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American Mathematical Society

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CONTEMPORARY MATHEMATICS

129

Oscillation and Dynamics in Delay Equations

Proceedings of an AMS Special Session
held January 16–19, 1991

John R. Graef
Jack K. Hale
Editors



American Mathematical Society
Providence, Rhode Island

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Preface

Oscillation theory and dynamical systems have long been rich and active areas of mathematical research. The papers in this volume are based on presentations at the Special Session of the same title held at the 97th Annual Meeting of the American Mathematical Society, in San Francisco, CA, in January 1991. With a special emphasis on delay equations, the papers cover a broad range of topics in ordinary, partial, and difference equations, and include applications to problems in commodity prices, biological modeling, and number theory. All the papers in this volume are in final form, and no versions of them will be submitted for publication elsewhere. The editors wish to express their appreciation to the authors of each paper for their assistance with the preparation of these proceedings.

John R. Graef

Jack K. Hale

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Oscillation and Dynamics in Delay Equations
John R. Graef and Jack K. Hale, Editors

Oscillation theory and dynamical systems have long been rich and active areas of research. Containing frontier contributions by some of the leaders in the field, this book brings together papers based on presentations at the AMS meeting in San Francisco in January 1991. With special emphasis on delay equations, the papers cover a broad range of topics in ordinary, partial, and difference equations and include applications to problems in commodity prices, biological modeling, and number theory. The book would be of interest to graduate students and researchers in mathematics or those in other fields who have an interest in delay equations and their applications.

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