

# CONTEMPORARY MATHEMATICS

148

## Index Theory and Operator Algebras

Proceedings of a  
CBMS Regional Conference  
on  $K$ -Homology and Index Theory  
held August 6–10, 1991  
in Boulder, Colorado

Jeffrey Fox  
Peter Haskell  
Editors



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## Preface

On August 6 - 10, 1991, a CBMS Regional Conference on  $K$ -Homology and Index Theory was held at the University of Colorado at Boulder. Paul Baum was the principal speaker. There were many other speakers at the conference. The papers in this volume are based on some of their talks.

The papers here provide both a careful exposition of new points of view on classical index theory and an introduction to currently active areas of the field. They reflect current interest in the topology of manifolds at infinity and in related questions concerning geometry and analysis associated with discrete groups. The papers use techniques derived from differential geometry and spectral theory to address questions arising in analysis on noncompact manifolds. Operator algebras play a central role as these papers extend our understanding of topics as diverse as operator theory, index theory on manifolds with boundary, and the representation theory of  $p$ -adic groups.

We thank the authors of these papers and all of the conference participants for their mathematical contributions to the success of the conference. We thank the National Science Foundation for their financial contribution and the American Mathematical Society for their patience and help in preparing this volume.

Jeffrey Fox  
Peter Haskell

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**Index Theory and Operator Algebras**  
Jeffrey Fox and Peter Haskell, Editors

This collection of papers by leading researchers provides a broad picture of current research directions in index theory. Based on lectures presented at the NSF-CBMS Regional Conference on  $K$ -Homology and Index Theory, held in August 1991 at the University of Colorado at Boulder, the book provides both a careful exposition of new perspectives in classical index theory and an introduction to currently active areas of the field. Presented here are two new proofs of the classical Atiyah-Singer Index Theorem, as well as index theorems for manifolds with boundary and open manifolds. Index theory for semi-simple  $p$ -adic groups and the geometry of discrete groups are also discussed. Throughout the book, the application of operator algebras emerges as a central theme. Aimed at graduate students and researchers, this book would be suitable as a text for an advanced graduate topics course on index theory.

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