The Penrose Transform and Analytic Cohomology in Representation Theory

AMS-IMS-SIAM Summer Research Conference
on The Penrose Transform and Analytic Cohomology in Representation Theory
June 27 to July 3, 1992
Mount Holyoke College, South Hadley, Massachusetts

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PREFACE

This volume contains papers presented at the conference "The Penrose Transform and Analytic Cohomology in Representation Theory," held 27th June to 3rd July, 1992, at Mount Holyoke College, South Hadley, Massachusetts. All articles have been refereed. The conference was one of a series of Summer Research Conferences sponsored jointly by the American Mathematical Society, the Institute of Mathematical Statistics, and the Society for Industrial and Applied Mathematics, and supported by a grant from the National Science Foundation. The organizing committee consisted of Robert J. Baston (Co-chair), Michael G. Eastwood (Co-chair), Victor W. Guillemin, Joseph A. Wolf, and Roger Zierau.

The main idea of this conference was to bring together representation theorists and differential geometers in order that they may profitably interact. In particular, it seemed that various integral transforms from representation theory, complex integral geometry, and mathematical physics were instances of the same general construction sometimes called the "Penrose transform." Speakers were asked to give talks bearing the diverse and non-specialist nature of their audience in mind. In this respect and also in informal discussion, the conference was very successful. There is considerable scope for further research and we hope that this conference and its proceedings will be seen as a useful catalyst.

Generally speaking, the programme was arranged with the more expository talks early on so that later speakers could refer to topics already discussed. The order has been maintained in these proceedings. Unfortunately, Victor Guillemin was unable to attend but has contributed a joint article with Meng-Kiat Chauh. Lisa Mantini kindly agreed to speak in his place. The programme and list of participants is given overleaf.

Special thanks are due to Carole Kohanski as Conference Coördinator and Donna Harmon for Editorial Assistance.

Michael Eastwood
Joseph Wolf
Roger Zierau
PREFACE

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D.A. Vogan, Jr.  "Unipotent Representations and Cohomological Induction"
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L. Barchini  "Intertwining Operators into Dolbeault Cohomology Representations"
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L. Mantini  "A Penrose-Radon Transform in $L^2$-cohomology for Massless Field Equations"
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D.M. Barbasch  "Unipotent Representations and Derived Functor Modules"
R. Howe  "Some Examples of the Infinitesimal Structure of Degenerate Principal Series"
R. Zierau  "Unitarity of Certain Dolbeault Cohomology Representations"

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This book contains refereed papers presented at the AMS-IMS-SIAM Summer Research Conference on the Penrose Transform and Analytic Cohomology in Representation Theory, held in the summer of 1992 at Mount Holyoke College. The conference brought together some of the top experts in representation theory and differential geometry. One of the issues explored at the conference was the fact that various integral transforms from representation theory, complex integral geometry, and mathematical physics appear to be instances of the same general construction, which is sometimes called the "Penrose transform". There is considerable scope for further research in this area, and this book would serve as an excellent introduction.