

CONTEMPORARY MATHEMATICS

154

The Penrose Transform and Analytic Cohomology in Representation Theory

AMS-IMS-SIAM Summer Research Conference
on The Penrose Transform and Analytic
Cohomology in Representation Theory
June 27 to July 3, 1992

Mount Holyoke College, South Hadley, Massachusetts

Michael Eastwood
Joseph Wolf
Roger Zierau
Editors



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CONTENTS

Preface	ix
Introduction to representations in analytic cohomology A. W. KNAPP	1
Admissible representations and geometry of flag manifolds JOSEPH A. WOLF	21
Unipotent representations and cohomological induction DAVID A. VOGAN, JR.	47
Introduction to Penrose transform MICHAEL EASTWOOD	71
Strongly harmonic differential forms on elliptic orbits L. BARCHINI	77
A finiteness theorem for quaternionic-Kähler manifolds with positive scalar curvature CLAUDE LEBRUN	89
Holomorphic language for $\bar{\partial}$ -cohomology and representations of real semisimple Lie groups SIMON GINDIKIN	103
Twistor theory for indefinite Kähler symmetric spaces EDWARD G. DUNNE AND ROGER ZIERAU	117
Algebraic \mathcal{D} -modules and representation theory of semisimple Lie groups DRAGAN MILIČIĆ	133
Parabolic invariant theory and geometry TOBY N. BAILEY	169
Kaehler structures on $K_{\mathbb{C}}/N$ MENG-KIAT CHUAH AND VICTOR GUILLEMIN	181

Cousin complexes and resolutions of representations JOHN W. RICE	197
Dolbeault cohomologies and Zuckerman modules H. W. WONG	217
Unipotent representations and derived functor modules DAN BARBASCH	225
Unitarity of certain Dolbeault cohomology representations ROGER ZIERAU	239

PREFACE

This volume contains papers presented at the conference “The Penrose Transform and Analytic Cohomology in Representation Theory,” held 27th June to 3rd July, 1992, at Mount Holyoke College, South Hadley, Massachusetts. All articles have been refereed. The conference was one of a series of Summer Research Conferences sponsored jointly by the American Mathematical Society, the Institute of Mathematical Statistics, and the Society for Industrial and Applied Mathematics, and supported by a grant from the National Science Foundation. The organizing committee consisted of Robert J. Baston (Co-chair), Michael G. Eastwood (Co-chair), Victor W. Guillemin, Joseph A. Wolf, and Roger Zierau.

The main idea of this conference was to bring together representation theorists and differential geometers in order that they may profitably interact. In particular, it seemed that various integral transforms from representation theory, complex integral geometry, and mathematical physics were instances of the same general construction sometimes called the “Penrose transform”. Speakers were asked to give talks bearing the diverse and non-specialist nature of their audience in mind. In this respect and also in informal discussion, the conference was very successful. There is considerable scope for further research and we hope that this conference and its proceedings will be seen as a useful catalyst.

Generally speaking, the programme was arranged with the more expository talks early on so that later speakers could refer to topics already discussed. The order has been maintained in these proceedings. Unfortunately, Victor Guillemin was unable to attend but has contributed a joint article with Meng-Kiat Chauh. Lisa Mantini kindly agreed to speak in his place. The programme and list of participants is given overleaf.

Special thanks are due to Carole Kohanski as Conference Coördinator and Donna Harmon for Editorial Assistance.

Michael Eastwood
Joseph Wolf
Roger Zierau

Programme

A.W. Knapp	“Introduction to Representations in Analytic Cohomology”
J.A. Wolf	“Geometric Quantization for Semisimple Lie Groups”
D.A. Vogan, Jr.	“Unipotent Representations and Cohomological Induction”
B. Kostant	“Nilpotent Coadjoint Orbits and Unipotent Representations of Semisimple Lie Groups”
M.G. Eastwood	“Introduction to Penrose Transform”
L. Barchini	“Intertwining Operators into Dolbeault Cohomology Representations”
C.R. LeBrun	“Holonomy Groups, the Penrose Transform, and Algebraic Geometry”
S. Gindikin	“Realizations of $\bar{\partial}$ -cohomology on Pseudo-Hermitian Symmetric Manifolds”
E.G. Dunne	“Twistor Theory for Indefinite Kähler Symmetric Spaces”
D. Milčić	“ \mathcal{D} -modules and the Classification of Harish-Chandra Modules”
T.N. Bailey	“Parabolic Invariant Theory and Geometry”
R.J. Baston	“Multiplicity One for Zuckerman’s Functors”
L. Mantini	“A Penrose-Radon Transform in L^2 -cohomology for Massless Field Equations”
J.E. Rice	“Cousin Complexes and Resolutions of Representations”
H.W. Wong	“Dolbeault Cohomologies and Zuckerman Modules”
W. Schmid	“ $G_{\mathbb{R}}$ -Equivariant Sheaves and Cohomology”
D.M. Barbasch	“Unipotent Representations and Derived Functor Modules”
R. Howe	“Some Examples of the Infinitesimal Structure of Degenerate Principal Series”
R. Zierau	“Unitarity of Certain Dolbeault Cohomology Representations”

Participants

T.N. Bailey, M.W. Baldoni, D.M. Barbasch, L. Barchini, R.J. Baston, A.O. Brega, R. Donley, E.G. Dunne, M.G. Eastwood, E. Galina, S. Gindikin, A.R. Gover, R. Howe, J. Huang, R. Keown, A.W. Knapp, T. Kobayashi, L. Koch, B. Kostant, R. Kunze, C.R. LeBrun, J.D. Lorch, L. Mantini, W. McGovern, R.C. Mclean, D. Milicic, J. Novak, J.W. Rice, W. Schmid, R.J. Stanton, R.W. Stokes, J.A. Tirao, D.A. Vogan, J.A. Wolf, H.W. Wong, H. Zheng, R. Zierau, Y.M. Zou.

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**The Penrose Transform and Analytic Cohomology
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Michael Eastwood, Joseph Wolf, and Roger Zierau, Editors

This book contains refereed papers presented at the AMS-IMS-SIAM Summer Research Conference on the Penrose Transform and Analytic Cohomology in Representation Theory, held in the summer of 1992 at Mount Holyoke College. The conference brought together some of the top experts in representation theory and differential geometry. One of the issues explored at the conference was the fact that various integral transforms from representation theory, complex integral geometry, and mathematical physics appear to be instances of the same general construction, which is sometimes called the "Penrose transform". There is considerable scope for further research in this area, and this book would serve as an excellent introduction.

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