomposable, more precisely, subscalar. Those hyponormal operators with spectra having empty interior may also have nontrivial invariant subspaces, but the proof is different from the case above since the scalar extension of a hyponormal operator is not clearly unconditionally decomposable.

Our final chapter is an expository survey of some research that has been done on the spectral decomposition of commuting systems of (bounded) operators in Banach space. This direction of study, initiated by E. Albrecht in the early 1970's, is based on J. L. Taylor's notion of joint spectrum. We include this material to give a certain completeness to our discussion.

Our point of view in writing the proofs in this book has been to make them accessible to the nonspecialist and to bring out the rich interplay between current operator theory and other branches of classical analysis. Thus the reader with a sound, rudimentary acquaintance with functional analysis (Banach space), function theory, harmonic analysis, elementary topology and homological algebra should be able to follow the arguments. To this degree, the volume is self-contained. But, of course, no work of mathematics can be truly "self-contained," so we have given the details of all proofs except those which would take the discussion clearly out of the essential line of development. In those few cases we cite an outside reference.

When the authors first prepared this volume, the second named author was visiting Central Michigan University and the University of Pittsburgh. He would like to express his thanks to Professors R. Fleming, W. Deskins, S. Hastings, K. S. Lau and H. Cohen for their kind invitations. He is also indebted to the committee of the National Natural Science Foundation of China for their valuable support.

The Authors

New Approaches in Spectral Decomposition
Ridgley Lange and Shengwang Wang

Aimed at a general mathematical audience, this book provides a careful exposition of recent developments in the theory of spectral decomposition. Bringing the reader from the basics up to the level of current research in the area, the authors present a readable account of the techniques used in the theory and applications of decomposable operators and related classes of operators. The book begins with a discussion of criteria for decomposable and related types of operators, and an analysis that relates and distinguishes among them. Perturbation theory of decomposable and other operators, applications to classical Hilbert space operators, quasisimilarity, and a new class of weakly decomposable operators are also discussed. The book closes with an exposition of some classical theories on invariant subspaces for subdecomposable and hyponormal operators, and a presentation of the parallel spectral theory of commuting systems.

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