

CONTEMPORARY MATHEMATICS

149

Doebelin and Modern Probability

Proceedings of the Doebelin Conference
held November 2–7, 1991
at the University of
Tubingen's Heinrich Fabri Institut,
Blaubeuren, Germany

Harry Cohn
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Photograph courtesy of Claude Doeblin.

Wolfgang Doeblin
1915–1940

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149

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Proceedings of the Doebelin Conference
"50 Years after Doebelin: Development in the Theory
of Markov Chains, Markov Processes,
and Sums of Random Variables"
held November 2–7, 1991, with support
from the Applied Probability Trust

Harry Cohn
Editor



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Proceedings of the Doeblin Conference “50 Years after Doeblin: Development in the Theory of Markov Chains, Markov Processes, and Sums of Random Variables” held November 2–7, 1991 at the University of Tübingen’s Heinrich Fabri Institut, Blaubeuren, Germany, through the efforts of Professors H. Hering and J. Gani, and with support from the Applied Probability Trust.

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PREFACE

J.L. DOOB

My first contact with Doebelin was a letter from him, protesting (justifiably) that I should have written him about a slip in his 1938 paper instead of simply noting the lapse in a later paper of my own. I was making so many errors in my own work, that the error in Doebelin's paper seemed no big deal to me! To do him justice, Doebelin held no grudge about my reference and sent me a card showing how to fix his proof. Thereafter, we exchanged reprints and letters; in particular he sent me his thesis. This thesis was published in 1937 in a rather inaccessible journal, and I thought so highly of it that I printed some of its results, with few changes, in my book "Stochastic Processes" in 1953. When the news spread that I had a copy of the thesis, I received so many requests for it in those pre Xerox days that I finally had it microfilmed to facilitate distribution.

In the late thirties, probability theory was in the process of modernization and was gradually being incorporated into the body of standard mathematics. Deep researchers like Doebelin delayed the acceptance of probability as mathematics by their virtuosity in dealing not only with probabilities, but also with the subtleties of sample functions. The latter seemed alien to classical analysts who while experiencing no difficulty in accepting analytical studies of conditional probabilities which led to difference equations, partial differential equations, and integral equations, in which the probabilistic background was hardly hinted at, found sample function contexts incomprehensible and therefore considered them outside mathematics.

I found it extraordinary that Doebelin could still do mathematical research while he was in the French army. The paragraph which I quote below, translated from a letter to me in French dated 16 April 1939 illustrates how he managed to work even in the army.

I had been intending since October to write out the above proof the following Sunday, but for many reasons I always deferred it. For the moment I spend rather little time on mathematics as military service scarcely leaves me any spare time. I have returned to my research on the set of powers of a probability law, which I hope to finish before October. I shall send you some reprints one of these days.

The following eloquent paragraph was written about Doebelin by his mentor Paul Lévy in his obituary; this is my translation:

One is always struck by the sureness and precision of his reasoning, and his extraordinary ability to solve the most varied difficulties either by frontal attack or by finding an indirect approach. I think I can say, to give an idea of the level at which he deserves to be placed, that one can count on the fingers of a single hand the mathematicians, who since Abel and Galois, died so young leaving such an important body of work. There is no doubt that his name should maintain an important place in the history of the calculus of probability.

It is to be hoped that the young probabilists of today will learn from this book how much they owe not only to the results obtained by Doebelin, but to the new approach to probability that he was instrumental in creating.

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INTRODUCTION

HARRY COHN

The present volume is based on the papers read at the conference **"50 years after Doeblin: Developments in the theory of Markov chains, Markov processes and sums of random variables"** held between 2-7 November 1991 at Blaubeuren.

The conference was dedicated to the memory of one of the greatest probabilists of this century, Wolfgang Doeblin, who died in action during World War II at the age of 25. Doeblin left behind several seminal contributions which have profoundly influenced many areas of probability and still continue to be a source of inspiration for modern developments.

A group of probabilists from a number of countries thought that a commemoration of his achievements by the community of scientists was in order 50 years after his death, and that the time was long overdue for a well documented account of his life and work. This led to a conference in Germany, where distinguished scientists from all over the world whose research has been influenced by Doeblin's papers presented lectures and engaged in discussions which were later to materialize in this book dedicated to Doeblin's memory.

An organizing committee was formed in 1989 consisting of A Blanc-Lapierre (France), KL Chung (USA), H Cohn (Australia), J Gani (Australia), H Hering (Germany) and M Iosifescu (Romania). In the final analysis, the conference became possible through the efforts of Professor H Hering who obtained funding for the accommodation of 35 participants in the modern building of the Heinrich Fabri Institute at Blaubeuren, a small picturesque town near Ulm. Some funds were also made available by the Applied Probability Trust through Professor J Gani. The participants, representing

16 countries, payed tribute to Doebelin's legacy in talks and discussions which covered many of the directions Doebelin had worked on during his short life. We were privileged to have Professor JL Doob (USA) attend the conference as guest of honour. His book "*Stochastic Processes*" published in 1953 was the first to establish Doebelin as a basic contributor to modern Probability Theory.

Although an article on Doebelin's life and work was written by Paul Lévy in 1955, there has been rather scanty information available on his life and only part of his 26 papers were known. There is however a well researched life of Doebelin's father, the noted novelist Alfred Doebelin, whose best known work "*Berlin, Alexanderplatz*" first published in 1929 has been internationally acclaimed and translated into many languages. Later this novel has become known to millions of people from Fassbinder's masterly adaptation to a TV series. Much unknown and very relevant material was also found in the Paris archives and brought to light through the efforts of Professors Blanc-Lapierre and Bru. We also wish to acknowledge the assistance of Doebelin's younger brother Claude, who lives in Nice, France, and kindly provided some material on Doebelin from the family archives, later to be incorporated in Bru's article.

Doebelin's life and work were illustrated by T Lindvall (Sweden) who gave a talk entitled *Wolfgang Doebelin: Snapshots of his life and work*, B Bru (France) who dealt with *W Doebelin: life and work from Parisian archives* and KL Chung (USA) who described how he became acquainted with Doebelin's work on Markov chains in *Discovering Doebelin's work*.

These talks were followed by a number of sessions where the participants discussed various contributions of Doebelin together with further developments in the current literature. Among the contributors were S Asmussen* (Denmark): *Light traffic theorems for random walks and queues*, KB Athreya (USA): *Continuous time stochastic gambling problems* E Bolthausen (Switzerland) *Large deviations and problems with long range interactions of the paths*, P Brémaud (France): *Maximal coupling and sensitivity analysis*, JE Cohen (USA): *Relative entropy under mappings by stochastic matrices*, H Cohn (Australia): *On a paper by Doebelin on finite non-homogeneous Markov chains*, M Csörgő (Canada): *Almost sure summability of partial sums of independent random variables*, I Cuculescu (Romania): *Applications of a construction of Markov processes*, Y Derriennick (France): *On the local limit theorem for some Markov chains on some unilateral random walks in random environments*,

*Some of these talks have not appeared as papers in this book, while others have had their titles altered when written up in their final forms.

M Gordin (Russia): *Homoclinic approach to the central limit theorem for dynamical systems*, J Hajnal (England): *A theory of shuffling*, A Hordijk (Netherlands): *The Doeblin condition and generalized strong ergodicity*, M Iosifescu (Romania): *A basic tool in mathematical chaos theory: the 1937 ergodic theorem of Doeblin and Fortet*, P Jagers (Sweden): *The Markov renewal structure of stable populations*, A Joffe (Canada): *Perturbations of homogeneous Markov chains*, S Kalpazidou (Greece): *On Doeblin-Fortet chains occurring in the number expansion theory*, JFK Kingman (England): *Poisson processes and random sets*, TM Liggett (USA): *The coupling technique in interacting particle systems*, S Meyn (USA): *Stability criteria for continuous time Markov processes*, A Mukherjea (USA): *Random walks on matrices and attractors*, P Ney (USA): *Regeneration for non-Markovian chains*, P Nüesch (Switzerland): *A multivariate look at ES Andersen's equivalence principle*, E Nummelin (Finland): *Duality for Markov operators with applications to interval maps*, M Peligrad (USA): *Limit theorems for dependent sequences*, P Révész (Austria): *Path properties of an infinite system of Wiener processes*, M Rosenblatt (USA): *The central limit theorem for Markov sequences*, H Thorisson (Iceland): *Shift-coupling*, R Tweedie (USA): *Conditions for Doeblin decomposition of general Markov chains*, E Seneta (Australia): *Applications of ergodicity coefficients to homogeneous Markov chains*.

Discussions followed each talk and often continued informally as the participants were accommodated in the same building and had their meals together. The talks revealed an amazing number of important contributions made by Doeblin; this was a surprise even for those participants, like Chung and Doob, who have written comprehensive books incorporating Doeblin's contributions. A number of Doeblin's articles published in obscure journals were known only to some of the participants; it turned out that very new developments such as chaos theory and random mappings transformations benefit from crucial contributions made by Doeblin more than 50 years ago.

Although a thorough analysis of Doeblin's contributions to probability has yet to be written, this book endeavours to highlight Doeblin's importance in the subject by emphasising the influence of his ideas on a number of contemporary research directions.

On completing this project, I feel gratitude to the organizing committee and the contributors to the Blaubeuren conference as well as to other colleagues who could not attend the conference but offered their support and advice. Special thanks are due to Kai Lai Chung who was enthusiastic about the project when it was first suggested and constantly offered very valuable advice, and my old friend Marius Iosifescu—one of the best con-

noisseurs of Doeblin's work. Last but not least, I am enormously grateful to Joe Gani whose help during all the stages of this project was essential in bringing it to fruition.

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Doebelin and Modern Probability
Harry Cohn, Editor

Wolfgang Doebelin, one of the greatest probabilists of this century, died in action during World War II at the age of twenty-five. He left behind several seminal contributions which have profoundly influenced the field and continue to provide inspiration for current research. This book is based on papers presented at the conference, "Fifty Years after Doebelin: Developments in the Theory of Markov Chains, Markov Processes, and Sums of Random Variables," held at Blaubeuren, Germany, in November 1991. Presented here for the first time is an account of Doebelin's life and work, revealing the circumstances of his tragic death in 1940. Organized into sections according to topic, the papers describe both Doebelin's original contributions as well as current developments. With contributions by top probabilists from sixteen countries, this book will be of interest both to researchers in probability and to historians of science.

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