CONTEMPORARY MATHEMATICS

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Lie Algebras, Cohomology, and New Applications to Quantum Mechanics

AMS Special Session on
Lie Algebras, Cohomology, and
New Applications to Quantum Mechanics
March 20–21, 1992
Southwest Missouri State University

Niky Kamran
Peter J. Olver
Editors

American Mathematical Society

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Erratum

Two figures were inadvertently omitted from "Quasi-exactly-solvable spectral problems and conformal field theory", by Mikhail A. Shifman, pp. 237–262. The figures are reproduced below.

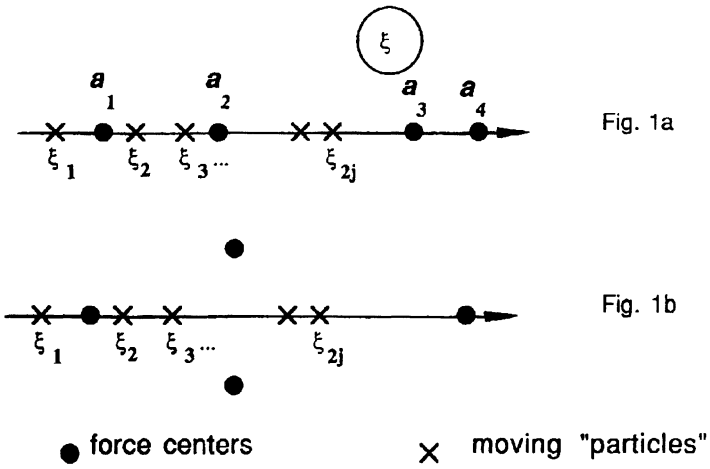

Figure 1

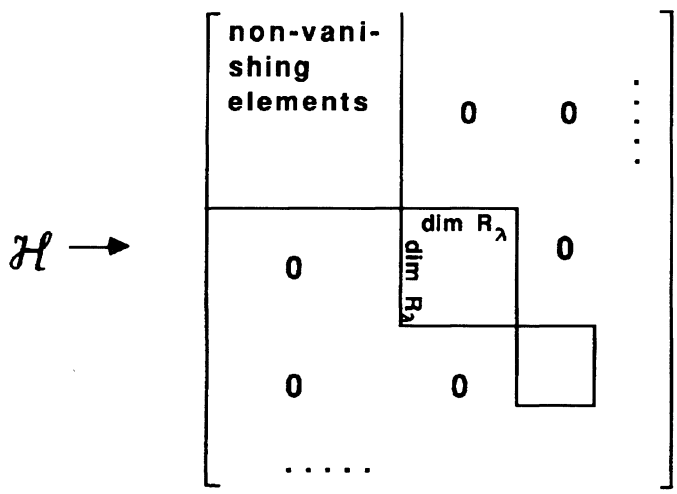

Figure 2

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Preface

The application of group theoretic methods to quantum mechanics has a long and distinguished history, dating back to the early investigations of Weyl, Wigner and Bargmann. In the "standard" approach, symmetries of the classical physical problem become symmetries (commuting operators) of the associated quantum mechanical Schrödinger operator. In favorable situations, this implies that the bound states of the associated stationary Schrödinger equation decompose into irreducible representations for the symmetry group, and the powerful theoretical methods from representation theory can be effectively used to analyze such problems; vice versa, the applications to quantum mechanics have themselves served as a powerful catalyst and motivation for the enormous amount of mathematical research effort that has gone into the study of representation theory.

The present volume is devoted to a range of important new directions and ideas concerning the applications of Lie groups and Lie algebras to Schrödinger operators and associated quantum mechanical systems. In these applications, the group does not appear as a standard symmetry group, but rather as a "hidden" symmetry group; nevertheless, the representation theory can still be effectively employed to analyze at least part of the spectrum of the operator. Such methods have their origin in the work of Dothan, Gell–Mann, Ne'eman, Barut and Bohm in the mid 1960's. The concept of a spectrum-generating algebra was introduced in nuclear physics, molecular physics, and scattering theory by Iachello, Arima, and Levine, in the 1970's and 80's, leading to the general concept of a Lie algebraic Schrödinger operator. In the mid 1980's, Turbiner, Shifman, Ushveridze, and collaborators introduced the more restrictive notion of a "quasi-exactly solvable" Schrödinger operator, for which the hidden symmetry algebra leads to the determination of a part of the spectrum by purely algebraic means. The synthesis and extensions of these seminal ideas has proved to be of great fruitfulness, not only in physical problems, but in the mathematical theory required to understand and classify the original examples.

In light of the rapidly developing subject, we decided that the time was ripe for bringing together, perhaps for the first time, a group of mathematicians and physicists working in areas closely related to these themes. As you can see from the contributions contained herein, a wide variety of physical applications and

mathematical methods come into play, including Lie group methods, Lie algebras and Lie algebra cohomology, representation theory, differential operators, differential geometry, special functions, the theory of orthogonal polynomials, q-series, conformal field theory, quantum groups, scattering theory, classical invariant theory, and so on. The subject is, in our opinion, extraordinarily rich and vibrant in its range of indisciplinary ideas and new applications.

All together, we believe, the papers assembled in this volume will provide the reader with an informative and challenging survey of the broad range of current research in this exciting field.

We would like to thank the speakers for having made this session successful, and for their contributions to the proceedings. We would also like to take this opportunity to thank Donna Harmon at the AMS for her help in the time-consuming task of assembling the volume together in a suitable form.

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