Recent Advances in Real Algebraic Geometry and Quadratic Forms


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Dedicated to Professor Donald Dubois
Recent Advances in Real Algebraic Geometry and Quadratic Forms


William B. Jacob
Tsit-Yuen Lam
Robert O. Robson
Editors

American Mathematical Society
Providence, Rhode Island
The papers in this volume grew out of a year long program in Real Algebraic Geometry and Quadratic Forms held at the University of California at Berkeley, during the academic year 1990–91 and a special session on the same topics held at the joint winter mathematics meeting in San Francisco during January of 1991. The special year came to be known as “RAGSQUAD”. Funding was provided by NSF and NSA/MSP grants DMS-9003386 and MDA90-H-1015.

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BILL JACOB, T. Y. LAM, ROBBY ROBSON, EDITORS

Preface

The papers in this volume grew out of a year long program in Real Algebraic Geometry and Quadratic Forms held at the University of California at Berkeley, during the academic year 1990–91 and a special session on the same topics held at the joint winter mathematics meetings in San Francisco during January of 1991. The program at Berkeley was organized by T. Y. Lam and Robby Robson, while the AMS special session was organized by Bill Jacob. The special year came to be known as “RAGSQUAD”.

A list of RAGSQUAD participants appears below.

This volume of papers is dedicated to Professor Donald Dubois. The lead article D. W. Dubois and the Pioneer Days of Real Algebraic Geometry, contributed by Carlos Andradas and Tomás Recio, touches on the recent history of conferences, meetings, and special years in Real Algebraic Geometry and Quadratic Forms.

There is a long list of persons and organizations we wish to thank for material, scientific, and moral support. The University of California provided visiting positions, “University professorships”, and academic exchanges for a number of participants. The Mathematical Sciences Research Institute provided research professorships, and both institutions provided support in many other ways as well. The AMS sponsored the special session noted above, and funding was provided by federal grants\(^1\). Many foreign participants received support from their own countries and institutions, and of course there were numerous graduate

\(^1\) NSF and NSA/MSP grants DMS-9003386 and MDA90-H-1015.
students and office workers without whose efforts the program would not have run.

This volume contains scientific works resulting from the spirit of international collaboration which characterized the RAGSQUAD year. In more human terms, RAGSQUAD became a community built upon a foundation of intense mathematical friendship, not to mention some wonderful international pot luck dinners and even a RAGSQUAD concert. We hope that both the high quality of the mathematics and the RAGSQUAD spirit are reflected in this volume.

**RAGSQUAD Participants**

### Longer Term Participants

| S. Akbulut | M. E. Alonso | C. Andradas |
| R. Berr   | L. Bröcker   | R. Brown   |
| G. Brumfiel | M. Buchner | J.-L. Colliot-Thélène |
| M. J. de la Puente | C. Delzell | M. Dickmann |
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| M. Kräskemper | K. H. Leung | T. Y. Lam |
| D. Leep | J. Madden | M. Marshall |
| J. Merzel | R. Parimala | A. Pfister |
| A. Prestel | R. Robson | J. Ruiz |
| W. Scharlau | C. Scheiderer | R. Sridharan |
| G. Stengle | N. Schwartz | T. Smith |
| A. Wadsworth | C. Yeomans |

### Additional AMS Special Session Participants and short–term visitors

| R. Aravire | R. Baeza | S. Barton |
| R. Brown | T. Craven | R. Elman |
| R. Fitzgerald | L. González-Vega | M. Hirsch |
| J. Hsia | W. Jacob | L. Legorreta |
| C. Mulcahy | I. Pays | R. Perlis |
| V. Powers | T. Récio | M.-F. Roy |
| D. Shapiro | J. Shick | J.-P. Tignol |
| L. van den Dries | R. Ware | J. Yucas |

### Student Participants

| M. J. Gonzalez-Lopéz | D. Hoffmann | Y.-S. Hwang |
| N. Kan | A. Klute | D. Tao |
Epilogue

Putting this volume together was an adventure in electronic communication and modern \TeX{}nology. In the end, all circuits led through Corvallis to Providence. I wish to personally thank both Donna Harmon and Christine Thivierge of the American Mathematical Society for their help, hard work, and necessary prodding, and Bill McMecham of Professional Editing in Corvallis for translating some papers into \verb|\ams-\TeX| and adding the necessary \verb|\ams-\TeX| formats to others.

–Robby Robson, Corvallis, June 1993–

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